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ON OPTIMAL OPERATING POLICIES OF EPQ MODEL WITH TIME-DEPENDENT REPLENISHMENT AND DETERIORATION HAVING SELLING PRICE DEPENDENT DEMAND

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ABSTRACT. EPQ Models are more important for scheduling the production processes and manufacturing units. In this paper, we study an Economic Production Quantity Model with the assumption that the replenishment is time-dependent. It is further assumed that the rate of deterioration is also linearly dependent on time. Using differential equations the instantaneous state of inventory is derived. With suitable cost considerations, the total cost function and profit rate function is obtained. By maximizing the profit rate function the optimal policies are derived. Through numerical illustrations, the solution procedure of the model is demonstrated. The sensitivity analysis of the model reveals that the replenishment parameters and deterioration parameters have a significant influence on the operating policies.

1 Introduction

Economic Production Quantity models play a dominant role in the analysis of Production process, Warehouse Management, Market yards etc., in various practical situations the economic production quantity is dependent

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on, through the constituent process of the model viz., 1.) Replenishment 2.) Demand 3.) Nature of the commodity. Much work has been reported regarding EPQ models with finite and infinite rates of replenishment. But in many practical situations the replenishment may or may not be finite but varying depending on time. Due to variations,in the procurement, transportation, storage capacity etc. Little work has been reported in the literature regarding EPQ models with time-dependent replenishment.

Mak (1982) developed a production lot size model which deals with an unfilled order backlog for an inventory model with exponential decaying items. Chowdhary and Chaudhuri (1983) obtained an inventory model for deteriorating items with a finite rate of production and the constant rate of deterioration for both deterministic and probabilistic cases. **Su, et al.** (1999) developed a deterministic production inventory model for deteriorating items with an exponential declining demand over a fixed time and production dependent demand. Mandal and Phaujdar (1989) obtained a single item stock control model with shortages for deteriorating items having a uniform rate of production and variable demand is dependent on instantaneous inventory level. Sujit and Goswami (2001) considered a replenishment policy for items with finite production rate and fuzzy deterioration rate. Here, the deterioration rate is considered to solve the Economic Production Quantity (EPQ) model. The production quantity due to the faulty/aged machine is also studied. **Sana, et al.** (2004) presented a production inventory model for deteriorating items over a finite planning horizon and a linear time-varying demand with a finite production rate, shortages. Balkhi (2001) considered a generalized production lot size inventory model for deteriorating items over finite planning where the demand, production, and deteriorating rates were assumed to be known and continuous function of time. Samanta and Roy (2004) developed a continuous production control inventory model for items deteriorating exponentially with time and where the two different rates of production are available and it is possible that production started at one rate and after some time it may be switched over to another rate. Lin and Gong (2006) studied a production inventory system of deteriorating items subjected to random machine breakdowns with fixed repair time allowing price discount using the permissible delay in payments. Chen (2008) discussed economic production run length and warranty period for products with Weibull lifetime. Miirzazadeh, et al. (2009) discussed an inventory model under uncertain inflationary conditions, finite production rate and inflation dependent demand rate for deteriorating items with shortages. Lee and Hsu (2009) developed a two-warehouse inventory model for deteriorating items with time-dependent demand. The variation in production cycle times to determine the number of production cycles and the times for replenishment during a finite planning horizon is considered. Manna and Chiang (2010) developed two deterministic economic production quantity (EPQ) models with and without shortages of Weibull distribution, deteriorating items with demand rate as a ramp type function of time. Sridevi, et al. (2010) developed and analyzed an inventory model with the assumption that the rate of production is random and follows a Weibull distribution and the demand is a function of the selling price. By maximizing the profit rate function they obtained the optimal ordering and pricing policies of the model. The sensitivity of the model with respect to the parameters and costs was also analyzed. Tripathy, et al (2010) developed an EPQ model with no shortages for items with trine varying holding cost, linear deterioration, and constant production and demand rates. Partially deteriorated items were allowed to float into the market with a discount. Manna and Chiang (2010) developed two deterministic Economic Production Quantity models for Weibull distribution deteriorating items with demand rate as ramp type function of time. Sarkar and Moon (2011) studied a finite time production inventory model for stochastic demand with shortages and the effect of inflation. A certain percentage of products were of imperfect quality and the lifetime of a defective item was assumed to follow a Weibull distribution. They derived profit function by using both a general distribution of demand and the uniform rectangular distribution of demand. The production rate of the inventory model is considered to be constant. Srinivasa Rao and Essey Kebede Muluneh (2012) developed a production inventory model for deteriorating items and analyzed with the assumption that the production rate is dependent on stock on hand. It is further assumed that the lifetime of the commodity is random and follows a three-parameter Weibull distribution and demand rate is a function of both selling price and time. Palanivel & Uthayakumar (2013), developed an Economic Production Quantity (EPQ) model with price and advertisement dependent demand under the effect of inflation and time value of money. The selling price of a unit is determined by a mark-up over the production cost. In this model, the holding cost per unit of the item per unit time is assumed to be an increasing linear function of time spent in storage. Hui-Ming Teng (2013), studied an economic production quantity model for deteriorating items in which backorder is allowed. The selling price of backorder depends on the customers that are willing to purchase the items under the condition that they receive their orders after a certain fraction of waiting time. Anand and Jagat Veer Singh (2013) developed a production inventory model for decaying items with multi-variate demand and variable holding cost. Demand rate function depends upon the present inventory level and the selling price per unit during the production phase. Shortages are permitted with partial backlogging. The backlogging rate is waiting time for the next replenishment. Sanjay Sharma and Singh (2013), developed a model with the concept of space restriction in which demand is exponential, and deterioration is time-dependent. Production is taken as a function of demand. Ashendra Kumar Saxena and Ravish Kumar Yadav (2013) studied an Economic Production Quantity (EPQ) model for the noninstantaneous deteriorating item in which production and demand rates are constant and the holding cost varies with quadratic in time. Srinivasa Rao, et al. (2013), developed and analyzed an EPQ model for deteriorating items with stock-dependent production rate and Pareto rate of decay. Meenakshi Srivastava & Ranjana Gupta (2013), studied continuous production inventory model for deteriorating items with time and price-dependent demand under markdown policy for both fresh and deteriorated units. Da Wen, Pan Ershun, Wang Ying and Liao Wenzhu (2014), integrated predictive maintenance into EPQ models in which autoregressive integrated moving average model is adopted to predict system's healthy indicator due to machine degradation. Palanivel & Uthayakumar (2014), developed an Economic Production Quantity (EPQ) model under the effect of inflation and time value of money. The selling price of a unit is determined by a mark up over the production cost. They have considered three types of continuous probabilistic deterioration function, and also considered that the holding cost of the item per unit time is assumed to be an increasing linear function of time spent in storage. Himani Dem, Singh, Jitendra Kumar (2014), Studied, an Economic Production Quantity model (EPQ) model for finite production rate and deteriorating items with time-dependent trapezoidal demand. An EPQ model is formulated for deteriorating items with time-dependent demand rate and time-dependent production rate. To retain the confidence of the buyers the machine reliability, flexibility and packaging cost are considered. Shortages are allowed and it is completely backlogged. A mathematical model has been presented to find the optimal order quantity and total cost. The holding cost, setup cost, deterioration cost, labour cost for packing, the material cost for packing, shortage cost, production cost involved in this model are taken as triangular fuzzy numbers. To validate the optimal solution, a numerical example is provided. To analyze the effect of variations in the optimal solution with respect to the change in one parameter at a time, sensitivity analysis is carried out. Ankit Bhojak, Gothi (2015), they developed an inventory model to determine the economic production quantity for deteriorating items under time dependent demand. Inventory holding cost was taken as a linear function depending upon time. A combination of two parameters and three-parameter Weibull distribution is considered for the deterioration of units over a period of time. For practical applicability, shortages were allowed to occur. Besides that, they have assumed that demand is a linear and also quadratic function of time at different time intervals. A numerical example is given for the developed model with its sensitivity analysis. Kirtan Parmar and Gothi (2015), they have analyzed a production inventory model for deteriorating items with time-dependent holding cost. Three parameters Weibull distribution was assumed for time to deterioration of items. Shortages were allowed to occur. The derived model was illustrated by a numerical example and its sensitivity analysis is carried out. Kousar Jaha Begum and Devendra (2016) developed and analyzed an E.P.Q model with the assumptions that the lifetime of a commodity is random and follows a Generalized Pareto Distribution. Dhir Singh & Singh (2017), developed economic production quantity (EPQ) model for deteriorating product with time dependent demand and the time dependent inventory carrying cost. There, it was assumed that the production rate at any instant depends on both the stock and the demand for the product. To make the model more realistic, the shortages were allowed and partially backlogged. The back ordering rate was taken as a decreasing function of waiting time for the next refill. Khedlekar, Namedeo, Nigwal (2018), in the paper, made an attempt to develop an economic production quantity model using optimization method for deteriorating items with production disruption. The disruption in a production system occurs due to labour problem, machines breakdown, strikes, political issue, and weather disturbance, etc. This leads to delay in the supply of the products, resulting customer to approach other dealers for the products. The optimal production and inventory plan were provided. So that the manufacturer can reduce the loss occurred due to disruption. Nita Shah and Chetansinh Vaghela (2018), developed this paper to establish an economic production quantity (EPQ) model for deteriorating items with both up-stream and down-stream trade credits. Here, In practice, trade credit induces more sales over time by allowing customers to purchase without immediate cash. Nita Shah, Mrudul Jani, Urmila Chaudhari (2018), In this article, a production inventory model with dynamic production rate and production time dependent selling price has been presented. They considered the product with constant deterioration rate which is a very realistic approach. It is also considered that the production rate is a decreasing function of the inverse efficiency of the system.

Hence, in this paper, we develop and analyze EPQ models with time dependent replenishment. Here, it is assumed that the replenishment is linearly dependent and is of the form R (t) = a+ bt, where 'a' and 'b' are two parameters. This replenishment also includes a constant rate of replenishment when b = 0. Another important consideration in EPQ models is demand. In this model, it is assumed that demand is the function of selling price and is of the form $\lambda(s) = (d - f\, s)$ where d and f are parameters and 's' is the selling price. Further, it is assumed that the lifetime of the commodity is finite and depends on time. This is represented with time dependent deterioration rate. Here, it is assumed that the instantaneous rate of deterioration is linearly dependent on time and is of the form $h(t) = \alpha + \beta t$. This includes the increasing/decreasing/constant rate of deterioration.

Using difference-differential equations the instantaneous state of inventory is derived. With suitable cost considerations, the total cost function and profit rate function is obtained. By maximizing the profit rate function the optimal pricing and ordering policies of the model are derived. The sensitivity of the model with respect to the costs and parameters are also studied. This model is extended to the case of without shortages.

2 ASSUMPTIONS

The following assumptions are made for developing the model.

i) The lifetime of a commodity is finite and dependent on time. The instantaneous rate of deterioration is

$$h(t) = \alpha + \beta t \tag{1}$$

where α and β are constants. If $\beta > 0$, it is an increasing rate of deterioration. If $\beta < 0$, it includes a decreasing rate of deterioration. If $\beta = 0$, it is a constant rate of deterioration.

ii) The demand rate $\lambda(s)$ is a linear function of unit selling price and it is of the form $\lambda(s) = (d - f\,s)$, where d and f are positive constants.

iii) The rate of production is time dependent and is of the form $R(t) = a + b$t such that $R(t) \geq 0$, where a and b are constants for $a > 0, b > 0$. This production rate includes increasing/decreasing/ constant rates of production for b > 0, b < 0 and b=0 respectively.

iv) Lead time is zero.

v) Cycle length, T is known and fixed.

vi) There is no repair or replacement of deteriorated item which occurs during the production cycle and the deteriorated item is thrown as scrap.

Notations

The following notations are used for developing the model.

A: Ordering cost

C: Cost per unit

h: Inventory holding cost per unit time

π: Shortage cost per unit time

Q: Total quantity of items produced in one cycle

s: Selling price of a unit

λ(s): demand rate

I (t): On hand inventory at time t, $0 \leq t \leq T$.

t_1: Time at which replenishment stops.

t_2: Time at which shortages start.

t_3: Time at which replenishment is restarted.

3 INVENTORY MODEL WITH SHORTAGES

This section, deals with an inventory system for deteriorating items in which the lifetime of the commodity is finite and time dependent. Here, it is assumed that shortages are allowed and fully backlogged. In this model, the stock level is initially zero at time t=0. Then it reaches its peak at time t_1. The inventory comes down gradually due to demand and deterioration in the interval (t_1, t_2). At time t_2 the inventory reaches zero and back orders accumulate during the period (t_2, t_3). At time t_3, the replenishment again starts and fulfils the backlog after satisfying the demand. During (t_3, T) the back orders are fulfilled and the inventory level reaches zero at the end of the cycle T. The schematic representation of the instantaneous state of inventory is shown in Figure 1.
Let I (t) denote the inventory level of the system at time t ($0 \leq t \leq T$)
The differential equations describing the instantaneous states of I (t) in the interval (0, T) are

$$\frac{d}{dt}I(t) + (\alpha + \beta t)I(t) = (a + bt) - \lambda(s); 0 \leq t \leq t_1 \tag{2}$$

$$\frac{d}{dt}I(t) + (\alpha + \beta t)I(t) = -\lambda(s) \; t_1 \leq t \leq t_2 \tag{3}$$

$$\frac{d}{dt}I(t) = -\lambda(s) \quad ; \quad t_2 \leq t \leq t_3 \tag{4}$$

$$\frac{d}{dt}I(t) = (a + bt) - \lambda(s) \; ; \quad t_3 \leq t \leq T \tag{5}$$

With the initial conditions $I(0) = 0, \; I(t_2) = 0, \; I(T) = 0$

Solving the differential equations (2) to (5) and using the boundary conditions, the instantaneous state of inventory at any time t, during the interval (0, T) is obtained as

$$I(t) = e^{-(\alpha t + \frac{\beta t^2}{2})} \int_0^t ((a + b t) - \lambda (s)) \, e^{(\alpha t + \frac{\beta t^2}{2})} d t \; ; \quad 0 \leq t \leq t_1 \tag{6}$$

$$I(t) = e^{-(\alpha t + \frac{\beta t^2}{2})} \int_t^{t_2} \lambda (s) \quad e^{(\alpha t + \frac{\beta t^2}{2})} \, d t \quad ; \quad t_1 < t \leq t_2 \tag{7}$$

$$I(t) = \int_{t_2}^{t} \lambda (s) \, d t \quad ; \quad t_2 \leq t \leq t_3 \tag{8}$$

$$I(t) = \int_t^T (a + b t) - \lambda (s) \, d t \quad ; \quad t_3 \leq t \leq T \tag{9}$$

The stock loss due to deterioration in the cycle of length T is given by

$$L(T) = a \, t_1 + b\frac{t_1^2}{2} - (d - f \, s) \, t_2 \tag{10}$$

The total production in the cycle of length T is

$$\begin{aligned} Q &= \int_0^{t_1} (a + b \, t) \, dt + \int_{t_3}^{T} (a + b \, t) \, d \, t \\ &= a \, (t_1 - t_3 + T) + \tfrac{b}{2} \, (t_1^2 + T^2 - t_3^2) \end{aligned} \tag{11}$$

From above equations, we get

$$I(t) = - (d - f \, s) \, (t - t_2) \tag{12}$$

$$I(t) = - \left[a \, (T - t) + b\frac{(T - t)^2}{2} - d \, (T - t) + f \, s \, (T - t) \right] \tag{13}$$

When t = t_3, the above equations become

$$I\,(t_3) = -\,(d - f\,s)\,(t_3 - t_2) \tag{14}$$

$$I\,(t_3) = -\left[a\,(T - t_3) + b\frac{(T-t_3)^2}{2} - d\;(T - t_3) + f\,s\,(T - t_3)\right] \tag{15}$$

On equating the equations (14) and (15), and by simplifying them and expressing t_2 in terms of t_3 as,

$$t_2 = t_3 - \frac{1}{d - f\,s}\left[a\,(T - t_3) + \frac{b}{2}\left(T^2 - t_3^2\right) + \left(d - f\,s\right)(T - t_3)\right] = y \quad \text{say} \tag{16}$$

Let K (t_1, t_2, t_3, s) be the total cost per unit. The total cost is the sum of the setup the cost per unit time, purchasing cost per unit time, holding cost per unit time and shortage cost per unit time. Then K (t_1, t_3, s) becomes

$$\begin{aligned} K(t_1, t_2, t_3, s) = \quad & \tfrac{A}{T} + \tfrac{CQ}{T} + \tfrac{h}{T}\left[\int_0^{t_1} I(t)\,dt + \int_{t_1}^{t_2} I(t)\,dt\right] \\ & + \tfrac{\pi}{T}\left[-\int_{t_2}^{t_3} I(t)\,dt - \int_{t_3}^{T} I(t)\,dt\right] \end{aligned}$$

Substituting the values of I (t) and Q, given in the above equations in Equation

$$\begin{aligned} K(t_1, t_3, s) = \; & \tfrac{A}{T} + \tfrac{C}{T}\left\{\left(a\,(t_1 - t_3 + T) + \tfrac{b}{2}\,(t_1^2 + T^2 - t_3^2)\right)\right\} \\ & + \tfrac{h}{T}\left\{\int_0^{t_1} e^{-(\alpha\,t + \beta\,\frac{t^2}{2})}\left(\int_0^{t}\left((a + b\,\mathrm{u}) - (d - f\,\mathrm{s})\right)\,e^{(\alpha\,\mathrm{u} + \beta\,\frac{\mathrm{u}^2}{2})}\,d\,u\right)\,d\,t\right. \\ & \left. + \int_{t_1}^{y(t)} e^{-(\alpha\,t + \beta\,\frac{t^2}{2})}\left(\int_t^{y(t)} (d - f\,\mathrm{s})\,e^{(\alpha\,\mathrm{u} + \beta\,\frac{u^2}{2})}\,d\,\mathrm{u}\right)\,d\,t\right] \\ & + \tfrac{\pi}{T}\left\{\int_{y(t)}^{t_3}\left(\int_{y(t)}^{t}(\,d - f\,\mathrm{s})\,d\,\mathrm{u}\right)d\,t + \int_{t_3}^{T}\left(\int_t^{T}\left((a + b\,\mathrm{u}) - (d - f\,\mathrm{s})\right)\,d\,\mathrm{u}\right)\,d\,t\right\} \end{aligned}$$

On integrating and simplifying the above equation we get

$$\begin{aligned} K(t_1, t_3, s) = \; & \tfrac{A}{T} + \tfrac{C}{T}\left\{a(t_1 - t_3 + T) + \tfrac{b}{2}(t_1^2 + T^2 - t_3^2)\right\} \\ & + \tfrac{h}{T}\left\{\left(a - \left(d - fs\right)\right)\left[\int_0^{t_1} e^{-(\alpha t + \beta\frac{t^2}{2})}\left(\int_0^{t} e^{(\alpha u + \beta\frac{u^2}{2})}\,du\right)dt\right]\right. \\ & + b\left[\int_0^{t_1} e^{-(\alpha t + \beta\frac{t^2}{2})}\left(\int_0^{t} \mathrm{u}\,e^{(\alpha u + \beta\frac{u^2}{2})}du\right)dt\right] \\ & \left. + (d - fs)\int_{t_1}^{y(t)} e^{-(\alpha t + \beta\frac{t^2}{2})}\left(\int_t^{y(t)} e^{(\alpha u + \beta\frac{u^2}{2})}du\right)dt\right\} \\ & + \tfrac{\pi}{T}\left\{\left(a - (d - fs)\right)\left(\tfrac{T^2}{2} - Tt_3 + \tfrac{t_3^2}{2}\right) + \mathrm{b}\left(\tfrac{T^3}{3} - \tfrac{T^2 t_3}{2} + \tfrac{t_3^3}{6}\right)\right\} \end{aligned}$$

Let P (t_1, t_3, s) be the profit rate function. Since the profit rate function is the total revenue per unit time minus total cost per unit time, we have

$$P\,(t_1,\; t_3,\; s) = s\,\lambda\,(s) - K\,(\,t_1,\; t_3,\; s\,) \tag{17}$$

Substituting the values of $K\,(t_1,\ t_3,\ s)$ given in above equation, we obtain the profit rate function $P(t_1, t_3,\ s)$ as

$$\begin{aligned}P(t_1,t_3,s) = {} & s\,(d-f\,\mathrm{s}) - \tfrac{A}{T} - \tfrac{C}{T}\ \left\{a\,(t_1-t_3+T)+\tfrac{b}{2}\,(t_1^2+T^2-t_3^2)\right\}\\ & -\tfrac{h}{T}\left\{\left(a-\left(d-f\,s\right)\right)\int_0^{t_1} e^{-(\alpha t+\beta\frac{t^2}{2})}\ \left(\int_0^t\ e^{(\alpha u+\beta\frac{u^2}{2})}du\right)\ dt\right.\\ & \left. +b\ \int_0^{t_1}\ e^{-(\alpha t+\beta\frac{t^2}{2})}\ \left(\int_0^t \mathrm{u}\ e^{(\alpha u+\beta\frac{u^2}{2})}du\right)\ dt\right\}\\ & +(d-f\ \mathrm{s})\ \int_{t_1}^{y(t)} e^{-(\alpha t+\beta\frac{t^2}{2})}\ \left(\int_t^{y(t)} e^{(\alpha u+\beta\frac{u^2}{2})}du\right)\ dt\Bigg\}\\ & -\tfrac{\pi}{T}\left\{\left(a-(d-f\ \mathrm{s})\right)\ \left(\tfrac{T^2}{2}-T\,t_3+\tfrac{t_3^2}{2}\right)\ +\mathrm{b}\left(\tfrac{T^3}{3}-\tfrac{T^2t_3}{2}+\tfrac{t_3^3}{6}\right)\right\}\end{aligned}$$

4 OPTIMAL PRICING AND ORDERING POLICIES OF THE MODEL

In this section, we obtain the optimal policies of the inventory system under study. To find the optimal values of $t_{1,}$ $t_{3,}$ and s, we obtain the first order partial derivatives of P ($t_{1,}$ t_3, s) given in equation above equation with respect to t_1, t_3 and s, and equate them to zero. The condition for maximization of P ($t_{1,}$ t_3, s) is

$$|D|\ \ =\ \ \begin{vmatrix}\frac{\partial^2P(t_1,t_3,s)}{\partial t_1{}^2} & \frac{\partial^2P(t_1,t_3,s)}{\partial t_1\partial t_3} & \frac{\partial^2P(t_1,t_3,s)}{\partial t_1\partial s}\\ \frac{\partial^2P(t_1,t_3,s)}{\partial t_1\partial t_3} & \frac{\partial^2P(t_1,t_3,s)}{\partial t_3{}^2} & \frac{\partial^2P(t_1,t_3,s)}{\partial t_3\partial s}\\ \frac{\partial^2P(t_1,t_3,s)}{\partial t_1\partial t_3} & \frac{\partial^2P(t_1,t_3,s)}{\partial t_3\partial s} & \frac{\partial^2P(t_1,t_3,s)}{\partial s^2}\end{vmatrix}\ <\ 0$$

Where D is the determinant of the Hessian matrix.

Differentiating $P(t_1, t_3, s)$ with respect to t_1 and equating it to zero, we get

$$\frac{c}{T}\ (a+b\ t_1)-\frac{h}{T}\ \left\{a+b\ \left(e^{-(\alpha t_1+\beta\frac{t_1^2}{2})}\ \int_0^{t_1} \mathrm{u}\ e^{(\alpha u+\beta\frac{u^2}{2})}\ du\right)\right\}=0 \tag{18}$$

Differentiating $P(t_1, t_3, s)$ with respect to t_3 and equating it to zero, we get

$$\begin{aligned}&\tfrac{c}{T}\ \ [a+b\ t_3]+\tfrac{h}{T}\left\{\left(2\left(d-fs\right)+(a+bt_3)\right)\left(e^{(\alpha y+\beta\frac{y^2}{2})}\right)\left(\int_{t_1}^{y} e^{-(\alpha t+\beta\frac{t^2}{2})}\ \ dt\right)\right\}\\ &\qquad -\tfrac{\pi}{T}\ \left\{\left(a-(d-f\ \mathrm{s}\,)\right)\ \ (t_3-T)+\tfrac{b}{2}\ \ \left(t_3{}^2-T^2\right)\right\}=0\end{aligned}$$

Differentiating $P(t_1, t_3, s)$ with respect to s, and equating it to zero, we get

$$(\mathrm{d}\ -2\ \mathrm{f}\ \ \mathrm{s}\ )-\ \frac{\pi}{T}\ \left\{\frac{f}{2}\ (\ T^2-2\ T\ t_3+t_3^2\ )\right\}\ \ =\ 0 \tag{19}$$

By solving above equations simultaneously, we obtain the optimal time at which the replenishment should be stopped i.e., $t_1{}^*$ of t_1, optimal time $t_3{}^*$ of t_3 at which replenishment is restarted after accumulation of backorders and the optimal selling price s* of s is obtained. The optimum ordering quantity Q^* of Q in the cycle of length T is obtained by substituting the optimal values of t_1 and t_3 in (11) as

$$\mathrm{Q}*\ =a\ (t_1{}^*-t_3{}^*+T)+\ \frac{b}{2}\ (t_1^{*^2}+T^2-t_3^{*^2}) \tag{20}$$

5 NUMERICAL ILLUSTRATIONS

Here, we deal with the solution procedure of the model through a numerical illustration by obtaining the replenishment (production) uptime, replenishment (production) downtime, optimal selling price, optimal quantity and profit of an inventory system. It is assumed that the commodity is of the deteriorating nature and shortages are allowed and fully backlogged. For demonstrating the solution procedure of the model the deteriorating parameter α is considered to vary between 0.3 to 0.8, the values of the other parameters and costs associated with model are: a = 3, 4, 5; b = 1, 1.5, 2; β = .1, .2, .3
A = 100, 200, 300; C = 2, 3, 4; d = 500, 600, 700; f = 10, 20, 30; h =0.012, 0.014, 0.02; π = .035, .04, .045; T = 12 months

Substituting these values, the optimal selling price, the optimal ordering quantity Q*, replenishment time, optimal value of time and total profit are computed and presented in Table 1.

As the demand parameter d is increasing, from 500 to 700, there is a decrease in optimal ordering quantity Q*, it decreases from 124.113 to 110.481, as the optimal value of $t_1{}^*$ increases from 1.089 to 3.138, the optimal replenishment time $t_3{}^*$ increases from 7.131 to 8.694, the optimal selling price s^* is increasing from 12.493 to 17.502 and the profit is increasing from 3069.16 to 6069.41. As the demand parameter f, is increasing from 10 to 30, the optimal ordering quantity Q* is decreasing from 124.126 to 124.100, the optimal value of $t_1{}^*$ increases from 1.088 to 1.091, the optimal replenishment time $t_3{}^*$ increases from 7.130 to 7.133, the optimal selling price s^* decreases from 24.993 to 8.326 and profit rate decreases from 6194.16 to 2027.50. As the ordering cost A increases, from 100 to 300, all the values remain constant, except for the profit. It decreases from 3077.50 to 3060.83. When the cost per unit C increases, from 2 to 4, there is an increase in optimal ordering quantity Q*, it increases from 107.363 to 124.113, the optimal value of $t_1{}^*$ decreases from 4.986 to 1.089, the optimal replenishment time $t_3{}^*$ decreases from 9.856 to 7.131, the optimal selling price s^* is decreasing from 12.504 to 12.493 and the profit decreases from 3088.16 to 3069.16. As the Production parameters 'a' increases, from 3 to 5, there is an increase in optimal ordering quantity Q*, from 112.986 to 124.113, the optimal value of $t_1{}^*$ decreases from 2.068 to 1.089, the optimal replenishment time $t_3{}^*$ decreases from 7.430 to 7.131, the optimal selling price $\mathbf{s}^*$ increases from 12.491 to 12.493 and the profit decreases from 3073.45 to 3069.16. When 'b' is increasing, from 1 to 2, there is an increase in optimal ordering quantity Q*, the increase is from 63.379 to 124.113, the optimal value of $t_1{}^*$ decreases from 3.830 to 1.089, the optimal replenishment time $t_3{}^*$ decreases from 9.670 to 7.131, the optimal selling price s^* is decreasing from 12.508 to 12.493 and the profit decreases from 3082.71 to 3069.16.

When the deteriorating parameter α increases, from 0.3 to 0.8, we observe that there is an increase in optimal ordering quantity Q*, from 124.113 to 126.168, the optimal value of $t_1{}^*$ increases from 1.089 to 1.677, the optimal replenishment time $t_3{}^*$ increases from 7.131 to 7.261 and the optimal selling price s^* is increasing from 12.493 to 12.495 and the profit decreases from 3069.16 to 3068.02. When β is increasing from 0.1 to 0.3, there is a decrease in optimal ordering quantity Q*, from 126.847 to 122.549, the optimal value of $t_1{}^*$ increases from 0.781 to 1.335, the optimal replenishment time $t_3{}^*$ increases from 6.876 to 7.306, the optimal selling price $\mathbf{s}^*$ is increasing from 12.489 to 12.496 and the profit is decreasing from 3069.96 to 3068.52. As the shortage cost per unit time π increases, from 0.035 to 0.045, there is a decrease in optimal ordering quantity Q*, from 131.283 to 117.963, the optimal value of $t_1{}^*$ increases from 0.422 to 1.673, the optimal replenishment time $t_3{}^*$ increases from 6.514 to 7.671, the optimal selling price s^* increases from 12.489 to 12.497 and the profit increases from 3068.47 to 3069.57.

Inventory holding cost per unit time h increases, from 0.012 to 0.02, there is an increase in the optimal ordering quantity Q*, from 124.113 to 129.452, the optimal value of $t_1{}^*$ increases from 1.089 to 1.716, the optimal replenishment time $t_3{}^*$ decreases from 7.131 to 7.108, the optimal selling price s^* increases from 124.113 to 129.452 and the profit is decreasing from 3069.16 to 3068.23.

6 SENSITIVITY ANALYSIS OF THE MODEL

Sensitivity analysis is carried to explore the effect of changes in model parameters and costs on the optimal policies, by varying each parameter (-15%,-10%, -5%, 0%, 5%, 10%, 15%) at a time for the model under study. The results are presented in Table 2.

As the demand parameter d decreases, the optimal value of $t_1{}^*$, the optimal replenishment time $t_3{}^*$, the optimal selling price s^* and profit rate are decreasing, the optimal ordering quantity Q* is increasing. If, 'd' increases, the optimal value of $t_1{}^*$, the optimal replenishment time $t_3{}^*$, the optimal selling price s^* and profit rate are increasing, the optimal ordering quantity Q* is decreasing. As 'f' decreases, the optimal value of $t_1{}^*$, the optimal replenishment time $t_3{}^*$ decreases and the optimal selling price s^*, the optimal ordering quantity Q* and profit rate are increasing. If, 'f' increases, the optimal value of $t_1{}^*$, the optimal replenishment time $t_3{}^*$ increases and the optimal selling price s^*, the optimal ordering quantity Q* and profit rate is decreasing. When the ordering cost 'A' decreases or increases, the optimal value of $t_1{}^*$, the optimal replenishment time $t_3{}^*$, the optimal selling price the optimal ordering quantity Q* and profit rate remain constant.

As the cost per unit 'C' is decreasing, the optimal value of $t_1{}^*$, the optimal replenishment time $t_3{}^*$, the optimal selling price s^* and profit rate are increasing and the optimal ordering quantity Q* is decreasing. When 'C' is increasing, the optimal value of $t_1{}^*$, the optimal replenishment time $t_3{}^*$, the optimal selling price s^* and profit rate are decreasing, the optimal ordering quantity Q* is increasing.

If the Production parameters 'a' decreases, the optimal value of $t_1{}^*$, the optimal replenishment time $t_3{}^*$, the optimal selling price s^* and profit rate are increasing, the optimal ordering quantity Q* is decreasing. If 'a' increases, the optimal value of $t_1{}^*$, the optimal replenishment time $t_3{}^*$, the optimal selling price s^* and profit rate are decreasing. The optimal ordering quantity Q* is increasing. If 'b' decreases, the optimal value of $t_1{}^*$, the optimal replenishment time $t_3{}^*$, the optimal selling price s^* and profit rate are increasing. The optimal ordering quantity Q* is decreasing. If 'b' increases, the optimal value of $t_1{}^*$, the optimal replenishment time $t_3{}^*$, the optimal selling price s^* and profit rate are decreasing. The optimal ordering quantity Q* is increasing.

As the deteriorating parameters 'α' decreases, the optimal value of $t_1{}^*$, the optimal replenishment time $t_3{}^*$, the optimal ordering quantity Q* are decreasing, the optimal selling price s^* and profit rate are increasing. If 'α' increases, the optimal value of $t_1{}^*$, the optimal replenishment time $t_3{}^*$, the optimal ordering quantity Q* are increasing, the optimal selling price s^* and profit rate are decreasing. If β decreases, the optimal value of $t_1{}^*$, the optimal replenishment time $t_3{}^*$ and the optimal selling price $s^*$are decreasing. The optimal ordering quantity $Q*$ and profit rate are increasing. If β increases, the optimal value of $t_1{}^*$, the optimal replenishment time $t_3{}^*$, the optimal selling price $s^*$are increasing, the optimal ordering quantity Q* and profit rate are decreasing.

When the shortage cost per unit π decreases, the optimal value of $t_1{}^*$, the optimal replenishment time $t_3{}^*$, the optimal selling price s^* and profit rate are decreasing, the optimal ordering quantity Q* is increasing. If π increases, the optimal value of $t_1{}^*$, the optimal replenishment time $t_3{}^*$, the optimal selling price s^* and profit rate are increasing. The optimal ordering quantity $Q*$ is decreasing.
As the holding cost per unit 'h' decreases, the optimal value of $t_1{}^*$, the optimal replenishment time $t_3{}^*$, the optimal ordering quantity Q* are decreasing, the optimal selling price s^* and profit rate are increasing. If 'h' increases, the optimal value of $t_1{}^*$, the optimal replenishment time $t_3{}^*$and the optimal ordering quantity Q* are increasing, the optimal selling price s^* and profit rate are decreasing.

7 INVENTORY MODEL WITHOUT SHORTAGES

In this section, the inventory model for deteriorating items without shortages is developed and analyzed. Here it is assumed that the shortages are not allowed and the stock level is zero at time t=0. The stock level increases during the period (0, t_1) due to excess replenishment after fulfilling the demand and deterioration. The replenishment stops at time t_1 when stock level reaches its peak. The inventory decreases gradually due to demand and deterioration in the interval ($t_{1,}$ T). At time T the inventory reaches zero. The schematic diagram representing the instantaneous state of inventory is given in Figure 3.
Let I (t) be the inventory level of the system at a time 't'$(0 \leq t \leq T)$.
Then the differential equations governing the instantaneous state of I (t) over the cycle of length T are

$$\frac{d}{dt}I(t) + (\alpha + \beta\ t)\ I(t) = (a + b\ t) - \lambda\ (s); \qquad 0 \leq t \leq t_1 \tag{21}$$

$$\frac{d}{dt}\ I(t) + (\alpha + \beta\ t)\ I(t) = -\lambda\ (s) \quad ; \qquad t_1 \leq t \leq T \tag{22}$$

With the boundary conditions I (T) = 0, I (0) = 0

The instantaneous state of inventory at any given time t during the interval (0, T) is

$$I(t) = e^{-(\alpha t+\beta t^2/2)}\ [\int_0^t [\ (a + bt) - \lambda(s)\ ]\ e^{(\alpha t+\beta t^2/2)} d\ t\ ; \qquad 0 \leq t \leq t_1 \tag{23}$$

$$I(t) = e^{-(\alpha\ t+\beta\ t^2/2)} \int_t^T \lambda\ (s)\ \ e^{(\alpha\ t+\beta\ t^2/2)}\ d\ t; \qquad t_1 \leq t \leq T \tag{24}$$

The production quantity during the cycle time (0, T) is given by

$$Q = at_1 + \frac{bt_1^2}{2} \tag{25}$$

Using equations (23) and (24), we obtain the stock loss due to deterioration in this interval (0, T) as the difference between the total quantity produced and the demand met during (0, T) and is given by

$$\mathrm{L\ (T)} = at_1 + b\,\frac{t_1^2}{2} - \lambda\ (\mathrm{s})\ \mathrm{T} \tag{26}$$

This amount of quantity is lost due to deterioration of commodity and is a waste. To obtain the optimal operating policies one must reduce the stock loss due to deterioration.

Let K (t_1, s) be the total cost per unit time. Since the total cost is the sum of the setup cost, cost of units, the inventory holding cost. Therefore the total cost is

$$K(t_1,s) = \frac{A}{T} + \frac{CQ}{T} + \frac{h}{T}\left[\int_0^{t_1} I(t)dt + \int_{t_1}^{T} I(t)dt\right] \tag{27}$$

Substituting the value of I (t) and Q given in equations (23), (24) and (25) in the equation (27) we obtain K (t_1, s) as

$$\begin{aligned} K(t_1,s) \;=\; & \tfrac{A}{T} + \tfrac{C}{T}\left(a\,t_1 + b\tfrac{t_1{}^2}{2}\right) + \tfrac{h}{T}\left\{\int_0^{t_1} e^{-(\alpha\,t+\beta\,\frac{t^2}{2})}\left(\int_0^t \left((a+bu)-(d-f\,s)\right) e^{(\alpha\,u+\beta\,\frac{u^2}{2})}\,d\,u\right)d\,t\right. \\ & \left.+\int_{t_1}^{T} e^{-(\alpha\,t+\beta\,\frac{t^2}{2})}\left(\int_t^T (d-f\,s)\; e^{(\alpha\,u+\beta\,\frac{u^2}{2})}\,d\,u\right)\,d\,t\right\} \end{aligned} \tag{28}$$

On integrating and simplifying equation (27) we get

$$\begin{aligned} K(t_1,s) \;=\; & \tfrac{A}{T}+\tfrac{C}{T}\left(at_1 + b\tfrac{t_1{}^2}{2}\right) + \tfrac{h}{T}\left\{\left(a-(d-f\,s)\right)\;\int_0^{t_1} e^{-(\alpha t+\beta\frac{t^2}{2})}\left(\int_0^t e^{(\alpha u+\beta\frac{u^2}{2})}du\right)dt\right. \\ & + b\;\int_0^{t_1} e^{-(\alpha t+\beta\frac{t^2}{2})}\left(\int_0^t \mathrm{u}\; e^{(\alpha u+\beta\frac{u^2}{2})}d\,u\right)d\,t \\ & \left.+ (d-f\,s)\;\int_{t_1}^{T}\; e^{-(\alpha t+\beta\frac{t^2}{2})}\;\left(\int_t^T e^{(\alpha u+\beta\frac{u^2}{2})}d\,u\right)d\,t\right\} \end{aligned} \tag{29}$$

Let $P(t_1,s)$ be the profit rate function. Since the profit rate function is the total revenue per unit time minus total cost per unit time, we have

$$\mathrm{P}(t_1,s) \;=\; \mathrm{s\,l\,(s)} - \mathrm{K}\,(t_1,s) \tag{30}$$

Substituting the value of $K(t_1,s)$given in equation (28), we obtain the profit rate function of $P(t_1,s)$ as

$$\begin{aligned} P(t_1,s) = \;& \mathrm{s\,(d\;-\;f\,s)} - \tfrac{A}{T} - \tfrac{C}{T}\;\left(a\;t_1 + b\tfrac{t_1{}^2}{2}\right)\; - \tfrac{h}{T}\left\{\left(a-(d-f\,s)\right)\int_0^{t_1} e^{-(\alpha t+\beta\frac{t^2}{2})}\left(\int_0^t e^{(\alpha u+\beta\frac{u^2}{2})}d\,u\right)d\,t\right. \\ & + b\int_0^{t_1} e^{-(\alpha t+\beta\frac{t^2}{2})}\left(\int_0^t u\;e^{(\alpha u+\beta\frac{u^2}{2})}d\,u\right)d\,t \\ & \left.+ (d-f\,s)\;\int_{t_1}^{T}\; e^{-(\alpha t+\beta\frac{t^2}{2})}\;\left(\int_t^T e^{(\alpha u+\beta\frac{u^2}{2})}d\,u\right)d\,t\right\} \end{aligned} \tag{31}$$

8 OPTIMAL PRICING AND ORDERING POLICIES OF THE MODEL

This section, we obtain the optimal policies of the inventory system under study. To find the optimal values of production time t_1 and the optimal unit selling price s. We equate the first order partial derivatives of P (t_1, s) with respect to t_1 and s, equate them to zero. The condition for maximization of P (t_1, s) is

$|D| = \begin{vmatrix} \frac{\partial^2 P(t_1,s)}{\partial t_1{}^2} & \frac{\partial^2 P(t_1,s)}{\partial t_1 \partial s} \\ \frac{\partial^2 P(t_1,s)}{\partial s \partial t_1} & \frac{\partial^2 P(t_1,s)}{\partial s^2} \end{vmatrix} < 0$ where D is the determinant of the Hessian matrix.

Differentiate P (t_1, s) with respect to t_1 and equating it to zero, we get

$$\begin{aligned} & \tfrac{C}{T}\,[a+bt_1] - \tfrac{h}{T}\left\{\left(a-\left(d-f\,s\right)\;\right)\;\left[e^{-(\alpha t_1+\beta\frac{t_1{}^2}{2})}\;\int_0^{t_1} e^{(\alpha u+\beta\frac{u^2}{2})}du\right]\right. \\ & + \;b\;\left[e^{-(\alpha t_1+\beta\frac{t_1{}^2}{2})}\;\int_0^{t_1} \mathrm{u}\;e^{(\alpha u+\beta\frac{u^2}{2})}\;du\right] \\ & \left.- \left(d-f\,s\right)\;\left[e^{-(\alpha t_1+\beta\frac{t_1{}^2}{2})}\int_{t_1}^{T} e^{(\alpha u+\beta\frac{u^2}{2})}\;du\right]\right\} = 0 \end{aligned}$$

(??) (??)

Differentiate P (t_1, s) with respect to s, and equating it to zero, we get

$$-\ \frac{h}{T}\left\{f\left[\int_0^{t_1} e^{-(\alpha t+\beta\frac{t^2}{2})}\ \int_0^{t} e^{(\alpha u+\beta\frac{u^2}{2})}du\right]dt\ +f\left[\int_{t_1}^{T} e^{-(\alpha t+\beta\frac{t^2}{2})}\ \int_t^{T} e^{(\alpha u+\beta\frac{u^2}{2})}d\,\mathrm{u}\right]dt\right\}=0 \tag{32}$$

Solving the equations (**??**) and (32) simultaneously, we obtain the optimal time at which the replenishment is to be stopped $t_1{}^*$ of t_1 and the optimal unit selling price s^* of s. The optimum ordering quantity Q^* of Q in the cycle of length T is obtained by substituting the optimal values of t_1 in (25) as

$$Q* \ = \ at_1{}^* + \frac{bt_1^{2*}}{2} \tag{33}$$

9 NUMERICAL ILLUSTRATIONS

In this section, we discuss a numerical illustration of the model. For demonstrating the solution procedure of the model, the deteriorating parameter 'α' is considered to vary .1 to .3, the values of other parameters and costs associated with the model are:

A = 100, 200, 300; a = 5, 6, 7; b = 1, 2, β =.01, .02, .03, C = 2, 3, 4; d = 400, 500, 600; f = 10, 20, 30; h = 0.011, 0.012, 0.013; T = 12 months

From the table 3, it is observed that the optimal values of $t_1{}^*$, s^*, Q^* are obtained for different values of parameters and costs are shown in table 3.

As the demand parameter d' is increasing from 400 to 600, there is an increase in optimal ordering quantity Q*, it increases from 78.887 to 100.006, the optimal value of $t_1{}^*$ increases from 6.727 to 7.808, the optimal selling price $s^*$is increasing from 10.005 to 14.998 and the profit is increasing from 1967.900 to 4467.64. If 'f' is increasing from 10 to 30, the optimal value of $t_1{}^*$ decreases from 7.322 to 7.320, the optimal selling price s^* is decreasing from 25.001 to 8.334, the optimal ordering quantity Q* decreases from 90.222 to 90.218, the profit rate is decreasing from 6217.59 to 2050.92.

As the ordering cost 'A' increases from 100 to 300, all the values remain constant, except the profit rate which decreases from 3100.92 to 3084.25. As the cost per unit C increases from 2 to 4, there is decrease in optimal ordering quantity Q*, it decreases from 90.220 to 57.743, the optimal value of $t_1{}^*$ decreases from 7.321 to 5.499, the optimal selling price s^* is increasing from 12.501 to 12.516 and the profit decreases from 3092.59 to 3080.66. As the Production parameter 'a' increases from 5 to 7, the optimal value of $t_1{}^*$ decreases from 7.321 to 7.055, the optimal selling price s^* increases from 12.501 to 12.503, the optimal ordering quantity Q* increases from 90.222 to 99.163, the profit rate is decreasing from to 3092.59 to 3090.16. As 'b' is increasing from 1 to 3. The optimal value of $t_1{}^*$ decreases from 8.626 to 6.502, the optimal selling price s^* increases from 12.493 to 12.507, the optimal ordering quantity Q* increases from 80.337 to 95.926, the profit rate is decreasing from 3097.83 to 3088.59. As the deteriorating parameters 'α' increases, the optimal value of $t_1{}^*$ increases from 6.929 to 7.715, the optimal selling price s^* is increasing from 12.500 to 12.502, the optimal ordering quantity Q* decreases from 82.667 to 98.097, the profit rate is decreasing from 3094.28 to 3091.07. If the deteriorating parameters 'β' increases, the optimal value of $t_1{}^*$ increases from 7.321 to 8.083, the optimal selling price s^* is decreasing from 12.501 to 12.499, the optimal ordering quantity Q* increases from 90.220 to 105.759, the profit rate is decreasing from 3092.59 to 3090.45.

As the inventory holding cost per unit 'h' is increasing from .011 to .013, the optimal value of $t_1{}^*$ increases from 7.093 to 7.532, the optimal selling price s^* is decreasing from 12.502 to 12.499, the optimal ordering quantity Q* increases from 85.799 to 94.394 and the profit rate is decreasing from 3092.68 to 3092.56.

10 SENSITIVITY ANALYSIS OF THE MODEL

The sensitivity analysis is carried to explore the effect of changes in model parameters and costs on the optimal policies, by varying each parameter (-15%,-10%,-5%, 0%, 5%, 10%, 15%) at a time for the model under study. The results are presented in Table 4.

As the demand parameter d decreases, the optimal value of $t_1{}^*$, the optimal selling price s^*, the optimal ordering quantity Q* and profit rate are decreasing. If 'd' increases, the optimal value of $t_1{}^*$, the optimal selling price s^*, the optimal ordering quantity Q* and profit rate are increasing. If 'f' decreases, the optimal value of $t_1{}^*$, the optimal selling price s^*, the optimal ordering quantity Q* and profit rate are increasing. As 'f' increases, the optimal value of $t_1{}^*$, the optimal selling price s^*, the optimal ordering quantity Q* and profit rate are decreasing.

As the ordering cost 'A' decreases, the optimal value of $t_1{}^*$, the optimal selling price $s^{*\prime}$ the optimal ordering quantity Q* remain constant and profit rate is increasing. If 'A' increases, the optimal value of $t_1{}^*$, the optimal selling price $s^{*\prime}$ the optimal ordering quantity Q* remain constant and profit rate is decreasing. When the cost per unit 'C' is decreasing, the optimal value of $t_1{}^*$, the optimal

ordering quantity Q* and profit rate are increasing and the optimal selling price s^* is decreasing. If 'C' is increasing, the optimal value of $t_1{}^*$, the optimal ordering quantity Q* and profit rate are decreasing. The optimal selling price s^* is increasing.

As the Production parameters 'a' decreases, the optimal value of $t_1{}^*$ and profit rate are increasing, the optimal selling price s^* and the optimal ordering quantity Q* are decreasing. If 'a' increases, the optimal value of $t_1{}^*$, and profit rate are decreasing, the optimal selling price $s^*$and the optimal ordering quantity Q* are increasing. If 'b' decreases, the optimal value of $t_1{}^*$ and the profit rate are increasing. The optimal ordering quantity Q* and the optimal selling price s^* are decreasing. If 'b' increases, the optimal value of $t_1{}^*$ and profit rate are decreasing, the optimal selling price s^*, the optimal ordering quantity Q* are increasing.

When the deteriorating parameters 'α' decreases, the optimal value of $t_1{}^*$, the optimal ordering quantity Q*, the optimal selling price s^* are decreasing and the profit rate is increasing. If 'α' increases, the optimal value of $t_1{}^*$, the optimal ordering quantity Q*, the optimal selling price s^* are increasing, and the profit rate is decreasing. If 'β' decreases, the optimal value of $t_1{}^*$, the optimal ordering quantity Q*are decreasing. The optimal selling price s^* and profit rate are increasing. If 'β' increases, the optimal value of $t_1{}^*$, the optimal ordering quantity Q* are increasing, the optimal selling price s^* and profit rate are decreasing. As the holding cost per unit 'h' decreases, the optimal value of $t_1{}^*$, the optimal ordering quantity Q* are decreasing, the optimal selling price s^* and profit rate are increasing. If 'h' increases, the optimal value of $t_1{}^*$, the optimal ordering quantity Q* are increasing, the optimal selling price s^* and profit rate are decreasing.

11 CONCLUSIONS

In this paper, an economic production quantity model for deteriorating items is developed and analyzed. Here, the replenishment (production) is time dependent. This production rate includes constant/increasing/ decreasing rates of production for different values of the parameter. It is considered that the lifetime of the commodity is finite and depends on time. The deterioration is a linear function of time. This rate of deterioration includes constant/increasing/ decreasing rates of deterioration for different values of the parameter. It is supposed that the demand is a function of the selling price. Supposing that the shortages are allowed and fully backlogged, the instantaneous state of inventory, the stock loss due to deterioration, the backlogged demand and shortage levels are obtained. With suitable cost considerations, the total profit rate function is obtained. By maximizing, the total profit rate function, the optimal values of production downtime, production uptime, selling price and optimal ordering quantity are derived. This model is extended to the case of without shortages. Results are illustrated numerically and sensitivity analysis is carried out with respect to parameters in both the models. In a model with shortages, the production rate parameters influence all the parameters except optimal ordering quantity. In the model without shortages, the production rate parameters have a significant influence on the optimal values of production schedule and profit. It is also seen that in both models the optimal values of production uptime and total profit are highly sensitive to the production rate parameters and cost per unit time. Here, it is considered when a single product is produced. For future scope, it is anticipated to extend the proposed model by considering the multi-commodity EPQ models with time dependent production and deterioration.

REFERENCES

[1]. Mak (1982). A Production lot size inventory model for deteriorating items, Computers & Industrial Engineering, Vol.6, No.4, pp 309- 317.

[2]. Chowdhary and Chaudari (1983). An order l evel inventory model for deteriorating items with a finite rate of replenishment, Opsearch, Vol.20, pp.99-106.

[3]. Su, Lin, and Tsai (1999). A deterministic production inventory model for deteriorating items with an exponential declining demand, OPSEARCH, Vol.36, No.2, pp 95-105.

[4.] Mandal and Phaujdar (1989). An inventory model for deteriorating items and Stock- dependent Consumption Rate, Journal of Operational Research Society, Vol. 40, No. 5, pp 483-488.

[5.] Sujit and Goswami (2001). A replenishment policy for items with finite production rate and Fuzzy deterioration rate, Opsearch, Vol.38, No.4, pp 419.

[6]. Sana, Goyal, and Chaudhuri (2004). A production-inventory model for a deteriorating item with trended demand and shortages, European Journal of Operational Research, Vol.157, No.2, pp 357-371.

[7]. Balkhi (2001). On a finite horizon production lot size inventory model for deteriorating items, European Journal of Operational Research Vol. 132, No.1, pp 210-223.

[8] Samanta and Ajanta Roy (2004). A Deterministic Inventory Model of Deteriorating Items with Two rates of production and Shortages, Tamsui Oxford Journal of Mathematics Sciences, Vol. 20, No.2, pp 205-218.

[9]. Lin, Gong, and Dah-Chuan (2006). On a production-inventory system of deteriorating items subject to random

machine breakdowns with a fixed repair time, Mathematical and computer modelling, Vol.43, No. 7-9, pp 920-932.

[10]. Chen and Chen (2008). Optimal pricing and replenishment schedule for deteriorating items over a finite planning horizon, International Journal of Revenue Management, Vol.2, No. 3, pp 215-233.

[11]. Mirzazadeh, Seyyed Esfahani and Ghomi (2009). An inventory model under certain inflationary conditions, finite production rate, and inflation –Dependent demand rate for deteriorating items with shortages, International Journal of System Sciences, Vol.40, No.1, pp 21-31.

[12] .Manna and Chiang (2010). Economic production quantity models for deteriorating items with ramp type demand, International Journal of Operational Research, Vol. 7, No. 4, pp 429-444.

[13]. Sridevi, Nirupama Devi and Srinivasa Rao (2010). An inventory model for deteriorating items with Weibull rate of replenishment and selling price dependent Demand, International Journal of Operational research, Vol.9, No.3, pp 329-349.

[14]. Tripathy, Pardhan, and Mishra (2010). An EPQ model for Linear deteriorating Item with Variable Holding Cost, International Journal of Computational and Applied Mathematics, Vol.5 No. 2, pp 209-215.

[15]. Manna and Chiang (2010). Economic production quantity models for deteriorating items with ramp type demand, International Journal of Operational Research, Vol. 7, No. 4, pp 429-444.

[16]. Sarkar and Moon (2011). An EPQ model with inflation in an imperfect production system, Applied Mathematics, and Computation, Vol. 217, pp 6159- 6167.

[17]. Srinivasa Rao and Essey Kebede Muluneh (2012). Inventory models for deteriorating items with stock-dependent production rate and Weibull decay, International Journal of Mathematical Archive- Vol.3, No.10, 2012, pp.3709-3723. Available online through www.ijma.info ISSN 2229 – 5046.

[18]. Palanivel & Uthayakumar (2013). An EPQ model for deteriorating items with variable production cost, time dependent holding cost and partial backlogging under inflation. OPSEARCH, Application Article, First Online: 28 December 2013, DOI: 10.1007/s12597- 013-0168-8.

[19]. Hui-Ming Teng (2013). EPQ models for deteriorating items with linearly discounted, back ordering under limited utilization of facility Vol.7 No.29, pp 2882-2889, 7 August 2013, DOI: 10.5897/AJBM12.1214, ISSN 1993-8233 © 2013 Academic Journals, http://www.academicjournals.org/AJBM.

[20]. Sanjay Sharma and Singh (2013). EPQ Model of Deteriorating Inventory with Exponential Demand Rate under Limited Storage, Africa Development and Resources Research Institute (ADRRI) Journal (www.adrri.org) ISSN: 2343-6662 Vol. I, No.1, pp 23-31, October 2013.

[21]. Ashendra Kumar Saxena and Ravish Kumar Yadav (2013). An Optimal Production Cycle for Non- Instantaneous Deteriorating Items in Which Holding Cost Varies Quadratic in Time. International Journal of Application or Innovation in Engineering & Management (IJAIEM), Web Site:www.ijaiem.org, Volume 2, No. 6, June 2013, Page 148 -153, ISSN 2319 –4847.

[22]. Srinivasa Rao, Srinivasa Rao, Kesava Rao (2013). On Optimal Production Scheduling of an EPQ Model with Stock Dependent Production Rate and Pareto Decay, Int. Journal of Applied Sciences and Engineering Research, Vol. 2, Issue 3, 2013, ISSN 2277 – 9442.

[23]. Meenakshi Srivastava & Ranjana Gupta (2013). An EPQ model for deteriorating items with time and price dependent demand under markdown policy, OPSEARCH (Jan–Mar 2014), Vol.51, No.1, pp.148–158.

[24]. Da Wen, Pan Ershun, Wang Ying and Liao Wenzhu (2014). An economic production quantity model for a deteriorating system integrated with predictive maintenance strategy, Journal of Intelligent Manufacturing ©Springer Science + Business Media New York 2014, 10.1007/s10845-014-0954-z.
[25]. Palanivel & Uthayakumar (2014). An EPQ model with variable production, probabilistic deterioration and partial backlogging under inflation, Journal of Management Analytics, Vol.1,issue.3,DOI:10.1080/23270012.2014.971889, published online: 30 Oct 2014.
[26]. Himani Dem, Singh and Jitendra Kumar (2014). An EPQ model with trapezoidal demand under volume flexibility, International Journal of Industrial Engineering Computations, Volume 5 No 1, pp. 127-138, ISSN 1923-2934 (Online) - ISSN 1923-2926 (Print) Quarterly Publication.
[27]. Ankit Bhojak, Gothi (2015), EPQ Model with Time-Dependent IHC and Weibull Distributed Deterioration under Shortages, International Journal of Innovative Research in Science, Engineering and Technology (An ISO 3297: 2007 Certified Organization) Vol. 4, Issue 11, November, 2015, ISSN(Online): 2319- 8753,ISSN (Print): 2347-6710
[28]. Kirtan Parmar and Gothi (2015), EPQ model for deteriorating items under three- parameter Weibull distribution and time dependent IHC with shortages. American Journal of Engineering Research (AJER) e- ISSN: 2320-0847 p-ISSN: 2320-0936 Volume- 4, Issue-7, pp-246-255.
[29]. Anitha, Parvathi(2016), An EPQ Model for Deteriorating Items with Time-Dependent Demand with Reliability and Flexibility in a Fuzzy Environment International Journal of Innovative Research in Computer and Communication Engineering An ISO 3297: 2007 Certified Organization, Vol. 4, Issue 2, February 2016. ISSN(Online): 2320-9801,ISSN (Print): 2320-9798.
[30]. Kousar Jaha Begum and P.Devendra(2016). E.P.Q Model for Deteriorating Items with Generalizes Pareto Decay Having Selling Price and Time-Dependent Demand, Research Journal of Mathematical and Statistical Sciences _ Vol. 4(**??**), Page No: 1-11, February 2016, E-ISSN 2320–6047.
[31]. Dhir Singh & Singh(2017).Development of an EPQ Model for Deteriorating Product with Stock and Demand Dependent Production rate under Variable Carrying Cost and Partial Backlogging, Volume: 5 Issue: 5, pp:478 – 497, ISSN: 2321-8169, IJRITCC | May 2017, Available @ http://www.ijritcc.org
[32]. Khedlekar, Namedeo, Nigwal (2018), Production-Inventory Model With Disruption Considering Shortage And Time Proportional Demand. Yugoslav Journal of Operations Research, Vol. 28 (2018), Number 1, pp-123-139, DOI:https://doi.org/10.2298/YJOR161118008K
[33]. Nitashah and Chetansinh Vaghela(2018), An epq model for deteriorating items with price dependent demand and two level trade credit financing, revista investigación operacional vol. 39, no. 2,pp: 170-180, 2018

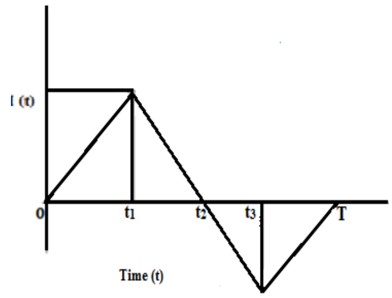

Figure 2.1 Instantaneous state of inventory for model

TABLE 1
OPTIMAL VALUES OF t_1^*, t_3^*, s^*, Q^* AND P^* FOR DIFFERENT VALUES OF PARAMETERS AND T=12

D	f	A	C	a	b	α	β	π	H	t_1^*	t_3^*	s^*	Q^*	P^*
500	20	200	4	5	2	0.3	0.2	0.04	0.012					
500										1.089	7.131	12.493	124.113	3069.16
600										2.228	8.010	14.999	115.881	4444.50
700										3.138	8.694	17.502	110.481	6069.41
	10									1.088	7.130	24.993	124.126	6194.16
	20									1.089	7.131	12.493	124.113	3069.16
	30									1.091	7.133	8.326	124.100	2027.50
		100								1.089	7.131	12.493	124.113	3077.50
		200								1.089	7.131	12.493	124.113	3069.16
		300								1.089	7.131	12.493	124.113	3060.83
			2							4.986	9.856	12.504	107.363	3088.16
			3							2.802	8.405	12.500	113.192	3079.04
			4							1.089	7.131	12.493	124.113	3069.16
				3						2.068	7.430	12.491	112.986	3073.45
				4						1.603	7.273	12.492	118.981	3071.25
				5						1.089	7.131	12.493	124.113	3069.16
					1					3.830	9.670	12.508	63.379	3082.71
					1.5					2.226	8.311	12.502	89.482	3076.60
					2					1.089	7.131	12.493	124.113	3069.16
						0.3				1.089	7.131	12.493	124.113	3069.16
						0.5				1.387	7.184	12.494	125.312	3068.64
						0.8				1.677	7.261	12.495	126.168	3068.02
							0.1			0.781	6.876	12.489	126.847	3069.96
							0.2			1.089	7.131	12.493	124.113	3069.16
							0.3			1.335	7.306	12.496	122.549	3068.52
								0.035		0.422	6.514	12.489	131.283	3068.47
								0.04		1.089	7.131	12.493	124.113	3069.16
								0.045		1.673	7.671	12.497	117.963	3069.57
									0.012	1.089	7.131	12.493	124.113	3069.16
									0.014	1.280	7.130	12.492	125.542	3068.85
									0.02	1.716	7.108	12.491	129.452	3068.23

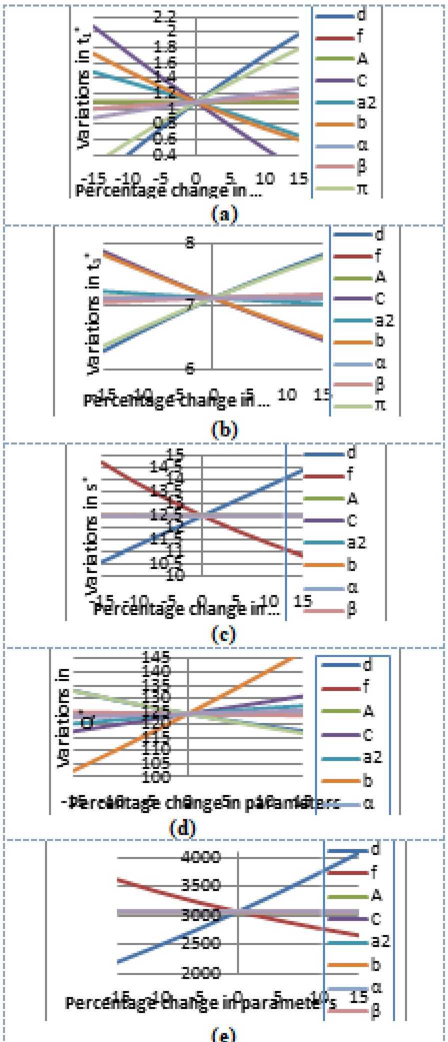

Fig 2: Relationship between parameters and optimal values with shortages

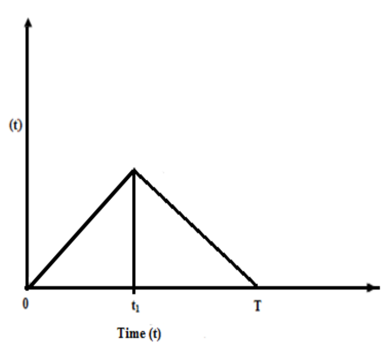

Figure 2.3 Instantaneous state of inventory for model

TABLE 3

OPTIMAL VALUES OF t_1^*, s^*, Q^* and P^* FOR DIFFERENT VALUES OF PARAMETERS AND T=12

d	f	A	c	a	b	α	B	h	t_1^*	s*	Q^*	P^*
500	20	200	2	5	2	0.2	0.01	0.012				
400									6.727	10.005	78.887	1967.90
500									7.321	12.501	90.220	3092.59
600									7.808	14.998	100.006	4467.64
	10								7.322	25.001	90.222	6217.59
	20								7.321	12.501	90.220	3092.53
	30								7.320	8.334	90.218	2050.92
		100							7.321	12.501	90.220	3100.92
		200							7.321	12.501	90.220	3092.59
		300							7.321	12.501	90.220	3084.25
			2						7.321	12.501	90.220	3092.59
			3						6.254	12.509	70.397	3085.97
			4						5.499	12.516	57.743	3080.66
				5					7.321	12.501	90.220	3092.59
				6					7.187	12.502	94.776	3091.36
				7					7.055	12.503	99.163	3090.16
					1				8.626	12.493	80.337	3097.87
					2				7.321	12.501	90.220	3092.59
					3				6.502	12.507	95.926	3088.59
						0.1			6.929	12.500	82.667	3094.28
						0.2			7.321	12.501	90.220	3092.59
						0.3			7.715	12.502	98.097	3091.07
							0.01		7.321	12.501	90.220	3092.59
							0.02		7.726	12.500	98.330	3091.49
							0.03		8.083	12.499	105.759	3090.45
								0.011	7.093	12.502	85.779	3092.68
								0.012	7.321	12.501	90.220	3092.59
								0.013	7.532	12.499	94.394	3092.56

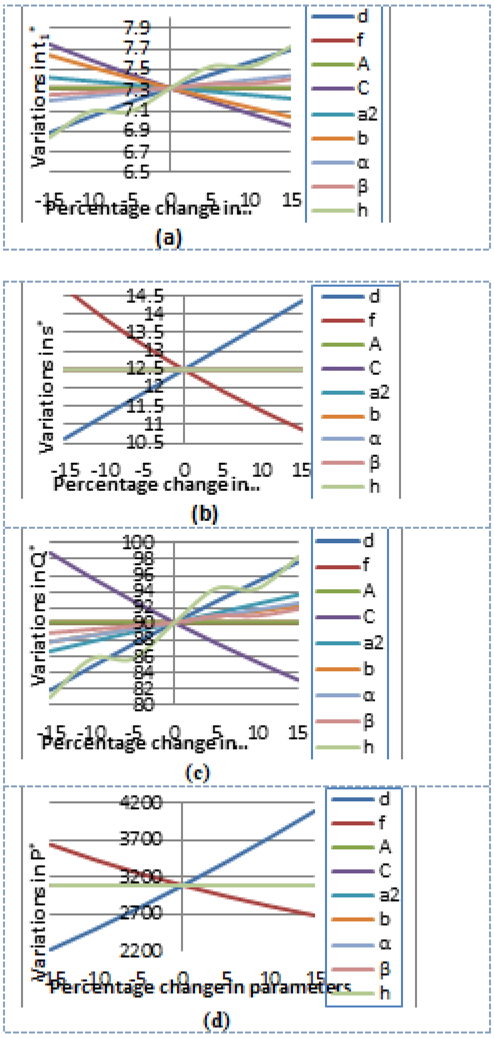

Fig 4: Relationship between parameters and optimal values without shortages

TABLE 4

SENSITIVITY ANALYSIS OF THE MODEL - WITHOUT SHORTAGES

Variation Parameters	Optimal policies	change in parameters						
		-15%	-10%	-5%	0%	5%	10%	15%
d	t1*	6.888	7.040	7.185	7.321	7.452	7.576	7.694
	s	10.629	11.253	11.877	12.501	13.125	13.749	14.373
	Q	81.895	84.780	87.552	90.220	92.793	95.278	97.680
	P	2225.59	2498.94	2787.94	3092.59	3412.88	3748.83	4100.41
f	t1*	7.322	7.322	7.321	7.321	7.321	7.320	7.320
	s	14.706	13.889	13.158	12.501	11.905	11.364	10.870
	Q	90.221	90.221	90.220	90.220	90.220	90.220	90.219
	P	3644.06	3439.81	3257.06	3092.59	2943.78	2808.50	2684.98
A	t1*	7.321	7.321	7.321	7.321	7.321	7.321	7.321
	s	12.501	12.501	12.501	12.501	12.501	12.501	12.501
	Q	90.220	90.220	90.220	90.220	90.220	90.220	90.220
	p	3095.09	3094.25	3093.42	3092.59	3091.75	3090.92	3090.09
C	t1*	7.748	7.598	7.456	7.321	7.193	7.071	6.954
	s	12.498	12.499	12.500	12.501	12.501	12.502	12.503
	Q	98.781	95.732	92.885	90.220	87.717	85.359	83.134
	p	3094.95	3094.14	3093.35	3092.59	3091.85	3091.12	3090.42
a	t1*	7.425	7.390	7.356	7.321	7.287	7.254	7.220
	s	12.500	12.500	12.501	12.501	12.501	12.502	12.502
	Q	86.690	87.878	89.054	90.220	91.375	92.519	93.653
	P	3093.52	3093.21	3092.90	3092.59	3092.28	3091.97	3091.67
b	t1*	7.641	7.529	7.423	7.321	7.224	7.131	7.042
	s	12.499	12.499	12.500	12.501	12.501	12.502	12.503
	Q	87.835	88.675	89.468	90.220	90.932	91.608	92.249
	P	3094.00	3093.52	3093.04	3092.59	3092.14	3091.71	3091.29
α	t1*	7.200	7.241	7.281	7.321	7.362	7.402	7.442
	s	12.500	12.500	12.501	12.501	12.501	12.502	12.502
	Q	87.857	88.641	89.429	90.220	91.014	91.808	92.603
	P	3093.08	3092.91	3092.75	3092.59	3092.43	3092.27	3092.11
β	t1*	7.256	7.278	7.300	7.321	7.364	7.364	7.406
	s	12.502	12.501	12.501	12.501	12.501	12.501	12.500
	Q	88.948	89.374	89.798	90.220	91.060	91.060	91.894
	P	3092.76	3092.70	3092.64	3092.59	3092.48	3092.48	3092.36
h	t1*	6.842	7.093	7.093	7.321	7.532	7.532	7.726
	s	12.503	12.502	12.502	12.501	12.499	12.499	12.498
	Q	81.031	85.779	85.779	90.220	94.394	94.394	98.333
	P	3092.84	3092.68	3092.68	3092.59	3092.56	3092.56	3092.58