International Journal of Advances in Mathematics
Volume 2019, Number 3, Pages 24-33, 2019
eISSN 2456-6098

On τ^*-Generalized β closed sets in Topological Spaces

C. Aruna* and R. Selvi[1]
*Research Centre of Mathematics, Sri Parasakthi College for Women, Courtallam-627802.
Affiliated to Manonmaniam Sundaranar University, Abishekapati,Tirunelveli-627012,India.
[1]Sri Parasakthi College for Women, Courtallam-627802, India.
*E-mail: arunarobert89@gmail.com

ABSTRACT. In this paper we introduce a new class of sets called τ^* generalized β closed sets in topological spaces (briefly $\tau^* g\beta$ closed set) and also we discussed some of their properties. Further we obtained the concept of τ^* generalized β continuity and τ^* generalized β irresolute. We introduced τ^* generalized β open map and τ^* generalized β homeomorphism by using τ^* generalized β open set. Also we studied a new class of compact and connected spaces by using τ^* generalized β closed set.

1 Introduction

In 1970, Levine introduced the concept of generalized closed set and discussed the properties of sets, closed and open maps, compactness, normal and separation axioms. Later in 1986 D. Andrijevic[3] gave a new type of generalized closed sets in topological space called semi pre open sets. In 1995,on generalizing semi pre open set is introduced by J. Dontchev. Dunham[2] introduced the concept of the closure operator cl^* and a new topology τ^* and studied some of their properties. A.Pushpalatha, S.Eswaran and P.RajaRubi[6] introduced a new class of sets called τ^* generalized closed sets and studied some of their properties. S.Eswaran and A.Pushpalatha, introduced τ^* generalized continuous maps,τ^* generalized compact spaces and τ^* generalized connected spaces in topological spaces.The concept of τ^* generalized homeomorphism in topological spaces is introduced by S.Eswaran, N. Nagaveni, The aim of this paper is to continue the study of τ^* generalized β closed sets and related concepts.

* Corresponding Author.
Received December 25, 2018; Revised February 22, 2019; Accepted February 26, 2019.
2010 Mathematics Subject Classification: 54A05.
Key words and phrases: τ^*-Generalized β closed sets.

2 Preliminaries

In this section, we recall the definitions.

Definition 2.1. A Subset A of a topological space X is called a β closed(or semi pre closed)[3] if $\inf(cl(int(A))) \subseteq A$.

Definition 2.2. A Subset A of a topological space X is called a generalized β closed (briefly, $g\beta$ closed)[4] if $\beta cl(A) \subseteq G$ whenever $A \subseteq G$ and G is open in X.

Definition 2.3.[2] For the subset A of a topological X, the generalized closure operator cl^* is defined by the intersection of all g-closed sets containing A.

Definition 2.4.[2] For the subset A of a topological X, the topology τ^* is defined by $\tau^* = \{G : cl^*(G^c) = G^c\}$.

3 On τ^* Generalized β closed sets

A subsets E of a topological space (X, τ^*) is called τ^* generalized β closed set if $cl^*[(int(cl(int(E)))] \subseteq W$ (briefly, $cl^*[(int(cl(int(E)))]$ denoted by $cl^*_\beta(E))$ whenever $E \subseteq W$ and W is τ^* open in X. The complement of τ^* generalized β closed set is called the τ^* generalized β open set.

Example 3.2. Let $X = \{a, b, c\}$ and $\tau = \{\phi, X, \{b\}\}$. Clearly X, τ^* is τ^*-generalized β closed.

Theorem 3.3. Every closed set in a topological space X is τ^* generalized β closed.

Proof: Let E be a closed set and $E \subseteq W$, where W is τ^*-open in X. Since E is closed, $Cl(E) = E \subseteq W$. But $cl^*[(int(cl(int(E)))] \subseteq Cl(E) \subseteq W$. Then $cl^*_\beta(E)) \subseteq W$ whenever $E \subseteq W$ and W is τ^*-open. Hence E is τ^* generalized β closed.

The converse of the above theorem needs not to be true as seen from the following example.

Example 3.4. Let $X = \{a, b, c\}$ be the topological space. Consider the topology $\tau = \{X, \varphi, \{a\}, \{a, c\}\}$. Here the generalized closed sets $\{X, \varphi, \{b\}, \{a, b\}, \{b, c\}\}$. Then, the set $\{c\}$ and $\{a, b\}$ are τ^* generalized β closed but not closed sets in X. **Theorem 3.5.** Every β closed set in a topological space X is τ^* generalized β closed.

Proof: Let E be a β closed set and $E \subseteq W$, where W is τ^*-open in X. Since E is β closed, $Cl[(int(cl(int(E)))] = E \subseteq W$. But $cl^*[(int(cl(int(E)))] \subseteq Cl[(int(cl(int(E)))] \subseteq W$. Thus, we have $cl^*_\beta(E)) \subseteq W$ whenever $E \subseteq W$ and W is τ^*-open. Therefore E is τ^* generalized β closed. The converse of the above theorem needs not to be true as seen from the following example.

Example 3.6. Let $X = \{a, b, c\}$ be the topological space. Consider the topology $\tau = \{X, \varphi, \{a, c\}\}$. Here the generalized closed sets $\{X, \varphi, \{b\}, \{a, b\}, \{b, c\}\}$. Then, the set $\{a, c\}$ are τ^* generalized β closed but not β closed set in X.

Theorem 3.7. Every generalized closed set in a topological space X is τ^* generalized β closed.

Proof: Let E be a generalized closed set. Let $E \subseteq W$, where W is τ^*-open in X. Since E is generalized closed, $Cl(E) \subseteq G$. But $cl^*_\beta(E)) \subseteq Cl(E) \subseteq W$. Then $cl^*_\beta(E)) \subseteq W$ whenever $E \subseteq W$ and W is τ^*-open. Thus E is τ^* generalized β closed.

The converse of the above theorem needs not to be true as seen from the following example.

Example 3.8. Let $X = \{a, b, c\}$ be the topological space. Consider the topology $\tau = \{X, \varphi, \{b\}, \{c\}, \{b, c\}\}$. Here

the generalized closed sets $\{X, \varphi, \{a\}, \{a,b\}, \{a,c\}\}$. Then, the set $\{b\}$ and $\{c\}$ are τ^* generalized β closed but not generalized closed.

Theorem 3.9. Every generalized β closed set in a topological space X is τ^* generalized β closed.

Proof : Let E be a generalized β closed set and $E \subseteq W$, where W is τ^* open in X. Since E is generalized β closed, $\beta Cl(E) \subseteq W$. But $cl^*_\beta(E)) \subseteq \beta Cl(E) \subseteq W$. Then $cl^*_\beta(E)) \subseteq W$ whenever $E \subseteq W$ and W is τ^*open. Therefore E is τ^* generalized β closed. The converse of the above theorem needs not to be true as seen from the following example.

Example 3.10. Consider the topological space $X = \{a,b,c\}$ with topology $\tau = \{X, \phi, \{b\}\}$. Then, the set $\{a,b\}$ is τ^* generalized β closed but not generalized β closed.

Theorem 3.11. Every τ^* closed set in a topological space X is τ^* generalized β closed.

Proof : Let A be a τ^* closed set and $E \subseteq W$, where W is τ^*-open in X. Since E is τ^* closed, $cl^*(E) = E \subseteq W$. But $cl^*_\beta(E) \subseteq cl^*(A) \subseteq G$. Then $cl^*_\beta(E) \subseteq W$ whenever $E \subseteq W$ and W is τ^*-open.

Therefore E is τ^* generalized β closed. The converse of the above theorem needs not to be true as seen from the following example.

Example 3.12. Let $X = \{a,b,c\}$ be the topological space. Consider the topology $\tau = \{X, \varphi, \{c\}\}$. Here the generalized closed sets $\{X, \varphi, \{a\}, \{b\}, \{a,b\}, \{a,c\}, \{b,c\}\}$. Then, the set $\{a\}, \{b\}, \{b,c\}$ and $\{a,c\}$ are τ^* generalized β closed but not τ^* closed.

Theorem 3.13. Every τ^* generalized closed set in a topological space X is τ^* generalized β closed.

Proof: Let E be a τ^* generalized closed set. Let $E \subseteq W$, where W is τ^*-open in X. Since E is τ^* generalized closed, $cl^*(E) \subseteq W$. But $cl^*_\beta(E) \subseteq cl^*(E) \subseteq W$. Then $cl^*_\beta(E) \subseteq W$ whenever $E \subseteq W$ and W is τ^*-open. Therefore E is τ^* generalized β closed.

Theorem 3.14. For any two sets E and F, $cl^*_\beta(E \cup F) = cl^*_\beta(E) \cup cl^*_\beta(F)$.

Proof: Since $E \subseteq E \cup F$, we have $cl^*_\beta(E) \subseteq cl^*_\beta(E \cup F)$ and Since $F \subseteq E \cup F$, we have $cl^*_\beta(E) \subseteq cl^*_\beta(E \cup F)$. Then $cl^*_\beta(E) \cup cl^*_\beta(E) \subseteq cl^*_\beta(E \cup F)$. Also, $cl^*_\beta(E)$ and $cl^*_\beta(F)$ are the closed sets. Therefore $cl^*_\beta(E) \cup cl^*_\beta(F)$ is also a closed set. Again, $E \subseteq cl^*_\beta(E)$ and $F \subseteq cl^*_\beta(F)$ Implies $E \cup F \subseteq cl^*_\beta(E) \cup cl^*_\beta(F)$. Thus, $cl^*_\beta(E) \cup cl^*_\beta(F)$ is a closed set containing $E \cup F$. Since $cl^*_\beta(E \cup F)$ is the smallest closed set containing $E \cup F$. We have $cl^*_\beta(E \cup F) = cl^*_\beta(E) \cup cl^*_\beta(F)$. Thus, $cl^*_\beta(E \cup F) = cl^*_\beta(E) \cup cl^*_\beta(F)$.

Theorem 3.15. The union of two τ^* generalized β closed subsets of X is also τ^* generalized β closed subset of X.

Proof: Assume that E and F are τ^* generalized β closed set in X. Let W is τ^* open in X such that $E \cup F \subset W$. Then $E \subset W$ and $F \subset W$. Since E and F are τ^* generalized β closed, $cl^*_\beta(E) \subset W$ and $cl^*_\beta(F) \subset W$. Hence by theorem 3.14, $cl^*_\beta(E \cup F) = cl^*_\beta(F) \subset cl^*_\beta(F) \subset W$. That is $cl^*_\beta(E \cup F) \subset W$. Therefore $E \cup F$ is τ^* generalized β closed set in X.

Theorem 3.16. A subsets A of X is τ^* generalized β closed if and only if $cl^*_\beta(E) - E$ contains no non empty τ^* closed set in X. Proof : Let E be a τ^* generalized β closed set. Suppose that F is a non empty τ^* closed subset of $cl^*_\beta(E) - E$. Now, $F \subseteq cl^*_\beta(E) - E$. Then $F \subseteq cl^*_\beta(E) \cap E^c$. Since $cl^*_\beta(E) - E = cl^*_\beta(E) \cap E^c$. Therefore $F \subseteq cl^*_\beta(E)$ and $F \subseteq E^c$. Since F^c is a τ^* open set and E is a τ^* generalized β closed, $cl^*_\beta(E) \subseteq F^c$. That is $F \subseteq cl^*_\beta(E) \cap [cl^*_\beta(E)]^c = \varphi$. That is $F = \varphi$, a contradiction. Thus, $cl^*_\beta(E) - E$ contains no non empty τ^* closed set in X. Conversely, assume that $cl^*_\beta(E) - E$ contains no non empty τ^* closed set. Let $E \subseteq W$, W is τ^* open. Suppose that $cl^*_\beta(A)$ is not con-

tained in W, then $cl^*_\beta(E) \cap W^c$ is a non- empty τ^* closed set of $cl^*_\beta(E) - E$ which is a contradiction. Therefore, $cl^*_\beta(E) - E \subseteq W$ and hence E is τ^* generalized β closed.

Corollary 3.17. A subsets A of X is τ^* generalized β closed if and only if $cl^*_\beta(E) - E$ contains no non empty closed set in X.

Proof: The proof follows from the theorem 3.16 and fact that every closed set is τ^* closed set in X.

Corollary 3.18. A subsets A of X is τ^* generalized β closed if and only if $cl^*_\beta(E) - E$ contains no non empty open set in X.

Proof: The proof follows from the theorem 3.16 and fact that every open set is τ^* open set in X.

Theorem 3.19. If a subset A of X is τ^* generalized β closed and $E \subseteq F \subseteq cl^*_\beta(E)$, then F is τ^* generalized β closed set in X.

Proof: Let E be a τ^* generalized β closed set such that $E \subseteq F \subseteq cl^*_\beta(E)$. Let W be a τ^* open set of X such that $F \subseteq W$. Since E is τ^* generalized β closed, we have $cl^*_\beta(E) \subseteq W$. Now, $cl^*_\beta(E) \subseteq cl^*_\beta(F) \subseteq cl^*(cl^*_\beta(E)) = cl^*_\beta(E) \subseteq W$. That is $cl^*_\beta(F) \subseteq W$,$W$ is τ^* open. Therefore F is τ^* generalized β closed set in X.

Theorem 3.20. Let E be a τ^* generalized β closed in X. Then E is generalized closed if and only if $cl^*_\beta(E) - E$ is τ^* open.

Proof: Suppose E is generalized closed in X. Then $cl^*_\beta(E) = A$ and so $cl^*_\beta(E) - E = \varphi$ which is τ^* open in X. Conversely, suppose $cl^*_\beta(E) - E$ is τ^* open in X. Since E is τ^* generalized β closed, by the theorem 3.16, $cl^*_\beta(E) - E$ contains no non empty closed set in X. So, $cl^*_\beta(E) - E = \varphi$. Hence E is generalized closed.

Theorem 3.21. For $x \in X$, the set $X - \{a\}$ is τ^* generalized β closed or τ^*-open.

Proof: Suppose $X\{a\}$ is not τ^*-open. Then X is the only τ^*-open set containing $X\{a\}$. This implies $cl^*_\beta(X\{a\}) \subseteq X$. Hence $X\{a\}$ is a τ^* generalized β closed in X.

Example 3.22.

1. Let $X = \{a, b, c, d\}$ be the topological space. Consider the topology $\tau = \{X, \varphi, \{a\}, \{a, b, c\}, \{a, b, d\}\}$. Then the set $\{b\}, \{b, c\}$ and $\{b, d\}$ are α closed but not τ^* generalized β closed.
2. Let $X = \{a, b, c\}$ be the topological space. Consider the topology $\tau = \{X, \varphi, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$. Here the generalized closed sets $\{X, \varphi, \{b\}, \{c\}, \{b, c\}, \{a, c\}\}$. Then , the set $\{a, b\}$ is semi closed but not τ^* generalized β closed.
3. Let $X = \{a, b, c\}$ be the topological space. Consider the topology $\tau = \{X, \varphi, \{b\}, \{a, b\}\}$. Here the generalized closed sets $\{X, \varphi, \{c\}, \{a, c\}\}$. Then , the set $\{a\}$ and $\{b, c\}$ are pre closed but not τ^* generalized β closed.
4. Let $X = \{a, b, c\}$ be the topological space. Consider the topology $\tau = \{X, \varphi, \{a\}\}$. Here the generalized closed sets $\{X, \varphi, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}\}$. Then, the set $\{a\}$ is b-closed but not τ^* generalized β closed.
5. Let $X = \{a, b, c\}$ be the topological space. Consider the topology $\tau = \{X, \varphi, \{a\}, \{a, b\}\}$. Here the generalized closed sets $\{X, \varphi, \{c\}, \{b, c\}, \{a, c\}\}$. Then, the set $\{a\}$ and $\{a, b\}$ are generalized semi closed but not τ^* generalized β closed.
6. Let $X = \{a, b, c\}$ be the topological space. Consider the topology $\tau = \{X, \varphi, \{a, b\}\}$. Here the general-

ized closed sets $\{X, \varphi, \{c\}, \{a, c\}, \{b, c\}\}$. Then, the set $\{a\}$ and $\{b\}$ are generalized pre closed but not τ^* generalized β closed.

7. Let $X = \{a, b, c\}$ be the topological space. Consider the topology $\tau = \{X, \varphi, \{b\}\}$. Here the generalized closed sets $\{X, \varphi, \{a\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$. Then, the set $\{b\}$ generalized b closed but not τ^* generalized β closed.

4 τ^*-Generalized β Irresolute Maps and Continuous Maps in Topological Spaces

Definition 4.1. A map $f : (X, \tau_1^*) \to (Y, \tau_1^*)$ is called τ^*-generalized β irresolute if the inverse image of every τ^* generalized β closed set in Y is τ^*-generalized β closed set in X.

Definition 4.2. A map $f : (X, \tau_1^*) \to (Y, \tau_1^*)$ is called τ^*-generalized β continuous if the inverse image of every closed set in Y is τ^*-generalized β closed set in X.

Theorem 4.3. A map $f : (X, \tau_1^*) \to (Y, \tau_1^*)$ is τ^*-generalized β irresolute if and only if the inverse image of every τ^* generalized β open set in Y is τ^*-generalized β open set in X.

Proof: Suppose $f : (X, \tau_1^*) \to (Y, \tau_1^*)$ is τ^*-generalized β irresolute and E is τ^*-generalized β open set in Y. Therefore E^c is τ^* generalized β closed set in Y. Then $f^{-1}(E^c)$ is τ^* generalized β closed set in X, since f is τ^*-generalized β irresolute. Now $f^{-1}(E^c) = Xf^{-1}(E)$ and hence $f^{-1}(E)$ is τ^* generalized β open set in X. Therefore, the inverse image of every τ^* generalized β open set in Y is τ^* generalized β open in X.

Conversely, suppose that the inverse image of every τ^* generalized β-open set in Y is τ^* generalized β open in X. Suppose E is τ^* generalized β closed set in Y, which implies E^c is τ^* generalized β open set in Y. Therefore $f^{-1}(E^c)$ is τ^* generalized β open in X, by assumption. Now $f^{-1}(E^c) = Xf^{-1}(E)$ and then $f^{-1}(E)$ is τ^* generalized β closed set in X. Hence, f is τ^*-generalized β irresolute.

Theorem 4.4. A map $f : (X, \tau_1^*) \to (Y, \tau_1^*)$ is τ^*-generalized β irresolute if and only if $f : (X, \tau_1^*) \to (Y, \tau_1^*)$ is τ^*-generalized β continuous.

Proof: Suppose $f : (X, \tau_1^*) \to (Y, \tau_1^*)$ is τ^*-generalized β irresolute and H is closed set in Y. Therefore, H is τ^*-generalized β closed set in Y, by theorem 3.3. Then $f^{-1}(H)$ is τ^* generalized β closed in X, since f is τ^*-generalized β irresolute. Hence, f is τ^*-generalized β continuous. Conversely, suppose that f is τ^*-generalized β continuous and H is closed set in Y. By theorem 3.3, H is τ^*-generalized β closed set in Y. Therefore, $f^{-1}(H)$ is τ^* generalized β closed in X, since f is τ^*-generalized β continuous. Hence f is τ^*-generalized β irresolute.

Theorem 4.5. Let X, Y and Z be any topological spaces. For any τ^*-generalized β irresolute map $q : (X, \tau_1^*) \to (Y, \tau_1^*)$, and any τ^*-generalized β continuous map $r : (Y, \tau_2^*) \to (Z, \tau_3^*)$, then the composition $r \circ q : (X, \tau_1^*) \to (Z, \tau_3^*)$ is *generalized continuous. Proof: Suppose H is closed set in Z. Therefore $r^{-1}(H)$ is τ^* generalized β closed in Y, since r is τ^* generalized β continuous. By assumption q is τ^* generalized β irresolute, then $q^{-1}(r^{-1}(H))$ is τ^* generalized β closed in X. Then $q^{-1}(r^{-1}(H)) = (r \circ q)^{-1}(H)$. Hence $r \circ q$ is τ^* generalized β continuous.

Theorem 4.6. Every continuous map is τ^* generalized β continuous. Proof: Suppose $f : X \to Y$ is continuous

map and F is closed set in Y. Then $f^{-1}(F)$ is closed in X, since f is continuous. By theorem 3.3, every closed set is τ^* generalized β closed which implies $f^{-1}(F)$ is τ^* generalized β closed in X. Hence, the inverse image of every closed set in Y is τ^* generalized β closed in X. Thus, every continuous map is τ^* generalized β continuous.

Theorem 4.7. Every generalized continuous map is τ^*-generalized β continuous.

Proof: Suppose $h : X \to Y$ is generalized continuous map and U is closed set in Y. Then $h^{-1}(U)$ is generalized closed in X, since f is generalized continuous. By theorem 3.7, every generalized closed set is τ^*-generalized β closed which implies $h^{-1}(U)$ is τ^*-generalized β closed in X. Hence, the inverse image of every generalized β closed set in Y is τ^*-generalized β closed in X. Thus, every generalized continuous map is τ^*-generalized β continuous.

Theorem 4.8. A map $f : (X, \tau_1^*) \to (Y, \tau_2^*)$ is τ^* generalized β continuous if and only if the inverse image of each generalized β open set in Y is τ^* generalized β open in X.

Proof: Suppose $f : (X, \tau_1^*) \to (Y, \tau_2^*)$ is τ^* generalized β continuous and K is open set in Y. Therefore K^c is closed set in Y. Then $f^{-1}(K^c)$ is τ^* generalized β closed set in X, since f is τ^* generalized β continuous. Now $f^{-1}(K^c) = Xf^{-1}(K)$ and hence $X - f - 1(K)$ is τ^* generalized β closed set in X and also $f^{-1}(K)$ is τ^* generalized β open in X. Therefore, the inverse image of every generalized β open set in Y is τ^* generalized β open in X.

Conversely, suppose that the inverse image of each open set in Y is τ^* generalized β open in X. Suppose L is closed set in Y, which implies L^c is open set in Y. Therefore $f^{-1}(L^c)$ is τ^* generalized β open in X, by assumption. Now $f^{-1}(L^c) = Xf^{-1}(L)$ and then $X - f^{-1}(L)$ is τ^* generalized β open set in X and also $f^{-1}(L)$ is τ^* generalized β closed in X. Hence, f is τ^*-generalized β continuous.

5 τ^*-Generalized open map in topological spaces

Definition 5.1. A map $h : (X, \tau_1^*) \to (Y, \tau_2^*)$ is called τ^*-generalized β open map if for each open V in X, $h(V)$ is a τ^*-generalized β open in Y.

Theorem 5.2. Every open map is τ^*-generalized β open map.

Proof: Suppose $h : (X, \tau_1^*) \to (Y, \tau_2^*)$ is an open map and V is any open set in X. Then, $h(V)$ is open in Y, since h is an open map. By theorem every open set is τ^*-generalized β open, therefore $h(V)$ is a τ^*-generalized β open in Y. Hence h is a τ^*-generalized β open map.

Theorem 5.3. Every generalized open map is τ^*-generalized β open map.

Proof: Suppose $h : (X, \tau_1^*) \to (Y, \tau_2^*)$ is generalized open map and G is any open set in X. Then, $h(G)$ is generalized open in Y, since h is generalized open map. By theorem every τ^*-generalized open is generalized β open, therefore $h(G)$ is a τ^*-generalized β open in Y. Hence h is a τ^*-generalized β open map.

Theorem 5.4. If $h : (X, \tau_1^*) \to (Y, \tau_2^*)$ is bijective , then the following statements are equivalent.

1. The inverse map h^{-1} is τ^*-generalized β continuous
2. h is a τ^*-generalized β open map.
3. h is a τ^*-generalized β closed map.

Proof: $(i) \to (ii)$ Suppose B is open set in X which implies that the inverse image of B under h^{-1} is τ^*-generalized β open in Y, since h^{-1} is τ^*-generalized β continuous. Now $(h-1)^{-1}(B) = h(B)$ is τ^*-generalized β open in Y and hence h is a τ^*-generalized β open. Therefore $(i) \to (ii)$.
$(ii) \to (iii)$ Suppose D is any closed set in X, which implies that D^c is open set in X. Then $h(D^c)$ is τ^*-generalized β open set in Y, since h is τ^*-generalized β open map. Now $h(D^c) = Yh(D)$ and hence $Yh(D)$ is τ^*-generalized β open set in Y, which implies that $h(D)$ is τ^*-generalized β closed in Y. Therefore h is a τ^*-generalized β closed map. Hence, $(ii) \to (iii)$. $(iii) \to (i)$ Suppose K is any closed set in X. Then $h(K)$ is τ^*-generalized β closed in Y, since h is a τ^*-generalized β closed map. Now, $h(K) = (h-1) - 1(K)$ which means the inverse map h^{-1} is τ^* generalized β continuous. Thus, $(iii) \to (i)$. Hence the above statements (i),(ii) and (iii) are equivalent.

6 τ^*-Generalized β Homeomorphism in topological spaces

Definition 6.1. A bijection $h : (X, \tau_1^*) \to (Y, \tau_2^*)$ is called τ^*-generalized β homeomorphism if h is both τ^* generalized β continuous map and τ^* generalized β open map.

Theorem 6.2.

1. Every homeomorphism is τ^*-generalized β homeomorphism.
2. Every generalized homeomorphism is τ^*-generalized β homeomorphism

Proof: (i) Suppose $h : (X, \tau_1^*) \to (Y, \tau_2^*)$ is homeomorphism which implies h is both continuous and open map. By theorem 4.6, every continuous map is τ^* generalized β continuous map and also by theorem 3.3, every open map is τ^* generalized β open map which implies that h is both τ^* generalized β continuous map and τ^* generalized β open map. Therefore h is τ^*-generalized β homeomorphism. Thus, every homeomorphism is τ^*-generalized β homeomorphism.
(ii) Suppose $h : (X, \tau_1^*) \to (Y, \tau_2^*)$ is generalized homeomorphism which implies h is both generalized continuous and generalized open map. By theorem 4.7 every generalized continuous map is τ^*-generalized β continuous map and also by theorem 5.3, every generalized open map is τ^* generalized β open map which implies that h is both τ^* generalized β continuous map and τ^* generalized β open map. Therefore h is τ^*-generalized β homeomorphism. Thus, every generalized homeomorphism is τ^*-generalized β homeomorphism.

Theorem 6.3. If $h : (X, \tau_1^*) \to (Y, \tau_2^*)$ is bijective and τ^*-generalized β continuous map , then the following statements are equivalent.

1. h is τ^*-generalized β open map,
2. h is τ^*-generalized β homeomorphism,
3. h is τ^*-generalized β closed map.

Proof: $(i) \to (ii)$ Suppose $h : (X, \tau_1^*) \to (Y, \tau_2^*)$ is bijective and τ^* generalized β continuous map and also h is τ^* generalized β open map. Obviously, h is τ^*-generalized β homeomorphism. Thus, $(i) \to (ii)$.
$(ii) \to (iii)$ Suppose h is τ^*-generalized β homeomorphism, which means h is bijective, τ^*-generalized β open and τ^*-generalized β continuous. By theorem 5.4, obviously h is τ^*-generalized β closed map. Thus, $(ii) \to (iii)$.

$(iii) \to (i)$ Suppose h is τ^*-generalized β closed map and bijective. By theorem 5.4, obviously h is τ^* generalized β open map. Thus, $(iii) \to (i)$. Hence, the following statements (i),(ii) and (iii) are equivalent.

7 τ^*-Generalized β Compact Spaces

Definition 7.1. A collection$\{E_i : i \in I\}$ of τ^* generalized β open sets in a topological space (X, τ^*) is called a τ^* generalized β open cover of a subset F if $F \subseteq \bigcup\{E_i : i \in I\}$

Definition 7.2. A topological space (X, τ^*) is called τ^* generalized β compact if every τ^* generalized β open cover of X has a finite sub cover.

Definition 7.3. A Subset F of a topological space (X, τ^*) is said to be τ^* generalized β compact relative to X, if for every collection $\{E_i : i \in I\}$ of τ^* generalized β open subsets of X such that $F \subseteq \bigcup\{E_i : i \in I\}$ there exists a finite subset I_0 of I such that $F \subseteq \bigcup\{E_i : i \in I\}$.

Theorem 7.4. A τ^* generalized β continuous image of a τ^* generalized β compact space is compact.

Proof: Suppose $f : X \to Y$ is τ^* generalized β continuous map from a τ^* generalized β compact space X onto a topological space Y. Consider $\{E_i : i \in I\}$ is an open cover of Y. Therefore $\{f^{-1}(Ei) : i \in I\}$ is a τ^*-generalized β open cover of X. We know that, X is τ^* generalized β compact, so it has a finite sub cover say, $\{f^{-1}(E_1), f^{-1}(E_2),, f^{-1}(E_n)\}$. Obviously $\{E_1, E_2, ..., E_n\}$ is an open cover of Y Hence, Y is compact.

Theorem 7.5. A τ^*-generalized β closed subset of τ^*-generalized β compact space is τ^*-generalized β compact relative to X.

Proof: Suppose E is τ^* generalized β closed subset of τ^* generalized β compact space, which implies that E^c is τ^* generalized β open in X. Suppose G is a cover of A by τ^* generalized β open sets in X. Therefore $\{G, E^c\}$ is a τ^* generalized β open cover of X. Then X has a finite sub cover, say $\{H_1, H_2, ..., H_n\}$, since X is τ^* generalized β compact. If the subcover, $\{H_1, H_2, ..., H_n\}$ contains E^c , we leave it . Otherwise omit, $\{H_1, H_2, ..., H_n\}$ as it is. Hence, we have obtained a finite τ^* generalized β open sub cover of A and hence A is τ^* generalized β compact relative to X.

Theorem 7.6. If a map $f : X \to Y$ is τ^* generalized β irresolute and a subset C is τ^* generalized β compact relative to X, then $f(C)$ is τ^* generalized β compact relative to Y.

Proof : Suppose $f : X \to Y$ is τ^* generalized β irresolute and C is τ^* generalized β compact relative to X. Suppose $\{E_i : i \in I\}$ is any collection of τ^* generalized β open cover of Y such that $f(C) \subseteq \bigcup\{E_i : i \in I\}$, which implies that $C \subseteq \bigcup\{f^{-1}(E_i) : i \in I\}$. By using assumption there exists a finite subset I_0 of I such that $C \subseteq \{f^{-1}(E_i) : i \in I\}$, which implies that $f(C) \subseteq \{E_i : i \in I_0\}$. Hence, $f(C)$ is τ^* generalized β compact relative to Y.

8 τ^*-Generalized β Connectedness

Definition 8.1. A topological space (X, τ^*) is said to be τ^* generalized β connected if X cannot be written as a disjoint union of two non empty τ^* generalized β open sets.

Definition 8.2. A subsets of X is τ^* generalized β connected if it is τ^* generalized β connected as a subspace.

Theorem 8.3. Every τ^* generalized β connected space is connected.

Proof: Suppose X is τ^* generalized β connected space. Now, suppose X is not connected which implies that X can be written as $X = E \cup F$ where E and F are disjoint non empty open sets in X. By theorem, every open set is τ^* generalized β open. Therefore, $X = E \cup F$ where E and F are disjoint non empty τ^* generalized β open sets in X, which contradicts the fact that X is τ^* generalized β connected. Hence, X is connected.

Definition 8.4. A topological space (X, τ^*) is called $T_{\tau^*g\beta}$ if every τ^* generalized β closed set in X is closed in X.

Theorem 8.5. Suppose that X is a $T_{\tau^*g\beta}$ space. Suppose X is connected if and only if X is τ^* generalized β connected. Proof: Suppose that X is $T_{\tau^*g\beta}$ space and connected. Now consider X is not τ^* generalized β connected implies that X can be written as $X = E \cup F$ where E and F are non empty disjoint τ^* generalized β open sets in X. By assumption, X is $T_{\tau^*g\beta}$ topological space which implies that $X = E \cup F$ where E and F are disjoint non empty open sets in X, which contradicts the assumption X is connected. Hence, X is τ^* generalized β connected. Conversely, assumed X is τ^* generalized β connected. By theorem 7.3, hence X is connected.

Theorem 8.6. For a topological space X the following are equivalent.

1. X is τ^* generalized β connected;
2. X and $\emptyset$ are the only subsets of X which are both τ^* generalized β open and τ^* generalized β closed;
3. Each τ^* generalized β continuous map of X into a discrete space Y with at least two points is a constant map.

Proof: $(i) \to (ii)$ Let G be any τ^* generalized β open and τ^* generalized β closed subset of X. Then G^c is both τ^* generalized β open and τ^* generalized β closed. Since X is disjoint union of the τ^* generalized β open sets G and G^c implies from the hypothesis of (i) that either $G = \emptyset$ *or* $G = X$.

$(ii) \to (i)$ Suppose that $X = E \cup F$ where E and F are disjoint non empty τ^* generalized β open subsets of X. Then E is both τ^* generalized β open and τ^* generalized β closed. By assumption $E = \emptyset$ *or* X . Therefore X is τ^* generalized connected.

$(ii) \to (iii)$ Suppose $f : X \to Y$ is a τ^* generalized β continuous map. Therefore X is covered by τ^* generalized β open and τ^* generalized β closed covering $\{f^{-1}(a) : a \in Y\}$. By assumption $f^{-1}(a) = \emptyset$ *or* X for each $a \in Y$. If $f^{-1}(a) = \emptyset$ for all $y \in Y$, then f fails to be a map. Then there exists only one point $a \in Y$ such that $f^{-1}(a) \neq \emptyset$ and hence $f^{-1}(a) = X$. This shows that f is a constant map.

$(iii) \to (ii)$ Let G be both τ^* generalized β open and τ^* generalized β closed in X. Suppose $G \neq \emptyset$. Let $f : X \to Y$ be a τ^* generalized β continuous map defined by $f(G) = \{u\}$ and $f(GC) = \{v\}$ for some distinct points u and v in Y. Since, f is constant. Therefore we have $G = X$.

Theorem 8.7. If $f : X \to Y$ is a τ^* generalized β continuous surjection and X is a τ^* generalized β connected, then Y is connected.

Proof: Suppose that Y is not connected. Let $Y = E \cup F$ where E and F are disjoint non-empty open set in Y. Since f is τ^* generalized β continuous and onto, $X = f^{-1}(E) \cup f^{-1}(F)$ where $f^{-1}(E)$ and $f^{-1}(F)$ are disjoint non-empty τ^* generalized β open sets in X. This contradicts the fact that X is τ^* generalized β connected. Hence Y is connected.

Theorem 8.8. If $f : X \to Y$ is a τ^* generalized β irresolute surjection and X is τ^* generalized β connected, then Y is τ^* generalized β connected. Proof: Suppose that Y is not τ^* generalized β connected. Let $Y = E \cup F$

where E and F are disjoint non-empty τ^* generalized β open set in Y. Since f is τ^* generalized β irresolute and onto, $X = f^{-1}(E) \cup f^{-1}(F)$ where $f^{-1}(E)$ and $f^{-1}(F)$ are non-empty τ^* generalized β open sets in X. Now $X = f^{-1}(Y) = f^{-1}(E \cup F) = f^{-1}(E) \cup f^{-1}(F)$. Therefore X is a union of disjoint non-empty τ^* generalized β open set in X which contradicts the assumption X is τ^* generalized β connected. Hence Y is τ^* generalized β connected.

References

[1] N. Levine, Generalized closed sets in topology, Rend.Circ.Mat.Palermo 19,(2)(1970),89-96.

[2] W.Dunham, A New Closure Operator For Non-T_1 Topologies, Kyungpook Math.J.22(1982),55-60.

[3] D.Andrijevic, Semi-Pre open Sets, Mat.Vesnik, 38 (1986), 24-32.

[4] J. Dontchev, On generalizing semi pre open sets, Mem, Fac. Sci .Kochi Uni .Ser A ,Math.,16(1995), 35-48.

[5] Maki.H., Sundaram.P and Balachandran.K, On generalized homeomorphisms in topological spaces,Bull.Fukuoka Univ.Ed.III 40 (1991) 13-21.

[6] A.Pushpalatha, S.Eswaran and P.Raja Rubi, τ^*-generalized closed sets in topological spaces,WCE 2009, July 1-3, 2009, London, U.K.