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Common Fixed Point Theorems for W-Compatible maps of type (P) in Intuitionistic Generalized Fuzzy Metric Spaces

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ABSTRACT. In this paper we introduce the concept of W-compatible maps of type (P) in intuitionistic generalized fuzzy metric space. Also we proved common fixed point theorems for four self maps in intuitionistic generalized fuzzy metric spaces.

1 Introduction and Mathematical Preliminaries

The concept of fuzzy sets, introduced by Zadeh [17] plays an important role in topology and analysis. Since then, there are many authors studied the fuzzy sets with applications. Especially Kramosil and Michlek put forward a new concept of fuzzy metric space. George and Veeramani [4] revised the notion of fuzzy metric space with the help of continuous t-norm. As a result, many fixed point theorems for various forms of mappings are obtained in fuzzy metric space. Dhage [3] introduced the definition of D-metric space and proved many new fixed point theorems in D-metric space. Recently, Mustafa and Sims [7,8] presented a new definition of G-metric space and

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made great contribution to the development of Dhage theory.

In 2010, Guangpeng Sun and Yang [14] introduced the notion of Q- fuzzy metric space. In this study, we introduce the notion of intuitionistic generalized fuzzy metric space, which can be considered as a generalization of fuzzy metric space. We proved some new fixed point theorems in such intuitionistic generalized fuzzy metric spaces. The results presented in this paper improve and extend some known results.

2 Intuitionistic Generalized Fuzzy Metric Spaces

Definition 2.1. A binary operation $* : [0,1] \times [0,1] \to [0,1]$ is a continuous t-norm if it satisfies the following conditions:

1. $*$ is associative and commutative,
2. $*$ is continuous,
3. $a * 1 = a$ for all $a \in [0,1]$,
4. $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$, for each, $a, b, c, d \in [0,1]$.

Definition 2.2. A binary operation $\diamond : [0,1] \times [0,1] \to [0,1]$ is said to be a continuous t-conorm if it satisfies the following conditions:

1. $\diamond$ is associative and commutative,
2. $\diamond$ is continuous,
3. $a \diamond 0 = a$ for all $a \in [0,1]$,
4. $a \diamond b \leq c \diamond d$ whenever $a \leq c$ and $b \leq d$, for each, $a, b, c, d \in [0,1]$.

Definition 2.3. A 3-tuple $(X, Q, *)$ is called a Q- fuzzy metric space if X is an arbitrary (non-empty) set, $*$ is a continuous t-norm, and Q is a fuzzy set on $X^3 \times (0, \infty)$, satisfying the following conditions for each $x, y, z, a \in X$ and $t, s > 0$:

1. $Q(x, x, y, t) > 0$ and $Q(x, x, y, t) \leq Q(x, y, z, t)$ for all $x, y, z \in X$ with $z \neq y$.
2. $Q(x, y, z, t) = 1$ if and only if $x = y = z$.
3. $Q(x, y, z, t) = Q\{p(x, y, z\}, t)$, (symmetry) where p is a permutation function.
4. $Q(x, a, a, t) * Q(a, y, z, s) \leq Q(x, y, z, t + s)$
5. $Q(x, y, z, .) : (0, \infty) \to [0,1]$ is continuous.

A Q-fuzzy metric space is said to be symmetric if $Q(x, y, y, t) = Q(x, x.y, t)$ for all $x, y \in X$.

Definition 2.4. A 5-tuple $(X, Q, H, *, \diamond)$ is said to be intuitionistic generalized fuzzy metric space (Shortly IGFM) if X is an arbitrary non-empty set,$*$ is a continuous t-norm, $\diamond$ is a continuous, t-conorm, Q and H are fuzzy sets on $X^3 \times (0, \infty)$ satisfying the following conditions. For every $x, y, z, a \in X$ and $t, s > 0$.

1. $Q(x,y,z,t) + H(x,y,z,t) \leq 1$,
2. $Q(x,x,y,t) > 0$ for $x \neq y$,
3. $Q(x,x,y,t) \leq Q(x,y,z,t)$ for $y \neq z$,
4. $Q(x,y,z,t) = 1$ if and only if $x = y = z$,
5. $Q(x,y,z,t) = Q(p(x,y,z),t)$, where p is a permutation function,
6. $Q(x,a,a,t) * Q(a,y,z,s) \leq Q(x,y,z,t+s)$,
7. $Q(x,y,z,.) : (0,\infty) \to [0,1]$ is continuous,
8. Q is a non-decreasing of $\Re^+$, $\lim_{t\to\infty} Q(x,y,z,t) = 1$ and $\lim_{t\to 0} Q(x,y,z,t) = 0$ for all $x,y,z, \in X, t > 0$,
9. $H(x,x,y.t) < 1$ for $x \neq y$,
10. $H(x,x,y,t) \geq H(x,y,z,t)$ for $y \neq z$,
11. $H(x,y,z,t) = 0$ if and only if $x = y = z$,
12. $H(x,y,z,t) = H(p(x,y,z),t)$, where p is a permutation function,
13. $H(x,a,a,t) \diamond H(a,y,z,s) \geq H(x,y,z,t+s)$,
14. $H(x,y,z,.) : (0,\infty) \to [0,1]$ is continuous,
15. H is a non-increasing function on R^+, $\lim_{t\to\infty} H(x,y,z,t) = 0$ and $\lim_{t\to 0} H(x,y,z,t) = 1$ for all $x,y,z, \in X, t > 0$.

In this case, the pair (Q,H) is called an intuitionistic generalized fuzzy metric on X.

Remark 2.5. In intuitionistic generalized fuzzy metric space $Q(x,y,z,.)$ is non- decreasing and $H(x,y,z,.)$ is non-increasing for all $x,y,z \in X$.

Definition 2.6. Let $(X,Q,H,*,\diamond)$ be an intuitionistic generalized fuzzy metric space, then

1. A sequence $\{x_n\}$ in X is said to be convergent to x if $\lim_{n\to\infty} Q(x_n,x_n,x,t) = 1$ and $\lim_{n\to\infty} H(x_n,x_n,x,t) = 0$.
2. A sequence $\{x_n\}$ in X is said to be cauchy sequence if $\lim_{n,m\to\infty} Q(x_n,x_n,x_m,t) = 1$ and $\lim_{n,m\to\infty} H(x_n,x_n,x_m,t) = 0$ that is, for any $\epsilon > 0$ and for each $t > 0$, there exists $n_0 \in N$ such that $Q(x_n,x_n,x_m,t) > 1-\epsilon$ and $H(x_n,x_n,x_m,t) < \epsilon$ for $n,m \geq n_0$.
3. A intuitionistic generalized fuzzy metric space $(X,Q,H,*,\diamond)$ is said to be complete if every cauchy sequence in X is convergent.

Definition 2.7. Two self maps A and S of a intuitionistic generalized fuzzy metric space $(X,Q,H,*,\diamond)$ are called jointly W-continuous if there exists a point $x \in X$ such that if $\lim_{n\to\infty} Q(x_n,x,x,t) = 1$ and $\lim_{n\to\infty} H(x_n,x,x,t) = 0$ then $\lim_{n\to\infty} Q(Ax_n,Sx,Sx,t) = 1$ and
$\lim_{n\to\infty} H(Ax_n,Sx,Sx,t) = 0$ whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Ax_n =$
$\lim_{n\to\infty} Sx_n = p$ for some $p \in X$.

Definition 2.8. Two self maps A and S of a intuitionistic generalized fuzzy metric space (X, Q, H, $*, \diamond$) are called W-compatible of type (P) if

1. The W-continuity of S implies $\lim_{n\to\infty} Q(AAx_n, Sp, Sp, t) = 1$ and $\lim_{n\to\infty} H(AAx_n, Sp, Sp, t) = 0$.
2. The W-continuity of A implies $\lim_{n\to\infty} Q(SSx_n, Ap, Ap, t) = 1$ and $\lim_{n\to\infty} H(SSx_n, Ap, Ap, t) = 0$.
   whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = p$ for some $p \in X$.

Definition 2.9. Two self maps A and S of a intuitionistic generalized fuzzy metric space $(X, Q, H, *, \diamond)$. The pair $(A, S)$is said to be compatible if $\lim_{n\to\infty} Q(ASx_n, SAx_n, SAx_n, t) = 1$ and $\lim_{n\to\infty} H(ASx_n, SAx_n, SAx_n, t) = 0$, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = p$ for some $p \in X$

Proposition 2.10. *Let A and S be self maps of a intuitionistic generalized fuzzy metric space $(X, Q, H, *, \diamond)$ which are W-compatible of type (P). If A and S are $W-$continuous then they are compatible.*

Proof. Let $\{x_n\}$ be a sequence in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = p$, since A and S are W- compatible of type (P), then $\lim_{n\to\infty} Q(AAx_n, Sp, Sp, t) = 1$, $\lim_{n\to\infty} H(AAx_n, Sp, Sp, t) = 0$, $\lim_{n\to\infty} Q(SSx_n, Ap, Ap, t) = 1$ and $\lim_{n\to\infty} H(SSx_n, Ap, Ap, t) = 0$.

But, A is W- continuous and $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = p$. This implies $\lim_{n\to\infty} Q(AAx_n, Ap, Ap, t) = 1$, $\lim_{n\to\infty} H(AAx_n, Ap, Ap, t) = 0$. Now,

$$\begin{aligned} Q(Ap, Sp, Sp, t) &\geq Q(Ap, AAx_n, AAx_n, t/2) * Q(AAx_n, Sp, Sp, t/2) \\ &= Q(AAx_n, Ap, AAx_n, t/2) * Q(AAx_n, Sp, Sp, t/2) \\ &\geq Q(AAx_n, Ap, Ap, t/4) * Q(Ap, Ap, AAx_n, t/4) * Q(AAx_n, Sp, Sp, t/2) \end{aligned}$$

and

$$\begin{aligned} H(Ap, Sp, Sp, t) &\leq H(Ap, AAx_n, AAx_n, t/2) \diamond H(AAx_n, Sp, Sp, t/2) \\ &= H(AAx_n, Ap, AAx_n, t/2) \diamond H(AAx_n, Sp, Sp, t/2) \\ &\leq H(AAx_n, Ap, Ap, t/4) \diamond H(Ap, Ap, AAx_n, t/4) \diamond H(AAx_n, Sp, Sp, t/2) \end{aligned}$$

Taking limit as $n \to \infty$, we obtain $Q(Ap, Sp, Sp, t) = 1, H(Ap, Sp, Sp, t) = 0$

Also, W- continuity of A and S implies

$\lim_{n\to\infty} Q(ASx_n, Ap, Ap, t) = 1$, $\lim_{n\to\infty} Q(SAx_n, Sp, Sp, t) = 1$ and

$\lim_{n\to\infty} H(ASx_n, Ap, Ap, t) = 0$, $\lim_{n\to\infty} H(SAx_n, Sp, Sp, t) = 0$. Therefore,

$$\begin{aligned} Q(ASx_n, SAx_n, SAx_n, t) &\geq Q\left(ASx_n, Ap, Ap, \frac{t}{2}\right) * Q\left(Ap, SAx_n, SAx_n, \frac{t}{2}\right) \\ &\geq Q\left(ASx_n, Ap, Ap, \frac{t}{2}\right) * Q\left(Ap, Sp, Sp, \frac{t}{4}\right) * Q\left(Sp, SAx_n, SAx_n, \frac{t}{4}\right) \end{aligned}$$

and

$$\begin{aligned} H(ASx_n, SAx_n, SAx_n, t) &\leq H\left(ASx_n, Ap, Ap, \frac{t}{2}\right) \diamond H\left(Ap, SAx_n, SAx_n, \frac{t}{2}\right) \\ &\leq H\left(ASx_n, Ap, Ap, \frac{t}{2}\right) \diamond H\left(Ap, Sp, Sp, \frac{t}{4}\right) \diamond H\left(Sp, SAx_n, SAx_n, \frac{t}{4}\right) \end{aligned}$$

Taking limit on both sides we get, $\lim_{n\to\infty} Q(ASx_n, SAx_n, SAx_n, t) = 1$ and $\lim_{n\to\infty} H(ASx_n, SAx_n, SAx_n, t) = 0$. Thus A and S are W-Compatible. □

Proposition 2.11. *Let A and S be W-continuous self maps of a intuitionistic generalized fuzzy metric space $(X, Q, H, *, \diamond)$ which are W- compatible of type (P) and $Ap = Sp$ then $AAp = SAp = ASp = SSp$.*

Proof. Let $\{x_n\}$ be a sequence in X such that $\lim_{n\to\infty} x_n = p$ for $n \in N$. Since $\{A, S\}$ in W- compatible of type (P), A and S are W- continuous. Thus $\lim_{n\to\infty} Ax_n = Ap$ and $\lim_{n\to\infty} Sx_n = Sp$.
Let $Ap = Sp = z$. Thus $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z$. By the definition of W- compatibility we have, $\lim_{n\to\infty} Q(AAx_n, Sz, Sz, t) = 1$, $\lim_{n\to\infty} Q(SSx_n, Az, Az, t) = 1$ and
$\lim_{n\to\infty} H(AAx_n, Sz, Sz, t) = 0$, $\lim_{n\to\infty} H(SSx_n, Az, Az, t) = 0$.
Again from $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z$ and W- continuity of A,
we have $\lim_{n\to\infty} Q(AAx_n, Az, Az, t) = 1$, $\lim_{n\to\infty} H(AAx_n, Az, Az, t) = 0$.
But,

$$\begin{aligned} Q(Az, Sz, Sz, t) &\geq Q(Az, AAx_n, AAx_n, t/2) * Q(AAx_n, Sz, Sz, t/2) \text{ and} \\ H(Az, Sz, Sz, t) &\leq H(Az, AAx_n, AAx_n, t/2) \diamond H(AAx_n, Sz, Sz, t/2) \end{aligned}$$

Taking limit as $n \to \infty$, we get $Az = Sz$. Therefore $AAp = SAp = ASp = SSp$. □

Example 2.12. Let $X = [0, \infty)$ with the usual metric and $*$ be the continuous t-norm, $\diamond$ be the continuous t- conorm defined by, a $*b = min\{a, b\}$, $a \diamond b = max\{a, b\}$ for all $a, b \in [0, 1]$.
For each $t \in (0, \infty)$ and $x, y \in X$ defined Q, H by

$$Q = \begin{cases} \frac{t}{t+|x-y|+|y-z|+|z-x|}, & t > 0 \\ 0, & t = 0 \end{cases}$$

and

$$H = \begin{cases} \frac{|x-y|+|y-z|+|z-x|}{t+|x-y|+|y-z|+|z-x|}, & t > 0 \\ 1, & t = 0. \end{cases}$$

Define A and S by

$$Ax = \begin{cases} 1, & 0 \leq x \leq 1 \\ 1 + x, & 1 < x < \infty \end{cases}$$

$$Sx = \begin{cases} 1 + x^2, & 0 \leq x \leq 1 \\ 1, & 1 \leq x < \infty. \end{cases}$$

Let $\{x_n\} \subset [0, 1]$ be such that $x_n \to 0$, then $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = 1$. Also $A0 = S0 = 1$. hence, A and S are jointly W- continuous, also $Q(Ax_n, A0, A0, t) = \frac{t}{t+|Ax_n-A0|+|A0-A0|+|A0-Ax_n|}$ and $H(Ax_n, A0, A0, t) = \frac{|Ax_n-A0|+|A0-A0|+|A0-Ax_n|}{t+|Ax_n-A0|+|A0-A0|+|A0-Ax_n|}$. Therefore, $Q(Ax_n, A0, A0, t) \to 1$ and $H(Ax_n, A0, A0, t) \to 0$ as $n \to \infty$. Hence, $\lim_{n\to\infty} AAx_n = 1 = S1$, $\lim_{n\to\infty} SSx_n = A1 = 1$.
Thus, $\{A, S\}$ are compatible of type (P).

3 Main Results

Theorem 3.1. *Let A, B, S and T be self maps on a complete intuitionistic generalized fuzzy metric space $(X, Q, H, *, \diamond)$ such that*

1. *$AX \subseteq TX$, and $BX \subseteq SX$,*
2. *$\{A, S\}$ and $\{B, T\}$ are W- continuous,*
3. *$\{A, S\}$ and $\{B, T\}$ are W- compatible of type (P),*
4. *$Q(Ax, By, Bz, t) \geq \phi(\min\{Q(Sx, Ty, Tz, t), Q(Sx, Ax, Ay, t), Q(Ty, By, Bz, t)\})$ and $H(Ax, By, Bz, t) \leq \psi(\max\{H(Sx, Ty, Tz, t), H(Sx, Ax, Ay, t), H(Ty, By, Bz, t)\})$, where $\phi, \psi : [0, 1] \to [0, 1]$ is a continuous function such that $\phi(s) > s$ and $\psi(s) < s$ for each $0 < s < 1$, $\phi(0) = 0, \phi(1) = 1$, $\psi(0) = 0$ and $\psi(1) = 1$.*

   Then, A, B, S and T have unique common fixed point.

Proof. Let $x_0 \in X$ be an arbitrary point. Then by condition (1), there exists $x_1, x_2 \in X$ such that $Ax_0 = Tx_1$ and $Bx_1 = Sx_2$. Therefore we can construct a sequence $\{y_n\}$ by induction such that $y_{2n} = Ax_{2n} = Tx_{2n+1}$, and $y_{2n+1} = Bx_{2n+1} = Sx_{2n+2}$, for $n = 0, 1, 2 \cdots$.

But, from (4) we have,

$$\begin{aligned}
& Q(y_{2n+1}, y_{2n+2}, y_{2n+3}, t) = Q(Ax_{2n+1}, Bx_{2n+2}, Bx_{2n+3}, t) \\
\geq\ & \phi\left(\min\{Q(Sx_{2n+1}, Tx_{2n+2}, Tx_{2n+3}, t), Q(Sx_{2n+1}, Ax_{2n+1}, Ax_{2n+2}, t), Q(Tx_{2n+2}, Bx_{2n+2}, Bx_{2n+3}, t)\}\right) \\
=\ & \phi\left(\min\{Q(y_{2n}, y_{2n+1}, y_{2n+2}, t), Q(y_{2n}, y_{2n+1}, y_{2n+2}, t), Q(y_{2n+1}, y_{2n+2}, y_{2n+3}, t)\}\right) \\
=\ & \phi\left(\min\{Q(y_{2n}, y_{2n+1}, y_{2n+2}, t), Q(y_{2n+1}, y_{2n+2}, y_{2n+3}, t)\}\right)
\end{aligned}$$

and

$$\begin{aligned}
& H(y_{2n+1}, y_{2n+2}, y_{2n+3}, t) = H(Ax_{2n+1}, Bx_{2n+2}, Bx_{2n+3}, t) \\
& \leq \psi(\max\{H(Sx_{2n+1}, Tx_{2n+2}, Tx_{2n+3}, t), H(Sx_{2n+1}, Ax_{2n+1}, Ax_{2n+2}, t), \\
& \qquad H(Tx_{2n+2}, Bx_{2n+2}, Bx_{2n+3}, t)\}) \\
& = \psi(\max\{H(y_{2n}, y_{2n+1}, y_{2n+2}, t), H(y_{2n}, y_{2n+1}, y_{2n+2}, t), H(y_{2n+1}, y_{2n+2}, y_{2n+3}, t)\}) \\
& = \psi(\max\{H(y_{2n}, y_{2n+1}, y_{2n+2}, t), H(y_{2n+1}, y_{2n+2}, y_{2n+3}, t)\}).
\end{aligned}$$

Let $M = \min\{Q(y_{2n}, y_{2n+1}, y_{2n+2}, t), Q(y_{2n+1}, y_{2n+2}, y_{2n+3}, t)\}$ and $N = \max\{H(y_{2n}, y_{2n+1}, y_{2n+2}, t), H(y_{2n+1}, y_{2n+2}, y_{2n+3}, t)\}$.

Now, suppose $M = Q(y_{2n+1}, y_{2n+2}y_{2n+3}, t)$, and $N = H(y_{2n+1}, y_{2n+2}, y_{2n+3}, t)$, then,

$$\begin{aligned} Q(y_{2n+1}, y_{2n+2}, y_{2n+3}, t) &\geq \phi(Q(y_{2n+1}, y_{2n+2}, y_{2n+3}, t)) \\ &> Q(y_{2n+1}, y_{2n+2}, y_{2n+3}, t) \end{aligned}$$

and

$$\begin{aligned} H(y_{2n+1}, y_{2n+2}, y_{2n+3}, t) &\leq \psi(H(y_{2n+1}, y_{2n+2}, y_{2n+3}, t)) \\ &< H(y_{2n+1}, y_{2n+2}, y_{2n+3}, t) \end{aligned}$$

which is a contradiction. Hence $M = Q(y_{2n}, y_{2n+1}, y_{2n+2}, t)$ and $N = H(y_{2n}, y_{2n+1}, y_{2n+2}, t)$, for $n = 0, 1, 2 \cdots$. In general we have,

$$Q(y_{n+1}, y_{n+2}, y_{n+3}, t) > Q(y_n, y_{n+1}, y_{n+2}, t)$$

and

$$H(y_{n+1}, y_{n+2}, y_{n+3}, t) < H(y_n, y_{n+1}, y_{n+2}, t).$$

Thus, $\{Q(y_{n+1}, y_{n+2}, y_{n+3}, t)\}$ is an increasing sequence of positive real numbers in $[0, 1]$ and tends to $l \leq 1$. Now, if $l < 1$, then by taking the limit of

$$Q(y_{n+1}, y_{n+2}, y_{n+3}, t) \geq \phi(Q(y_n, y_{n+1}, y_{n+2}, t)) > Q(y_n, y_{n+1}, y_{n+2}, t)$$

We get $l \geq \phi(l) > l$ which contradicts the properties of ϕ, hence $l = 1$.

Also, $\{H(y_{n+1}, y_{n+2}, y_{n+3}, t)\}$ is a decreasing sequence of positive real numbers in $[0, 1]$ and tends to 0 as $n \to \infty$

Hence, we have $\lim_{n\to\infty} Q(y_n, y_{n+1}, y_{n+2}, t) = 1$ and $\lim_{n\to\infty} H(y_n, y_{n+1}, y_{n+2}, t) = 0$. Now for any positive integer p and for $t > 0$, we obtain, $\lim_{n\to\infty} Q(y_n, y_{n+p}, y_{n+p}, t) \geq 1 * 1 * ... * 1 = 1$ and $\lim_{n\to\infty} H(y_n, y_{n+p}, y_{n+p}, t) \leq 0 \diamond 0 \diamond ... \diamond 0 = 0$.

This implies that $\{y_n\}$ is a Cauchy sequence in X. Since X is complete, then there exist $u \in X$ such that $y_n \to u$. Therefore, $Ax_{2n} \to u$, and $Bx_{2n} \to u, Sx_{2n+2} \to u$, and $Tx_{2n+1} \to u$.

Given that, $\{A, S\}$ is W- compatible of type (P) and W- continuity of S implies that $\lim_{n\to\infty} Q(AAx_n, Au, Au, t) = 1$ and $\lim_{n\to\infty} H(AAx_n, Au, Au, , t) = 0$. Now we have $Ax_n \to u$, $Sx_n \to u$. Then W- continuity of A implies that $\lim_{n\to\infty} Q(AAx_n, Su, Su, t) = 1$ and $\lim_{n\to\infty} H(AAx_n, Su, Su, , t) = 0$.

Now, $Q(Au, Su, Su, t) \geq Q(Au, AAx_{2n}, AAx_{2n}, t/2) * Q(AAx_{2n}, Su, Su, t/2)$ by taking limit as $n \to \infty$ we get $Q(Au, Su, Su, t) \geq 1 * 1 = 1$. Also, we have

$$H(Au, Su, Su, t) \leq H(Au, AAx_{2n}, AAx_{2n}, t/2) \diamond H(AAx_{2n}, Su, Su, t/2)$$

by taking limit as $n \to \infty$ we get $H(Au, Su, Su, t) \leq 0 \diamond 0 = 0$.

Therefore, $Au = Su$. Since $\{B, T\}$ is W- compatible of type (P), then in similar manner we get $Bu = Tu$,.

Now we claim that $u = Au$. Suppose $u \neq Au$, then

$$\begin{aligned} Q(Au, y_{2n+2}, y_{2n+3}, t) &= Q(Au, Bx_{2n+2}, Bx_{2n+3}, t) \\ &\geq \phi(\min[Q(Su, Tx_{2n+2}, Tx_{2n+3}, t), Q(Su, Au, Ax_{2n+2}, t), Q(Tx_{2n+2}, Bx_{2n+2}, Bx_{2n+3}, t)]) \\ &= \phi(\min[Q(Au, Tx_{2n+2}, Tx_{2n+3}, t), Q(Au, Au, Ax_{2n+2}, t), Q(Tx_{2n+2}, Bx_{2n+2}, Bx_{2n+3}, t)]) \end{aligned}$$

letting $n \to \infty$, in the above equation we get,

$$Q(Au, u, u, t) \geq \phi(\min[Q(Au, u, u, t), Q(Au, u, u, t), Q(u, u, u, t)]). \tag{3.1}$$

Also,

$$\begin{aligned} H(Au, y_{2n+2}, y_{2n+3}, t) &= H(Au, Bx_{2n+2}, Bx_{2n+3}, t) \\ &\leq \psi(max[H(Su, Tx_{2n+2}, Tx_{2n+3}, t), H(Su, Au, Ax_{2n+2}, t), H(Tx_{2n+2}, Bx_{2n+2}, Bx_{2n+3}, t)]) \\ &= \psi(max[H(Au, Tx_{2n+2}, Tx_{2n+3}, t), H(Au, Au, Ax_{2n+2}, t), H(Tx_{2n+2}, Bx_{2n+2}, Bx_{2n+3}, t)]) \end{aligned}$$

letting $n \to \infty$, in the above equation we get,

$$H(Au, u, u, t) \leq \psi(max[H(Au, u, u, t), H(Au, u, u, t), H(u, u, u, t)]). \tag{3.2}$$

But from properties of ϕ and ψ we deduce that equations (3.1) and (3.2) becomes,

$$Q(Au, u, u, t) > Q(Au, u, u, t) \text{ and } H(Au, u, u, t) < H(Au, u, u, t)$$

a contradiction, hence we get $Au = Su = u$.

Now, we shall show that $u = Bu$. Suppose on the contrary $u \neq Bu$, then

$$\begin{aligned} Q(y_{2n+1}, Bu, Bu, t) &= Q(Ax_{2n+1}, Bu, Bu, t) \\ &\geq \phi(min[Q(Sx_{2n+1}, Tu, Tu, t), Q(Sx_{2n+1}, Ax_{2n+1}, Au, t), Q(Tu, Bu, Bu, t)]) \\ &= \phi(min[Q(Sx_{2n+1}, Bu, Bu, t), Q(Sx_{2n+1}, Ax_{2n+1}, Au, t), Q(Bu, Bu, Bu, t)]) \end{aligned}$$

Letting $n \to \infty$, we get

$$Q(u, Bu, Bu, t) \geq \phi\{min[Q(u, Bu, Bu, t), Q(u, u, u, t), 1]\} > Q(u, Bu, Bu, t).$$

Also,

$$\begin{aligned} H(y_{2n+1}, Bu, Bu, t) &= H(Ax_{2n+1}, Bu, Bu, t) \\ &\leq \phi(max[H(Sx_{2n+1}, Tu, Tu, t), H(Sx_{2n+1}, Ax_{2n+1}, Au, t), H(Tu, Bu, Bu, t)]) \\ &= \psi(max[H(Sx_{2n+1}, Bu, Bu, t), H(Sx_{2n+1}, Ax_{2n+1}, Au, t), H(Bu, Bu, Bu, t)]) \end{aligned}$$

Letting $n \to \infty$, we get

$$H(u, Bu, Bu, t) \leq \psi\{max[H(u, Bu, Bu, t), H(u, u, u, t), 0]\} < H(u, Bu, Bu, t)$$

a contraction. Thus $u = Bu$. Hence $Au = Bu = Su = Tu = u$.

Uniqueness:

let w be another common fixed point of A, B, S and T.

$$\begin{aligned} Q(u,w,w,t) &= Q(Au,Bw,Bw,t) \\ &\geq \phi(min[Q(Su,Tw,Tw,t),Q(Su,Au,Aw,t),Q(Tw,Bw,Bw,t)]) \\ &= \phi(min[Q(u,w,w,t),Q(u,u,w,t),Q(w,w,w,t)]) \\ &> Q(u,w,w,t) \end{aligned}$$

and

$$\begin{aligned} H(u,w,w,t) &= H(Au,Bw,Bw,t) \\ &\leq \psi(max[H(Su,Tw,Tw,t),H(Su,Au,Aw,t),H(Tw,Bw,Bw,t)]) \\ &= \psi(max[H(u,w,w,t),H(u,u,w,t),H(w,w,w,t)]) \\ &< H(u,w,w,t). \end{aligned}$$

a contradiction. Thus $u = w$. That is A, B, S and T have unique common fixed point. □

Corollary 3.2. *Let A, S and T be self maps on a complete intuitionistic generalized fuzzy metric space $(X, Q, H, *, \diamond)$ such that*

1. *$AX \subseteq TX$, and $AX \subseteq SX$,*
2. *$\{A, S\}$ and $\{A, T\}$ are W- continuous,*
3. *$\{A, S\}$ and $\{A, T\}$ are W- compatible of type (P),*
4. *$Q(Ax, Ay, Az, t) \geq \phi(min[Q(Sx, Ty, Tz, t), Q(Sx, Ax, Ay, t), Q(Ty, Ay, Az, t)])$ and $H(Ax, Ay, Az, t) \leq \psi(max[H(Sx, Ty, Tz, t), H(Sx, Ax, Ay, t), H(Ty, Ay, Az, t)])$ where $\phi, \psi : [0, 1] \to [0, 1]$ is a continuous function such that $\phi(s) > s, \psi(s) < s$ for each $0 < s < 1$. $\phi(0) = 0, \phi(1) = 1, \psi(0) = 0$ and $\psi(1) = 1$. Then A, B, S and T have unique common fixed point.*

Proof. The proof is obvious □

The following corollary is obtained by substituting $S = T$ in Theorem 3.1.

Corollary 3.3. *Let A, B and S be self maps on a complete intuitionisitc generalized fuzzy metric space $(X, Q, H, *, \diamond)$ such that*

1. *$AX \subseteq SX, \quad AX \subseteq SX$,*
2. *$\{A, S\}$ and $\{B, S\}$ are W- continuous,*
3. *$\{A, S\}$ and $\{B, S\}$ are W- compatible of type (P),*
4. *$Q(Ax, By, Bz, t) \geq \phi(min[Q(Sx, Sy, Sz, t), Q(Sx, Ay, Az, t), Q(Sy, By, Bz, t)])$ and $H(Ax, By, Bz, t) \leq \psi(max[H(Sx, Sy, Sz, t), H(Sx, Ay, Az, t), H(Sy, By, Bz, t)])$ where $\phi, \psi : [0, 1] \to [0, 1]$ is a continuous function such that $\phi(s) > s, \psi(s) < s$ for each $0 < s < 1$. $\phi(0) = 0, \phi(1) = 1, \psi(0) = 0, \psi(1) = 1$. Then A, B, S have unique common fixed point.*

Substitute $A = B$ and $S = T$ in Theorem 3.1. Then we obtain the following corollary.

Corollary 3.4. *Let A, B and S be self maps on a complete intuitionisitc generalized fuzzy metric space $(X, Q, H, *, \diamond)$ such that*

1. $AX \subseteq SX$,
2. $\{A, S\}$ *is W- continuous,*
3. $\{A, S\}$ *is W- compatible of type (P).*
4. $Q(Ax, Ay, Az, t) \geq \phi(min[Q(Sx, Sy, Sz, t), Q(Sx, Ax, Ay, t), Q(Sy, Ay, Az, t)])$ *and* $H(Ax, Ay, Az, t) \leq \psi(max[H(Sx, Sy, Sz, t), H(Sx, Ax, Ay, t), H(Sy, Ay, Az, t)])$ *where* $\phi, \psi : [0, 1] \to [0, 1]$ *is a continuous function such that* $\phi(s) > s, \psi(s) < s$ *for each* $0 < s < 1$, $\phi(0) = 0, \phi(1) = 1, \psi(0) = 0, \psi(1) = 1$. *Then A and S have unique common fixed point.*

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