International Journal of Advances in Mathematics
Volume 2019, Number 1, Pages 72-82, 2019
eISSN 2456-6098

Some fixed point theorems in dislocated quasi b-metric spaces

K.P.R. Sastry[1] , L.Vijaya kumar*
[1] 8-28-8/1, Tamil Street, Chinna Waltair, Visakhapatnam - 530 017, India.
* Faculty of Mathematics, Dr.B.R.Ambedkar University, Srikakulam-A.P - 530 003, India.
*E-mail: vijju.kumar64@gmail.com

ABSTRACT. In this paper, we establish the common fixed point theorems in dislocated quasi b-metric spaces.Incidentally we obtain results of Aage and Golhare as corollaries.

1. Introduction

Many generalization of metric space have been introduced[2,3,4]and were studied with reference to fixed point theorem. The concept of b-metric space was introduced by Bahtin[5] and was used by Czerick[6] to study contraction mapping in b-metric space. Fixed point theory is one of the most important topics in the department of non-linear analysis. Also fixed point theory is widely applicable in many branches of science.The concept of dislocated quasi b-metric space is introduced by F.M. Zeyada,G.H.Hasson and M.A.Ahmed[8] and they proved some fixed theorems on it.The purpose of this paper is to obtain some new fixed point theorems in dislocated quasi b-metric space. Also we will study the recent developments in dislocated metric spaces[17].

2. Section 2

We begin with some known definitions

Definition 2.1. *Frechet*[1]: A metric on a non empty set X is a function $D : X \times X \to [0, \infty)$ such that for all $x, y, z \in X$ the following conditions are hold:

(i) $D(x, x) = 0, \forall x \in X;$

(ii) $D(x, y) = 0 = D(y, x) \Longrightarrow x = y, \forall x, y \in X.$

* Corresponding Author.
Received September 19, 2018; Revised December 07, 2018; Accepted December 12, 2018.
2010 Mathematics Subject Classification: 47H110, 54H25.
Key words and phrases: Fixed point,dislocated metric,b-metric space,dislocated b-metric space,Banach Contraction, Kannan contraction.

(iii) $D(x,y) = D(y,x), \forall x,y \in X;$

(iv) $D(x,y) \le D(x,z) + D(z,y), \forall x,y,z \in X$. The pair (X,D) is called metric space.

Definition 2.2. $(Bakhtin$ [5]$)$: A metric on a non empty set X is a function $D : X \times X \to [0,\infty)$ such that for all $x,y,z \in X$ the following conditions are hold:

(i) $D(x,y) = 0 = D(y,x) \iff x = y, \forall x,y \in X;$

(ii) $D(x,y) = D(y,x), \forall x,y \in X;$

(iii) $D(x,y) \le D(x,z) + D(z,y), \forall x,y,z \in X$. The pair (X,D) is called dislocated metric space.

Definition 2.3. $(Frechet$[1]$)$: A metric on a non empty set X is a function $D : X \times X \to [0,\infty)$ such that for all $x,y,z \in X$ the following conditions are hold:

(i) $D(x,x) = 0, \forall x \in X$

(ii) $D(x,y) = 0 = D(y,x) \iff x = y, \forall x,y \in X;$

(iii) $D(x,y) \le D(x,z) + D(z,y), \forall x,y,z \in X$. The pair (X,D) is called quasi-metric space.

Definition 2.4. $(Bakhtin$ [5]$)$: Let X be a non empty set. Let $D : X \times X \to [0,\infty)$ be a mapping and $k \ge 1$ be a constant such that:

(i) $D(x,y) = 0 = D(y,x) \iff x = y, \forall x,y \in X;$

(ii) $D(x,y) = D(y,x), \forall x,y \in X;$

(ii) $D(x,y) \le k(D(x,z) + D(z,y)), \forall x,y,z \in X$. The pair (X,D) is called b-metric space.

Definition 2.5. $(Shah\ and\ Huassain$ [15]$)$: Let X be a non empty set. Let $D : X \times X \to [0,\infty)$ be a mapping and $k \ge 1$ be a constant such that:

(i) $D(x,y) = 0 = D(y,x) \iff x = y, \forall x,y \in X;$

(ii) $D(x,y) \le k(D(x,z) + D(z,y)), \forall x,y,z \in X$. The pair (X,D) is called quasi b-metric space.

Definition 2.6. $(Shah\ and\ Huassain$ [15]$)$: Let X be a non empty set. Let $D : X \times X \to [0,\infty)$ be a mapping and $k \ge 1$ be a constant such that:

(i) $D(x,y) = 0 \iff x = y, \forall x,y \in X;$

(ii) $D(x,y) = D(y,x), \forall x,y \in X;$

(iii) $D(x,y) \le k(D(x,z) + D(z,y)), \forall x,y,z \in X$. The pair (X,D) is called quasi b-metric like space.

Definition 2.7. $(Chakkrid\ and\ Cholatis$ [13]$)$: Let X be a non empty set. Let $D : X \times X \to [0,\infty)$ be a mapping and $k \ge 1$ be a constant such that:

(i) $D(x,y) = 0 = D(y,x) \iff x = y, \forall x,y \in X;$

(ii) $D(x,y) \le k(D(x,z) + D(z,y)), \forall x,y,z \in X$. The pair (X,D) is called dislocated quasi b-metric like space.

Note. The constant k is called coefficient of (X,D). It is clear that b-metric spaces, quasi b-metric spaces and b-metric like spaces are dqb-metric spaces but converse is not true.

Example. Let $X = R^+$ and for $p > 1, D : X \times X \to [0,\infty)$ defined by $D(x,y) = |x-y|^p + |x|^p, \forall x,y \in X$. Then (X,D) is dqb-metric space with $k = 2^p > 1$. But (X,D) is not b-metric like space and also not dislocated quasi metric space.

Definition 2.8. $(Chakkrid\ and\ Cholatis$ [13]$)$: Let (X,D) be a dqb-metric spaces. Let $\{x_n\}$ be a sequence in X and $x \in X$. We say that $\{x_n\}$ is $dqb-$converges to x if $\lim_{n\to\infty} D(x_n,x) = 0 = \lim_{n\to\infty} D(x,x_n)$. Here x is called a dqb-limit of x_n and written as $x_n \to x$.

Definition 2.9. $(Chakkrid\ and\ Cholatis$ [13]$)$: Let (X,D) be a dqb-metric spaces. Let $\{x_n\}$ be a sequence in X. We say that $\{x_n\}$ is $dqb-$Cauchy sequence if $\lim_{n,m\to\infty} D(x_n,x_m) = 0 = \lim_{n,m\to\infty} D(x_m,x_n)$.

Definition 2.10. (*Chakkrid and Cholatis* [13]): A dqb-metric spaces(X, D) is said to be dqb-complete if every dqb-Cauchy sequence in it is dqb-convergent in X.

Lemma 2.11. (*Chakkrid and Cholatis* [13]): Every subsequence of a dqb-convergent sequence in a dqb-metric space (X, D) is a dqb-convergent sequence.

Lemma 2.12. (*Chakkrid and Cholatis* [13]): Every subsequence of a dqb-Cauchy sequence in a dqb-metric space (X, D) is a dqb-convergent sequence.

Lemma 2.13. (*Chakkrid and Cholatis* [13]): If (X, D) is a dqb-metric space then a function $f : X \to X$ is continuous if and only if $x_n \to x \Longrightarrow fx_n \to fx$.

Lemma 2.14. (*Chakkrid and Cholatis* [13]): Let (X, D) be a dqb-metric space and $\{x_n\}$ be a sequence in it such that, $D(x_n, x_n + 1) \leq \alpha D(x_n - 1, x_n), n = 1, 2, 3....$ and $0 \leq \alpha < 1, \alpha \in [0, 1)$ where k is coefficient of (X, D), then $\{x_n\}$ is a Cauchy sequence in X.

Lemma 2.15. (*Chakkrid and Cholatis* [13]): If x is a limit point of some dqb-convergent sequence in a dqb-metric space (X, D) then $D(x, x) = 0$.

Lemma 2.16. (*Chakkrid and Cholatis* [13]): Every dqb-convergent sequence in a dqb-metric space (X, D) is dqb-Cauchy sequence.

Definition 2.17. (*Jungck and Rhoades* [18]): Let f and g be self maps of a set X. If $w = fx = gx$ for some $x \in X$ is called a coincidence point of f and g, and w is called a point of coincidence of f and g.

Definition 2.18. (*Jungck and Rhoades* [18]): Let f and g be self maps of a set X. Then f and g are said to be weakly compatible if they commute at their coincidence point.

Lemma 2.19. (*Abbas and Jungck*[19]): Let f and g are weakly compatible self maps of a set X. If f and g have unique point of coincidence $w = fx = gx$, then w is the unique common fixed point of f and g.

Definition 2.20. (*Jungck and Rhoades* [18]): Let $f : X \to X$ be a self mappings of a set X, f is said to be sub sequentially convergent if for every sequence $\{x_n,\}$ if fx_n is dqb-convergent then $\{x_n,\}$ has a dqb-convergent subsequence in X.

Definition 2.21. (*Jungck and Rhoades* [18]): Let $f : X \to X$ be a self mappings of a set X, f is said to be sequentially convergent if for every sequence $\{x_n,\}$ if fx_n is dqb-convergent then $\{x_n,\}$ is also dqb-convergent in X.

Definition 2.22. (*Samet and Vetro* [2o]): Let T be a self on a set X, and $\alpha : X \times X \to [0, \infty)$ be a function. We say that T is α-admissible mapping, if $x, y \in X$ then

$$\alpha(x, y) \geq 1 \Longrightarrow \alpha(Tx, Ty) \geq 1$$

Definition 2.23. (*Jungck and Rhoades* [18]): Let (X, D) be a dqb-metric space and $T, S : X \to X$ be two mappings then the mapping S is called $T-$Banach contraction if $\exists\ \alpha \in [0, 1) \ni D(TSx, TSy) \leq \alpha D(Tx, Ty) \forall x, y \in X$

Definition 2.24. (*Jungck and Rhoades* [18]): Let (X, D) be a dqb-metric space and $T, S : X \to X$ be two mappings then the mapping S is called $T-$Kannan contraction if $\exists\ \alpha \in [0, 1/2) \ni D(TSx, TSy) \leq \alpha[D(Tx, TSx) + D(Ty, TSy)], \forall x, y \in X$

3. Main Results

Theorem 3.1. *Let (X, D) be a dqb-complete metric space and $f, g : X \to X$ be self mappings satisfying the inequality $D(fx, fy) \leq \phi(D(gx, gy)), \forall x, y \in X$ where ϕ is altering distance function and $\phi(t) < \frac{t}{k}$ if $t > 0, k \geq 1$. If $f(X) \subseteq g(X)$*

and $g(X)$ is dqb-complete subspace of X, then f and g have unique point of coincidence in X. In addition if f and g are weakly compatible, then f and g have common unique fixed point in X.

Proof. Let x_0 be any arbitrary point in X. As $f(X) \subseteq g(X)$. Choose $x_1 \in X$ such that $fx_0 = gx_1$. Again Choose $x_2 \in X$ such that $fx_1 = gx_2$. continuing this procedure, we get a sequence $x_n \in X$ such that $fx_n = gx_n + 1$ for $n = 0, 1, 2......$ consider $D(gx_{n+1}, gx_n) = D(fx_n, fx_{n-1}) \leq \phi(D(gx_n, gx_{n-1})) < D(gx_n, gx_{n-1})$

$$Therefore,\ D(gx_{n+1}, gx_n) \leq \phi(D(gx_n, gx_{n-1})) \leq (\phi(\phi(D(gx_{n-1}, gx_{n-2}))) = \phi^2(D(gx_{n-1}, gx_{n-2}))$$

and repeatedly, then we get,

$$D(gx_{n+1}, gx_n) \leq \phi^n(D(gx_1, gx_0)) \leq \frac{1}{k^n} D(gx_1, gx_0)$$

Similarly $D(gx_n, gx_{n+1}) \leq \phi^n(D(gx_0, gx_1)) \leq \frac{1}{k^n} D(gx_0, gx_1)$

Now for $m, n \in N$ and $n > m, n = m + s$

Consider $D(gx_n, gx_m) = D(gx_{m+s}, gx_m)$

$$\leq k(D(gx_{m+s}, gx_{m+s-1}) + D(gx_{m+s-1}, gx_m))$$
$$\leq k\frac{1}{k^{m+s-1}}(D(gx_1, gx_0) + kD(gx_{m+s-1}, gx_m))$$

$$Therefore,\ D(gx_n, gx_m) = D(gx_{m+s}, gx_m) \leq (\frac{1}{k^{m+s-2}} + \frac{1}{k^{m+s-1}} + \frac{1}{k^{m+s}} + ... + \frac{1}{k^m} + \frac{1}{k^{m-1}})D(gx_1, gx_0)$$

take $\alpha = \frac{1}{k}$ then

$D(gx_n, gx_m) = D(gx_{m+s}, gx_m) \leq (\alpha^{m+s-2} + \alpha^{m+s-1} + ... + \alpha^m + \alpha^{m-1})D(gx_1, gx_0) \longrightarrow 0\ as\ m \longrightarrow \infty$

Similarly $D(gx_m, gx_n) \longrightarrow 0\ as\ m \longrightarrow \infty$

$$Therefore,\ \lim_{n,m\to\infty} D(x_n, x_m) = \lim_{n,m\to\infty} D(x_m, x_n) = 0$$

Hence $\{gx_n\}$ is a dqb-Cauchy sequence in X.

Since $g(X)$ is dqb-complete, $\exists v \in g(X) \ni gx_n \longrightarrow v\ as\ n \longrightarrow \infty$

Since $v \in g(X)$, we can find $u \in X \ni gu = v$

Now $D(gx_n, fu) = D(fx_{n-1}, fu) \leq \phi(D(gx_{n-1}, gu))$

Letting $n \longrightarrow \infty$ $\lim_{n\to\infty} D(gx_n, fu) \leq \phi \lim_{n\to\infty} D(gx_{n-1}, gu) = 0$

$\lim_{n\to\infty} D(gx_n, fu) = 0$

Similarly $\lim_{n\to\infty} D(fu, gx_n) = 0$

Hence we conclude that $gx_n \longrightarrow fu\ as\ n \longrightarrow \infty$ By uniqueness of limit in dqb-metric space,

we get $fu = gu$

$\Longrightarrow fu = gu$ is point of coincidence of f and g in X.

Now we claim that the point of coincidence of f and g in X is unique.

On the contrary we assume that $\exists w \in X \ni fw = gw$

Now $D(gu, gw) = d(fu, fw) \leq \phi D(gu, gw) < D(gu, gw)$

which is contradiction unless $D(gw, gu) = 0$

Similarly $D(gw, gu) = 0$

$Therefore,\ D(gu, gw) = D(gw, gu) = 0$

$gu = gw$ and point of coincidence of f and g in X is unique.

By Lemma [19] the mapping f and g have unique common fixed point in X. □

Corollary 3.2.(Aage and Golhare[17]) *Let (X, D) be a dqb-complete metric space and $f, g : X \to X$ be self mappings satisfying the inequality $D(fx, fy) \leq \phi(D(gx, gy)), \forall x, y \in X$ where $\alpha \in [0, 1)$ such that $\alpha k \leq 1$ and k is coefficient of (X, D). If $f(X) \subseteq g(X)$ and $g(X)$ is dqb-complete subspace of X, then f and g have unique point of coincidence in X. In addition if f and g are weakly compatible, then f and g have common unique fixed point in X.*

Lemma 3.3 *If $x_n \longrightarrow x$ then $D(x, x) = 0$.*

Proof. $D(x,x) \leq kD(x,x_n) + kD(x_n,x) \longrightarrow 0.$

$Therefore, \ D(x,x) = 0.$

Theorem 3.4. *Let (X,D) be a dqb-complete metric space and $f,g : X \to X$ be self mappings satisfying the inequality $D(fx,fy) \leq \phi(\frac{D(fx,gx)+D(fy,gy)}{2}), \forall x,y \in X$ where ϕ is altering distance function and $\phi(t) < \frac{t}{k}$, if $t > 0, k > 1$. If $f(X) \subseteq g(X)$ and $g(X)$ is dqb-complete subspace of X, then f and g have unique point of coincidence in X. In addition if f and g are weakly compatible, then f and g have common unique fixed point in X.*

Proof. Let x_0 be any arbitrary point in X. As $f(X) \subseteq g(X)$.
We can choose that $x_1 \in X \ni fx_0 = gx_1$
Again we can choose $x_2 \in X \ni fx_1 = fx_2$
repeating in the same manner, for $x_n \in X$
we can choose $x_{n+1} \in X \ni fx_n = gx_{n+1}$ for $n = 0,1,2.....$
Now consider $D(gx_{n+1}, gx_n) = D(fx_n, fx_{n-1})$

$$\leq \phi(\frac{D(fx_n,gx_n)+D(fx_{n-1},g_{n-1})}{2})$$
$$= \phi(\frac{D(gx_{n+1},gx_n)+D(gx_n,g_{n-1})}{2})$$
$$< (\frac{D(gx_{n+1},gx_n)+D(gx_n,g_{n-1})}{2k})$$

$Therefore, \ 2kD(gx_{n+1}, gx_n) \leq D(gx_{n+1}, gx_n) + D(gx_n, g_{n-1})$

$(2k-1)D(gx_{n+1}, gx_n) \leq D(gx_n, g_{n-1})$

$Therefore, \ D(gx_{n+1}, gx_n) \leq \frac{1}{2k-1}D(gx_n, g_{n-1}) \leq \frac{1}{k}D(gx_n, gx_{n-1})$

By lemma, $\{x_n\}$ is dqb-Cauchy sequence in X.
Since $g(X)$ is dqb-complete subspace of X.
So $\exists v \in g(X) \ni gx_n \longrightarrow v$ as $n \longrightarrow \infty$
Since $v \in g(X)$, we can find $u \in X \ni gu = v$.
Now $D(gx_n, fu) = D(fx_{n-1}, fu)$

$$\leq \phi(\frac{D(fx_{n-1},gx_{n-1})+D(fu,gu)}{2})$$
$$= \phi(\frac{D(gx_n,gx_{n-1})+D(fu,gu)}{2})$$

$Therefore, \ D(v, fu) \leq \phi(\frac{D(v,v)+D(fu,v)}{2})$

$$\leq \phi(\frac{D(fu,v)}{2}) < \frac{D(fu,v)}{2k}$$

Similarly $D(fu,v) \leq \frac{D(v,fu)}{2k}$

$Therefore, \ D(v, fu) < \frac{D(fu,v)}{2k} \leq \frac{D(v,fu)}{4k^2}$

Which is a contradiction always
$Therefore, \ D(v, fu) = 0$ and $D(fu, v) = 0$

$fu = v$

$gu = v$

$fu = gu = v$

$Therefore,$ This show that u is a point of coincidence of f and g in X.
Now we claim that this point of coincidence of f and g in X is unique.
On the contrary we assume that there exist $w \in X$ such that $fw = gw$
$D(gu, gw) = D(fu, fw) \leq \phi(\frac{D(fu,gu)+D(fw,gw)}{2})$

$$\leq \phi(\frac{D(v,v)+D(fw,gw)}{2})$$

$$\leq \frac{D(fw,gw)}{2k}$$

$\leq \frac{kD(fw,fu)+kD(fu,gw)}{2k}$

$D(gu,gw) \leq \frac{D(fw,fu)+D(fu,gw)}{2}$

$2D(gu,gw) \leq D(fw,fu) + D(fu,gw)$

$2D(gu,gw) \leq D(fw,fu) + D(gu,gw)$

$D(gu,gw) \leq D(fw,fu)$

$= D(fw,v) \longrightarrow 0$

$= D(gu,gw) = 0$

Similarly $D(gw,gu) = 0$

Thus $D(gu,gw) = D(gw,gu) = 0$

$\Longrightarrow gu = gw$ *and point of coincidence of f and g in X is unique.*

Hence by the proposition f and g unique common fixed point in X. □

Corollary 3.5.(Aage and Golhare[17]) *Let* (X,D) *be a dqb-complete metric space and* $f,g : X \to X$ *be self mappings satisfying the inequality* $D(fx,fy) \leq \alpha[D(fx,gx) + D(fy,gy)], \forall x,y \in X$ *where* $\alpha \in [0,1/2)$ *such that* $k\frac{\alpha}{1-\alpha} < 1$ *and k is coefficient of* (X,D). *If* $f(X) \subseteq g(X)$ *and* $g(X)$ *is dqb-complete subspace of X, then f and g have unique point of coincidence in X. In addition if f and g are weakly compatible, then f and g have common unique fixed point in X.*

Theorem 3.6. *Let* (X,D) *be a dqb-complete metric space with coefficients* $k > 1$. *Let* $f,T : X \longrightarrow X$ *be self-mappings satisfying T is one-one and T(X) is closed* $T\phi-$ *contraction then f has unique fixed point.*

Proof. *Let* $x_0 \in X$, *define* $\{x_n\}$ *by* $x_1 = fx_0, x_2 = fx_1, ...x_{n+1} = fx_n = f^n x_0 \quad n = 0,1,2.....$

Since f is $T\phi-$*contraction*

$D(y_{n+1},y_n) \leq \phi D(y_n,y_{n-1})$

$\leq \phi^2 D(y_{n-1},y_{n-2}) .... \leq \phi^n D(y_0,y_1)$

write $t = D(y_0,y_1)$

$D(y_{n+1},y_n) = \phi^n(t) < \frac{t}{k^n}$

$D(y_{n+1},y_n) \leq \frac{t}{k^n}$

$D(y_{n+2},y_n) \leq kD(y_{n+2},y_{n+1}) + kD(y_{n+1},y_n)$

$\leq k\phi D(y_{n+1},y_n) + kD(y_{n+1},y_n)$

$< k\,\phi\ \ \phi^{n+1}(t) + k\phi^n(t)$

$= k\,\phi^{n+2}(t) + k\phi^n(t)$

$< k\,\frac{t}{k^{n+1}} + k\,\frac{t}{k^n}$

$= t(\frac{1}{k^n} + \frac{t}{k^{n-1}})$

$Therefore, \ \ D(y_{n+2},y_n) \leq t(\frac{1}{k^n} + \frac{t}{k^{n-1}})$

$Therefore, \ \ D(y_{n+3},y_n) \leq kD(y_{n+3},y_{n+2}) + kD(y_{n+2},y_n)$

$\leq k\phi(D(y_{n+2},y_{n+1}) + kD(y_{n+2},y_n)$

$< k\frac{t}{k^{n+2}} + kt(\frac{1}{k^n} + \frac{1}{k^{n-1}})$

$= \frac{t}{k^{n+1}} + \frac{t}{k^{n-1}} + \frac{t}{k^{n-2}}$

$= t(\frac{1}{k^{n+1}} + \frac{1}{k^{n-1}} + \frac{1}{k^{n-2}})$

$D(y_{n+4},y_n) \leq kD(y_{n+4},y_{n+3}) + kD(y_{n+3},y_n)$

$< k\frac{t}{k^{n+3}} + t(\frac{1}{k^n} + \frac{1}{k^{n-2}} + \frac{1}{k^{n-3}})$

$= \frac{t}{k^{n+2}} + t(\frac{1}{k^n} + \frac{1}{k^{n-2}} + \frac{1}{k^{n-3}})$

$= t(\frac{1}{k^{n+2}} + \frac{1}{k^n} + \frac{1}{k^{n-2}} + \frac{1}{k^{n-3}})$

$D(y_{n+s},y_n) \leq t(\frac{1}{k^{n+s-2}} + \frac{1}{k^{n+s-4}} + \frac{1}{k^{n+s-6}} + \frac{1}{k^{n+s-7}})$

$$< t(\frac{1}{k^{n+s-2}} + \frac{1}{k^{n+s-3}} + \frac{1}{k^{n+s-4}} + ... + \frac{1}{k^{n-2}})$$

$$D(y_{n+s+1}, y_n) \leq kD(y_{n+s+1}, y_{n+s}) + kD(y_{n+s}, y_n)$$

$$< k\frac{t}{k^{n+s}} + t(\frac{1}{k^{n+s-2}} + \frac{1}{k^{n+s-3}} + ... + \frac{1}{k^{n-2}})$$

$$= t(\frac{1}{k^{n+s-1}} + \frac{1}{k^{n+s-2}} + ... + \frac{1}{k^{n-2}})$$

$Therefore,\ D(y_{n+s}, y_n) \longrightarrow 0\ as\ n \longrightarrow \infty$

$Therefore,\ D(y_n, y_{n+s}) \longrightarrow 0\ as\ n \longrightarrow \infty$

$\{y_n\}$ is dqb-Cauchy sequence in X.

so $\exists\, v \in X \ni y_n \longrightarrow v$

Suppose $T(X)$ is closed.

$Therefore,\ y_n \longrightarrow v \Longrightarrow \exists\, u \in X \ni Tu = v$

So that $Tf^n x_0 \longrightarrow Tu$

$$D(Tf^{n+1}x_0, Tfu) = D(Tf(f^n)x_0, Tfu)$$

$$\leq \phi(D(Tf^n x_0, Tu)) \quad \longrightarrow 0 \text{ as } n \longrightarrow \infty$$

$Therefore,\ D(Tf^{n+1}x_0, Tfu) \longrightarrow 0 \text{ as } n \longrightarrow \infty$

$Therefore,\ y_{n+1} \longrightarrow Tfu$

$Therefore,\ Tfu = v = Tu$

$Therefore,\ Tfu = Tu$

$Therefore,\ fu = u \quad (\because T \text{ is one} - \text{one})$

Now we prove that fixed point of f is unique.

Assume that $w \in X$ is another fixed point

$i.e fw = w$

Now $D(Tu, Tw) = D(Tfu, Tfw) \leq \phi(D(Tu, Tw))$

$$< D\frac{(Tu, Tw)}{k}$$

Which is a contradiction

$Tu = Tw$

$u = w(\because T \text{ is one} - \text{one})$ □

Corollary 3.7.(Aage and Golhare[17]) *Let (X, D) be a dqb-complete metric space with coefficients $k \leq 1$. Let $f, T : X \longrightarrow X$ be self-mappings such that T is continuous, one-one and f is continuous T− Banach contraction with $k\alpha \leq 1$. If T is dqb-sub-sequentially convergent then f has unique fixed point in X.*

Theorem 3.8. *Let (X, D) be a dqb-complete metric space with coefficients $k > 1$. Let $f, T : X \longrightarrow X$ be self-mappings satisfying T is one-one and $T(X)$ is closed T− Kannan contraction then f has unique fixed point.*

Proof.Let $x_0 \in X$ define $\{x_n\}$ by $x_1 = fx_0, x_2 = fx_1.......x_{n+1} = fx_n = f^{x_0} n = 0, 1, 2.....$

Since f is T−Kannan Contraction

Write $y_n = Tf^{x_0}$

$D(Tfx_0, Tf^2x_0) \leq \phi(max\{D(Tx_0, Tfx_0), D(Tfx_0, Tf^2x_0)\})$

$D(y_1, y_2) \leq \phi(max\{D(y_0, y_1), D(y_1, y_2)\})$

Suppose $max\{D(y_0, y_1), D(y_1, y_2)\} = D(y_1, y_2)$

$D(y_1, y_2) \leq \phi(D(y_1, y_2))$ *if* $D(y_0, y_1) \leq D(y_1, y_2)$

$D(y_1, y_2) < D(y_1, y_2)$ *if* $D(y_1, y_2) > 0$

Which is a contradiction

$D(y_1, y_2) = 0$

$max\{D(y_0, y_1), D(y_1, y_2)\} = D(y_1, y_2)$

$Therefore\ \ D(y_1, y_2)\} \leq \phi(D(y_0, y_1))$

$D(y_2, y_3) \leq \phi(max\{D(y_1, y_2), D(y_2, y_3)\})$

Suppose $max\{D(y_1, y_2), D(y_2, y_3)\} = D(y_2, y_3)$

$D(y_2, y_3) \leq \phi(D(y_2, y_3))$ *if* $D(y_1, y_2) \leq D(y_2, y_3)$

$D(y_2, y_3) < D(y_2, y_3)$ *if* $D(y_2, y_3) > 0$

Which is a contradiction

$D(y_2, y_3) = 0$

$max\{D(y_1, y_2), D(y_2, y_3)\} = D(y_1, y_2)$

Therefore $D(y_2, y_3)\} \leq \phi(D(y_1, y_2))$

$D(y_2, y_3)\} = \phi^2(D(y_0, y_1))$

In general $D(y_n, y_{n+1})\} = \phi^n(D(y_0, y_1)) \longrightarrow 0$ *as* $n \longrightarrow \infty$

For $m, n \in N, \quad n + s = m$

Write $t = D(y_0, y_1)$ *and* $\phi(t) = \frac{t}{k}$

$D(y_n, y_{n+1}) \leq \phi^n D(y_0, y_1) = \phi^n(t) < \frac{t}{k^n}$

$$\begin{aligned}
D(y_n, y_{n+2}) &\leq kD(y_n, y_{n+1}) + kD(y_{n+1}, y_{n+2})\\
&\leq k\phi^n D(y_0, y_1) + k\phi^{n+1} D(y_0, y_1)\\
&= k\phi^n(t) + k\phi^{n+1}(t)\\
&< k\frac{t}{k^n} + k\frac{t}{k^{n+1}}\\
&= t(\frac{1}{k^{n-1}} + \frac{1}{k^n})
\end{aligned}$$

$$\begin{aligned}
D(y_n, y_{n+3}) &\leq kD(y_n, y_{n+2}) + kD(y_{n+2}, y_{n+3})\\
&\leq k(k\phi^n(t) + k\phi^{n+1}(t) + k\phi^{n+2}(t))\\
&< k(k\frac{t}{k^n} + k\frac{t}{k^{n+1}} + k\frac{t}{k^{n+2}})\\
&= t(\frac{1}{k^{n-2}} + \frac{1}{k^{n-1}} + \frac{1}{k^{n+1}})\\
&< t(\frac{1}{k^{n-2}} + \frac{1}{k^{n-1}} + \frac{1}{k^n} + \frac{1}{k^{n+1}})
\end{aligned}$$

Therefore $D(y_n, y_{n+s}) \leq k^{s-1}(\phi^n(t) + \phi^{n+1}(t) + ... + \phi^{n+s-1}(t))$

$$\begin{aligned}
&< k^{s-1}(\frac{t}{k^n} + \frac{t}{k^{n+1}} + ... + \frac{t}{k^{n+s-1}})\\
&= k^{s-1}\frac{t}{k^n}(1 + \frac{1}{k} + \frac{1}{k^2} + ... + \frac{1}{k^{s-1}})\\
&\leq k^{s-1}\frac{t}{k^n}(1 - \frac{1}{k})^{-1} \qquad = \frac{t}{k^{n-s+1}}(1 - \frac{1}{k})^{-1} \longrightarrow 0 \text{ as } n \longrightarrow \infty
\end{aligned}$$

Therefore $\{y_n\}$ *is dqb-Cauchy sequence in* X.

Since (X, D) *is dqb-complete metric space in* X

so $\exists\, v \in X \ni y_n \longrightarrow v$

Suppose $T(X)$ *is closed therefore* $y_n \longrightarrow v \Longrightarrow \exists u \in X \ni tu = v$

So that $Tf^n x_0 \longrightarrow Tu$

$$\begin{aligned}
D(Tf^{n+1}x_0, Tfu) &= D(Tf(f^n x_0), Tfu)\\
&\leq \phi(max\{D(Tf^n x_0, Tf^{n+1}x_0), D(Tu, Tfu)\})\\
&\leq \phi(max\{D(Tf^n x_0, Tfu), D(Tu, Tfu)\})\\
&\leq \phi(max\{D(Tu, Tfu), D(Tu, Tfu)\})
\end{aligned}$$

$D(Tu, Tfu) \leq \phi(D(Tu, Tfu))$

Therefore $D(Tu, Tfu) = 0$

Therefore $Tu = Tfu$

Therefore $u = fu$ *(Since T is one-one)*

Now we prove that fixed point of f *is unique.*

Assume that $w \in X$ *is another fixed point*

$i.e fw = w$

Now $D(Tu, Tw) = D(Tfu, Tfw)$

$\leq \phi(max\{D(Tu, Tfu), D(Tw, Tfw)\})$
$= \phi(max\{D(Tu, Tu), D(Tw, Tw)\})$
$= 0$
Therefore $D(Tu, Tw) = 0$
Similarly $D(Tw, Tu) = 0$
Therefore $Tu = Tw$
Therefore $u = w$ *(since T is one-one)* □

Corollary 3.9.(Aage and Golhare[17]) *Let* (X, D) *be a dqb-complete metric space with coefficients* $k \geq 1$. *Let* $f, T : X \longrightarrow X$ *be self-mappings such that T is continuous, one-one and f is continuous T− Kannan contraction with* $k\alpha \leq 1$. *If T is dqb-sub-sequentially convergent then f has unique fixed point.*

Theorem 3.10. *Let* (X, D) *be a dqb-complete metric space with coefficients* $k > 1$. *Let* $f : X \longrightarrow X$ *be self-mapping satisfying* $D(fx, fy) \leq \phi\{D(x, y), D(fx, x), D(y, fy)\}\forall x, y \in X$ *where* ϕ *is altering distance function. Then f has unique fixed point in X.*

Proof. *Let* $x_0 \in X$ *Define* $\{x_n\}$ *by* $x_1 = fx_0, x_2 = fx_1, ...x_n = fx_{n-1}, x_{n+1} = fx_n.......$ *for* $n = 0, 1, 2.....$
Consider $D(x_1, x_2) = D(fx_0, fx_1) \leq \phi(max\{D(x_0, x_1), D(fx_0, x_0), D(x_1, fx_1)\})$
$= \phi(max\{D(x_0, x_1), D(x_1, x_0), D(x_1, x_2)\})$
$=\leq \phi(max\{D(x_0, x_1), D(x_1, x_0)\}) = \phi(\alpha_1)$ *where* $\alpha_1 = max\{D(x_0, x_1), D(x_1, x_0)\}$
$D(x_2, x_1) = D(fx_1, fx_0) \leq \phi(max\{D(x_1, x_0), D(fx_1, x_1), D(x_0, fx_0)\})$
$= \phi(max\{D(x_1, x_0), D(x_2, x_1), D(x_0, x_1)\})$
$\leq \phi(max\{D(x_1, x_0), D(x_0, x_1)\}) = \phi(\alpha_1)$ *where* $\alpha_1 = max\{D(x_0, x_1), D(x_1, x_0)\}$
$D(x_2, x_3) = D(fx_1, fx_2) \leq \phi(max\{D(x_1, x_2), D(fx_1, x_1), D(x_2, fx_2)\})$
$= \phi(max\{D(x_1, x_2), D(x_2, x_1), D(x_2, x_3)\})$
$\leq \phi(max\{D(x_1, x_2), D(x_2, x_1)\}) = \phi(\alpha_2)$ *where* $\alpha_2 = max\{D(x_1, x_2), D(x_2, x_1)\}$
similarly $D(x_3, x_2) \leq \phi(max\{D(x_2, x_1), D(x_1, x_2)\}) = \phi(\alpha_2)$ *where* $\alpha_2 = max\{D(x_1, x_2), D(x_2, x_1)\}$
Therefore $\alpha_2 \leq \phi(\alpha_1)$ *where* $\alpha_2 = max\{D(x_1, x_2), D(x_2, x_1)\}$
Therefore $\alpha_3 \leq \phi(\alpha_2) \leq \phi^2(\alpha_1)$
In general $\alpha_n \longrightarrow 0$ *as* $n \longrightarrow \infty$
Therefore $\alpha_{n+1} \leq \phi^n(\alpha_1) \longrightarrow$ *as* $n \longrightarrow \infty$
Therefore $\alpha_n \longrightarrow 0$ *as* $n \longrightarrow \infty$
$max\{D(x_{n-1}, x_n), D(x_n, x_{n-1})\} \longrightarrow 0$ *as* $n \longrightarrow \infty$
$D(x_{n-1}, x_n) \longrightarrow 0$ *as* $n \longrightarrow \infty$
$D(x_n, x_{n-1}) \longrightarrow 0$ *as* $n \longrightarrow \infty$
Consider for $m, n \in N,\ n > m,\ m = n + s$
Now $D(x_{n+1}, x_n) = D(fx_n, fx_{n-1}) \leq \phi(max\{D(x_n, x_{n-1}), D(fx_n, x_n), D(x_{n-1}, fx_{n-1})\})$
$= \phi(max\{D(x_n, x_{n-1}), D(x_{n+1}, x_n), D(x_{n-1}, x_n)\})$
$= \phi(max\{D(x_n, x_{n-1}), D(x_{n-1}, x_n)\})$
Now $D(x_n, x_{n+1}) = D(fx_{n-1}, fx_n) \leq \phi(max\{D(x_{n-1}, x_n), D(fx_{n-1}, x_{n-1}), D(x_n, fx_n)\})$

$= \phi(max\{D(x_{n-1}, x_n), D(x_n, x_{n-1}), D(x_n, x_{n+1})\})$
$= \phi(max\{D(x_{n-1}, x_n), D(x_n, x_{n-1})\})$
$D(x_{n+2}, x_n) \leq kD(x_{n+2}, x_{n+1}) + kD(x_{n+1}, x_n)$
$k\alpha_{n+1} + k\alpha_n$

$k\phi^n(\alpha_1) + k\phi^{n-1}(\alpha_1)$

$D(x_n, x_{n+2}) \leq kD(x_n, x_{n+1}) + kD(x_{n+1}, x_{n+2})$

$\leq k\alpha_n + k\alpha_{n+1}$

$\leq k\phi^{n-1}(\alpha_1) + k\phi^n(\alpha_1)$

$D(x_{n+3}, x_n) \leq kD(x_{n+3}, x_{n+2}) + kD(x_{n+2}, x_n)$

$\leq k\alpha_{n+2} + k(k\phi^n(\alpha_1) + k\phi^{n-1}(\alpha_1))$

$\leq k\phi^{n+1}(\alpha_1) + k^2\phi^n(\alpha_1) + k^2\phi^{n-1}(\alpha_1))$

$< k^2\phi^{n+1}(\alpha_1) + k^2\phi^n(\alpha_1) + k^2\phi^{n-1}(\alpha_1))$

$D(x_n, x_{n+3}) \leq kD(x_n, x_{n+2}) + kD(x_{n+2}, x_{n+3})$

$= k(k\phi^{n-1}(\alpha_1) + k\phi^n(\alpha_1)) + k\alpha_{n+2}$

$= k^2\phi^{n-1}(\alpha_1) + k^2\phi^n(\alpha_1) + k\phi^{n+1}(\alpha_1))$

$< k^2\phi^{n-1}(\alpha_1) + k^2\phi^n(\alpha_1) + k^2\phi^{n+1}(\alpha_1))$

$= k^2\frac{\alpha_1}{k^{n-1}} + k^2\frac{\alpha_1}{k^n} + k^2\frac{\alpha_1}{k^{n+1}}$

$= \frac{\alpha_1}{k^{n-3}} + \frac{\alpha_1}{k^{n-2}} + \frac{\alpha_1}{k^{n-1}}$

Therefore $D(x_{n+s}, x_n) \leq k^{s-1}\phi^{n-1}(\alpha_1) + k^{s-2}\phi^n(\alpha_1) + k^{s-3}\phi^{n+1}(\alpha_1)) + ... + k\phi^{n+s-3}(\alpha_1)$

Therefore $D(x_n, x_{n+s+1}) \leq k^s\phi^{n-1}(\alpha_1) + k^{s-1}\phi^n(\alpha_1) + k^{s-2}\phi^{n+1}(\alpha_1)) + ... + k\phi^{n+s-2}(\alpha_1)$

Therefore $D(x_{n+s}, x_n) \leq \alpha_1[\frac{1}{k^{(n-s)}} + \frac{1}{k^{(n-s)+1}} + \frac{1}{k^{(n-s)+2}} + ... + \frac{1}{k^{(n-2)}} + \frac{1}{k^{(n-1)}}] \longrightarrow 0$ *as* $n \longrightarrow \infty$

Therefore $\{y_n\}$ *is dqb-cauchy sequence in* X

Therefore $\exists v \in X \ni x_n \longrightarrow v$

Now $D(fx_n, fv) = D(x_{n+1}, fv) \leq \phi(max\{D(x_n, v), D(fx_n, v), D(v, fv)\})$

$= \phi(max\{D(x_n, v), D(x_{n+1}, v), D(v, fv)\})$

Letting $n \longrightarrow \infty$

$D(v, fv) \leq \phi(max\{D(v, v), D(v, v), D(v, fv)\})$

$= \phi(max\{D(v, v), D(v, fv)\})$

similarly $D(fv, v) \leq \phi(max\{D(v, v), D(v, v), D(fv, v)\})$

Since $x_n \longrightarrow v \Longrightarrow D(v, v) = 0$

Therefore $D(fv, v) = 0$

Therefore $D(v, fv) = 0$

Therefore $fv = v$

v *is a fixed point of* f

Now we prove that fixed point of f *is unique.*

Assume that $w \in X$ *is another fixed point*

i.e $fw = w$ *Now* $D(v, w) = D(fv, fw) = \phi(max\{D(v, w), D(fv, v), D(w, fw)\})$

$\leq \phi(max\{D(v, w), D(v, v), D(w, w)\})$

By lemma we get $D(v, w) = 0$

Similarly $D(w, v) = 0$

$v = w.$ □

Corollary 3.11.(Aage and Golhare[17]) *Let* (X, D) *be a dqb-complete metric space with coefficients* $k \geq 1$. *Let* $f : X \longrightarrow X$ *be self-mapping satisfying* $D(fx, fy) \leq \alpha\{D(x, y), D(fx, x), D(y, fy)\} \forall x, y \in X$ *where* $\alpha \in [0, 1)$ *such that* $k\alpha \leq 1$.*Then* f *has unique fixed point in X.*

Conclusion:*Finally this article can be concluded with the following observations. In[8], where the concept of dislocated quasi* $b-$ *metric space was introduced, many applications of this concept were presented and some theorems were also proved. The purpose of this paper to obtain some new fixed point theorems in dislocated quasi* $b-$ *metric space was fulfilled. Incidentally we have obtained the results of Aage and Golhare[17] as corollaries.*

Acknowledgements: *I am very much grateful to University authorities for giving me the necessary facilities to carry on this research work and also very much thankful to referees who have helped me in developing this paper.*

References

[1] *Frechet., Sur quelques points duo calcul fonctionel,. Rendiconti del Circolo Mahematicodi. Palermo. 22: 1-74, 1906.*

[2] *J.Liu, M.Song, common fixed point theorems for three maps under nonlinear contraction of cycle from in partially ordered G-metric spaces, Adv. Fixed Point Theory 5 (2015), 293-309.*

[3] *K.S. Eke, Some fixed and coincidence point results for expansive mapping on G-partial metric spaces,Adv. Fixed Point Theory 5 (2015), 369-386.*

[4] *Z. Mustafa, B. Simis, A new approach to a generalized metric spaces, J.Nonlinear convex Anal. 7 (2007),289-297.*

[5] *I.A. Bakhtin, The Contraction mapping principle in quasimetric spaces, In: Function Analysis, UIyanovsk(1989).*

[6] *K.P.R.Sastry, L.V.Kumar, P.K.Kumari,K.Sujatha, Common fixed point theorems for self maps on a partially ordered b-metric-like spaces,Bul of Math and Statistics Research,Vol.4. S1.2016;PP 161-168.ISSN:2348-0580.*

[7] *K.P.R.Sastry, L.V.Kumar, PS Kumar, Common fixed point theorems for F-Contractions on generalized metric spaces,Advance in Mathematics Vol.2018, Number 1, Pp 44-49, 2018.*

[8] *S.Czersik, Contraction mapping in b-metric spaces,ActMath:Infor, Univ,Ostrav.1(1993),5-11*

[9] *Frechet., Sur quelques points duo calcul fonctionel,. Rendiconti del Circolo Mahematicodi. Palermo. 22: 1-74, 1906.*

[10] *F.M.Zeyada, G H Hassan and M A Ahmed, A generalization of a fixed point theorem due to Hitzler and Seda in doslocated quasi-metric spaces, The Arabian J.Sci. Engg., 31(1A)(2006),111-114.*

[11] *C.T.Aage,J.N.Salunke, The results on fixed points in dislocated and dislocated quasi-metric space, Applied Mathematical Sciences, 59(2),(2008), 2941-2948.*

[12] *C.T.Aage,J.N.Salunke,Some results of fixed point theorems in dislocated metric spaces, Bulletin of the Marathwada Mathematical society,9(2),(2008),1-5.*

[13] *Chakkrid Klin-eam and Cholatis Suanoom, Dislocated quasi-b-metric spaces and fixed point theorem for cyclic contractions, Fixed point theory and applications, (2015) 2015:74,DOI 10.1186/s13663-015-0325-2.*

[14] *E Karapinar, P.Salimi, Dislocated metric space to metric space with some fixed point theorems, Fixed point theory Appl.2013, 222(2013).*

[15] *M H Saha and N Hussain, Nonlinear contraction in partially ordered quasi b-metric space, Commun. Korean Math. Soc., 27(1), (2012),117-128.*

[16] *L.Pasicki, Dislocated metric and fixed point theorems, Fixed point theory Appl., 82,(2015).*

[17] *CT Aage and PG Golhare, On fixed point theorems in dislocated quasi b-metric spaces,Advance in Mathematics Vol.2016, Number 1, Pp 55-70, 2016.*

[18] *G Jungck and B E Rhoads, Fixed point for set valued functions without continuity, Indian J.Pure. Appl.29(3),(1998),227-238.*

[19] *M Abbas and G Jungck, Common fixed point results for noncommuting mappings without continuity in cone metric spaces, J.Math. Anal. Appl. bf341(2008),416-420.*

[20] *B.Samet,C.Vetro and P.Vetro, Fixed point theorem for* $\alpha - \psi$ *contractive type mappings,Nonlinear Anal.,75(2012),2154-2165.*