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On Topological Indices of the Line Graph of Uniformly Subdivided Graph

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Abstract. In this paper we have discussed the first and second K Banhatti indices, first and second K-hyper Banhatti indices of the line graph of uniformly subdivided helm graph. In addition we also computed the Atom bond connectivity index, Geometric-arithmetic index, first and second Zagreb indices for the defined graph.

1 Introduction

Let $G(V, E)$ be a simple connected graph having no loops and multiple edges, with vertex set $V(G)$ and edge set $E(G)$ having order $|V(G)|$ and size $|E(G)|$. A graph is said to be connected if there exist a path between each pair of its vertices. The degree $d_G(u)$ of a vertex u is the number of edges adjacent to u. The edge connecting the vertices u and v will be denoted by uv. Let the degree of an edge e in G is denoted by $d_G(e)$, defined as, $d_G(e) = d_G(u) + d_G(v) - 2$ with $e = uv$ [10].

Topological indices are numerical descriptors that define the topology of a graph. They are real valued functions and assign a real number to a graph, being associated to a structural graph of a molecule they do not have any dependency on its pictorial representation because they remain unaffected with the change of size and shape of underlying structure. Topological indices play a vital role in structural chemistry. They correlate chemical compounds with various physical properties, chemical structures or bioactivity etc. Topological indices are

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used in QSAR/QSPR studies to predict bioactivity of chemical compounds [1, 2]. The atom bond connectivity index (ABC), Geometric-arithmetic index (GA) and $Randic'$ index are of vital importance in this regard. There are some major classes of topological indices, especially based on vertices and edge distances. The concept of topological index came from wiener in 1947 when he was working on boiling point of paraffin, for more details of Wiener index see [3].

The very first and oldest degree based topological invariant is $Randic'$ index which was introduced by Milan $Randic'$ in 1975, [4, 5]. A well- known topological index introduced by Estrada et al [6] is atom-bond connectivity index (ABC) and is defined as

$$ABC(G) = \sum_{uv\varepsilon E(G)} \sqrt{\frac{d_G(u) + d_G(v) - 2}{d_G(u)d_G(v)}}$$

The geometric-arithmetic (GA) index was introduced by Vukičvić and Furtula in 2009 [7] and is defined as,

$$GA(G) = \sum_{uv\epsilon E(G)} \frac{2\sqrt{d_G(u)d_G(v)}}{d_G(u) + d_G(v)}$$

Ivan Gutman introduced another very important index denoted by $M_1(G)$, named as Zagreb index more precisely first Zagreb index and is defined as,

$$M_1(G) = \sum_{uv\varepsilon E(G)} (d_G(u) + d_G(v))$$

The second Zagreb index is denoted by $M_2(G)$ and is defined as,

$$M_2(G) = \sum_{uv\varepsilon E(G)} (d_G(u) \times d_G(v))$$

For more results on Zagreb indices see [8, 9]. New topological indices were put forward by Kulli [10, 11] namely first and second k-indices. Z. Jie et al. computed k-indices for certain nano structures [12]. The first and second K-Banhatti indices are defined respectively as [10],

$$B_1(G) = \sum_{uv\varepsilon} [d_G(u) + d_G(e)]$$

$$B_2(G) = \sum_{uv\varepsilon E(G)} [d_G(u)d_G(e)]$$

The first and second K-hyper Banhatti indices are defined respectively as [11],

$$HB_1(G) = \sum_{uv\varepsilon E(G)} [d_G(u) + d_G(e)]^2$$

$$HB_2(G) = \sum_{uv\varepsilon E(G)} [d_G(u)d_G(e)]^2$$

In [14], [17], [18], [19], [21] and [22] the authors have studied a variety of indices for certain nanostructures. Farahani et al, [13] and [16] stated indices for circumcoronene series of benzenoid. More results on indices regarding different chemical structure can be found in [15], [20], [23] and [24].

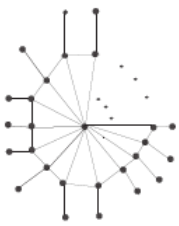

Figure 1: the helm graph (H_n).

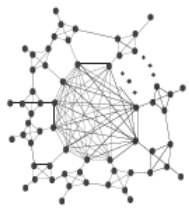

Figure 2: line graph of subdivided helm graph $L(S(H_n))$.

For a graph G the subdivision graph $S(G)$ is a graph obtained by inserting an additional vertex to each edge of a graph G. The Line graph $L(S(G))$ of a Subdivision graph is the graph whose vertices are the edges of $S(G)$, two vertices in $L(S(G))$ will be incident if and only if they have a common vertex in $S(G)$. The graph constructed from a Wheel graph by joining a pendent edge at each vertex of its cycle is called a Helm graph (H_n). It has $2n+1$ number of vertices and $3n$ number of edges.

In structural chemistry the usage of line graph started from its very beginning. In 1981, Bretz introduced the first topological index based on line graph [25]. In 2015 Nadeem et al. made calculations on topological indices of the line graph of subdivision graphs of certain nanostructures in [26] and extended his work in [27]. Ranjini et al. computed some topological indices of subdivison graphs and line graphs of different structures in [28, 29]. Farahani et al found expressions on some topological indices of line graph of $CNC_K[n]$ nanocones [30].

2 Results and Discussions

Let $G = L(S(H_n))$ be the line graph of subdivided helm graph. By algebraic method we get $|V(G)| = 6n$ and $|E(G)| = \frac{17n+n^2}{2}$. We have four partitions of the edge set $E(G)$ of $L(S(H_n))$ as in table-1.

$(d_G(u), d_G(v))/uv\epsilon E(G)$	(1,4)	(4,4)	(4,n)	(n,n)
$\|E\|$	n	7n	n	$\frac{n(n-1)}{2}$
$d_G(e)$	3	6	2+n	2n-2

Table-1 Types of edges for $L(S(H_n))$

Theorem 1: Let $G = L(S(H_n))$ be the line graph of uniformly subdivided helm graph. Then first and second K- Banhatti indices are;

$$B_1(G) = n(3n^2 - 2n + 161) \quad (1)$$

$$B_2(G) = n(2n^3 - 3n^2 + 8n + 359) \quad (2)$$

Proof. By using the definition of first K- Banhatti index and information in table 1 we have,

$$\begin{aligned}
B_1(G) &= \sum_{uv\varepsilon E(G)} [d_G(u) + d_G(e)] \\
&= \sum_{e=uv\varepsilon E_{(1,4)}} [(d_G(u) + d_G(e)) + (d_G(v) + d_G(e))] \\
&\quad + \sum_{e=uv\varepsilon E_{(4,4)}} [(d_G(u) + d_G(e)) + (d_G(v) + d_G(e))] \\
&\quad + \sum_{e=uv\varepsilon E_{(4,n)}} [(d_G(u) + d_G(e)) + (d_G(v) + d_G(e))] \\
&\quad + \sum_{e=uv\varepsilon E_{(n,n)}} [(d_G(u) + d_G(e)) + (d_G(v) + d_G(e))] \\
&= n[(1+3)+(4+3)] + 7n[(4+6)+(4+6)] + n[(4+2+n)+(n+2+n)] \\
&\quad + \frac{n(n-1)}{2}[(n+2n-2)+(n+2n-2)] \\
&= n(3n^2 - 2n + 161)
\end{aligned}$$

Similarly, by using the definition of second K- Banhatti index and information in table 1 we have,

$$\begin{aligned}
B_2(G) &= \sum_{uv\varepsilon E(G)} [d_G(u) \times d_G(e)] \\
&= \sum_{e=uv\varepsilon E_{(1,4)}} [(d_G(u)d_G(e)) + (d_G(v)d_G(e))] \\
&\quad + \sum_{e=uv\varepsilon E_{(4,4)}} [(d_G(u)d_G(e)) + (d_G(v)d_G(e))] \\
&\quad + \sum_{e=uv\varepsilon E_{(4,n)}} [(d_G(u)d_G(e)) + (d_G(v)d_G(e))] \\
&\quad + \sum_{e=uv\varepsilon E_{(n,n)}} [(d_G(u)d_G(e)) + (d_G(v)d_G(e))] \\
&= n[(1.3)+(4.3)] + 7n[(4.6)+(4.6)] + n[4.(2+n)+n.(2+n)] \\
&\quad + \frac{n(n-1)}{2}[n.(2n-2)+n.(2n-2)] \\
&= n(2n^3 - 3n^2 + 8n + 359)
\end{aligned}$$

Theorem 2: Let $G = L(S(H_n))$ be the line graph of uniformly subdivided helm graph. Then first and second K- hyper Banhatti indices are;

$$HB_1(G) = n(9n^3 - 16n^2 + 36n + 1501) \tag{3}$$

$$HB_2(G) = \frac{n}{2}(8n^5 - 22n^4 + 32n^3 + 32n^2 + 128n + 16562) \tag{4}$$

Proof. By using the definition of first K- hyper Banhatti index and information in table 1 we have,

$$\begin{aligned}
HB_1(G) &= \sum_{uv\varepsilon E(G)} [d_G(u)+d_G(e)]^2 \\
&= \sum_{e=uv\varepsilon E_{(1,4)}} [(d_G(u)+d_G(e))^2+(d_G(v)+d_G(e))^2] \\
&\quad + \sum_{e=uv\varepsilon E_{(4,4)}} [(d_G(u)+d_G(e))^2+(d_G(v)+d_G(e))^2] \\
&\quad + \sum_{e=uv\varepsilon E_{(4,n)}} [(d_G(u)+d_G(e))^2+(d_G(v)+d_G(e))^2] \\
&\quad + \sum_{e=uv\varepsilon E_{(n,n)}} [(d_G(u)+d_G(e))^2+(d_G(v)+d_G(e))^2] \\
&= n[(1+3)^2+(4+3)^2]+7n[(4+6)^2+(4+6)^2] \\
&\quad +n[(4+2+n)^2+(n+2+n)^2] \\
&\quad +\frac{n(n-1)}{2}[(n+2n-2)^2+(n+2n-2)^2 \\
&= n(9n^3-16n^2+36n+1501)
\end{aligned}$$

Similarly, by using the definition of second K- hyper Banhatti index and information in table 1 we have,

$$\begin{aligned}
HB_2(G) &= \sum_{uv\varepsilon E(G)} [d_G(u)\times d_G(e)]^2 \\
&= \sum_{e=uv\varepsilon E_{(1,4)}} [(d_G(u)d_G(e))^2+(d_G(v)d_G(e))^2] \\
&\quad + \sum_{e=uv\varepsilon E_{(4,4)}} [(d_G(u)d_G(e))^2+(d_G(v)d_G(e))^2] \\
&\quad + \sum_{e=uv\varepsilon E_{(4,n)}} [(d_G(u)d_G(e))^2+(d_G(v)d_G(e))^2] \\
&\quad + \sum_{e=uv\varepsilon E_{(n,n)}} [(d_G(u)d_G(e))^2+(d_G(v)d_G(e))^2] \\
&= n[(1+3)^2+(4+3)^2]+7n[(4+6)^2+(4+6)^2] \\
&\quad +n[(4+2+n)^2+(n+2+n)^2] \\
&\quad +\frac{n(n-1)}{2}[(n+2n-2)^2+(n+2n-2)^2] \\
&= \frac{n}{2}(8n^5-22n^4+32n^3+32n^2+128n+16562)
\end{aligned}$$

Theorem 3: Let $G = L(S(H_n))$ be the line graph of uniformly subdivided helm graph. Then the value of its atom-bond connectivity index (ABC) is;

$$ABC(G) = \frac{n}{2}\big(\sqrt{3}+7\frac{\sqrt{6}}{2}+\sqrt{\frac{2+n}{n}}+(1-\frac{1}{n})\sqrt{2n-2}\big)$$

Proof. By using the definition of atom bond connectivity index and information in table 1 we have

$$\begin{aligned}
ABC(G) &= \sum_{uv\varepsilon E(G)} \sqrt{\frac{d_G(u)+d_G(v)-2}{d_G(u)d_G(v)}} \\
&= \sum_{e=uv\varepsilon E_{(1,4)}} \sqrt{\frac{d_G(u)+d_G(v)-2}{d_G(u)d_G(v)}} + \sum_{e=uv\varepsilon E_{(4,4)}} \sqrt{\frac{d_G(u)+d_G(v)-2}{d_G(u)d_G(v)}} \\
&\quad + \sum_{e=uv\varepsilon E_{(4,n)}} \sqrt{\frac{d_G(u)+d_G(v)-2}{d_G(u)d_G(v)}} + \sum_{e=uv\varepsilon E_{(n,n)}} \sqrt{\frac{d_G(u)+d_G(v)-2}{d_G(u)d_G(v)}} \\
&= n\sqrt{\frac{1+4-2}{1.4}} + 7n\sqrt{\frac{4+4-2}{4.4}} + n\sqrt{\frac{4+n-2}{4n}} \\
&\quad + \frac{n(n-1)}{2}\sqrt{\frac{n+n-2}{n^2}} \\
&= \frac{n}{2}\big(\sqrt{3} + 7\frac{\sqrt{6}}{2} + \sqrt{\frac{2+n}{n}} + (1-\frac{1}{n})\sqrt{2n-2}\big)
\end{aligned}$$

Theorem 4: Let $G = L(S(H_n))$ be the line graph of uniformly subdivided helm graph. Then;

$$GA(G) = n\big(\frac{39}{5} + \frac{4\sqrt{n}}{4+n} + \frac{n-1}{2}\big) \tag{5}$$

Proof. By using the definition of geometric arithmetic index and information in table 1 we will get the result 5.

Theorem 5: Let $G = L(S(H_n))$ be the line graph of subdivided helm graph. Then its first and second Zagreb indices are;

$$M_1(G) = n(n^2+65) \tag{6}$$

$$M_2(G) = n\big(\frac{n^2(n-1)}{2} + 4n + 116\big) \tag{7}$$

Proof. By using the definition of first and second Zagreb indices and information in table 1 we will get the results 6 and 7.

3 Conclusion

In this article we computed various degree based topological indices such as K-Banhatti indices, K-hyper Banhatti indices, Atom-bond connectivity index, Geometric-arithmetic index, first and second Zagreb indices for the line graph of uniformly subdivided helm graph. These topological invariants perform a substantial role to predict biological attributes of certain chemical compounds in cheminformatics in development of QSPR and QSAR. These indices are also helpful for determining thermodynamics attributes of chemical compounds and in the study of stability and strain energy of alkane and cycloalkanes too.

4 Conflicts Of Interest

The authors declare no conflicts of interest.

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