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ON FUZZY SIMPLY* LINDELOF SPACES

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ABSTRACT. In this paper, the concept of fuzzy simply* Lindelof spaces is introduced and several characterizations of fuzzy simply* Lindelof spaces are given. The conditions under which fuzzy simply* Lindelof spaces become fuzzy simply Lindelof spaces, are obtained.

1. Introduction:

The concept of fuzzy sets was introduced by L.A. Zadeh [14]in 1965,as a new approach for modeling uncertainities. Since then this concept has invaded nearly all branches of Mathematics. C.L.Chang [4] introduced and developed the theory of fuzzy topological spaces in 1968.The paper of Chang paved the way for the subsequent tremendous growth of the numerous fuzzy topological concepts.

Among the various covering properties of fuzzy topological spaces, a lot of attention has been given to those covers which involve fuzzy open and fuzzy regular open sets. A.S.Bin Shahna[3] introduced the notion of fuzzy Lindelof spaces and investigated some of their properties. G.Thangaraj and G.Balasubramanian[6] introduced and studied fuzzy nearly Lindelof spaces, fuzzy almost Lindelof spaces,fuzzy weakly Lindelof spaces.The concept of fuzzy simply* open sets was introduced and studied by G.Thangaraj and K.Dinakaran[9]. The purpose of this paper is to introduce and study fuzzy simply* Lindelof spaces by means of fuzzy simply* open sets. Several characterizations of fuzzy simply* Lindelof spaces are established. Examples are given to illustrate the concepts introduced in this paper.

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2. Preliminaries:

In order to make the exposition self-contained, some basic notions and results used in the sequel are given. In this work by (X,T) or simply by X, we will denote a fuzzy topological space due to Chang (1968). Let X be a non-empty set and I, the unit interval [0,1]. A fuzzy set λ in X is a function from X into I. The null set 0 is the function from X into I which assumes only the value 0 and the whole fuzzy set 1 is the function from X into I which takes 1 only. The complement of a fuzzy set λ on X is given by $\lambda^c = 1 - \lambda$. The elementary operations on fuzzy sets on X, are given by

(i). $(\bigvee_{i \in I} \lambda_i)(x) = sup\{\lambda_i(x) : i \in J\}, \forall x \in X$

(ii). $(\bigwedge_{i \in I} \lambda_i)(x) = inf\{\lambda_i(x) : i \in J\}, \forall x \in X$, where j denotes an arbitrary index.

Definition 2.1. [4] Let (X,T) be a fuzzy topological space and λ be any fuzzy set in (X,T). The interior and the closure of λ are defined as follows:

(i) $int(\lambda) = \vee\{\mu/\mu \leq \lambda, \mu \in T\}$

(ii) $cl(\lambda) = \wedge\{\mu/\lambda \leq \mu, 1-\mu \in T\}$.

Lemma 2.1. [1] Let λ be any fuzzy set in a fuzzy topological space (X,T). Then $1 - cl(\lambda) = int(1-\lambda)$ and $1 - int(\lambda) = cl(1-\lambda)$.

Definition 2.2. [5] A fuzzy set λ in a fuzzy topological space (X,T), is called fuzzy dense if there exists no fuzzy closed set μ in (X,T) such that $\lambda < \mu < 1$. That is, $cl(\lambda) = 1$, in (X,T).

Definition 2.3. [5] A fuzzy set λ in a fuzzy topological space (X,T) is called fuzzy nowhere dense if there exists no non-zero fuzzy open set μ in (X,T) such that $\mu < cl(\lambda)$. That is, $int[cl(\lambda)] = 0$, in (X,T).

Definition 2.4. [5] A fuzzy set λ in a fuzzy topological space (X,T) is called a fuzzy first category set if $\lambda = \vee_{\alpha=1}^{\infty}\{\lambda_\alpha\}$ where $\{\lambda_\alpha\}$'s are fuzzy nowhere dense sets in (X,T). Any other fuzzy set in (X,T) is said to be of fuzzy second category.

Definition.2.5:[8] A fuzzy set λ in a fuzzy topological space (X,T) is called a fuzzy simply open set if $Bd(\lambda)$ is a fuzzy nowhere dense set in (X,T). That is, λ is a fuzzy simply open set in (X,T) if $intcl[cl(\lambda) \wedge cl(1-\lambda)] = 0$ in (X,T).

Definition 2.6.[9] A fuzzy set λ in a fuzzy topological space (X,T), is called a fuzzy simply* open set if $\lambda = \mu \vee \delta$, where μ is a fuzzy open set and δ is a fuzzy nowhere dense set in $(X,T)$and $1-\lambda$ is called a fuzzy simply* closed set in (X,T).

Definition.2.7:[3] A fuzzy topological space (X,T) is said to be fuzzy Lindelof if every fuzzy open cover of X has a countable subcover.That is, for every fuzzy open cover $\{\lambda_\alpha\}_{\alpha \in \Delta}$ of X,there exists a countable subcover $\{\lambda_{\alpha_n}\}_{n \in N}$ such that $\vee_{n \in N}\{\lambda_{\alpha_n}\} = 1$.

Definition.2.8:[11]: A fuzzy topological space (X,T) is said to be fuzzy simply Lindelof if each cover of X by fuzzy simply open sets has a countable subcover.That is, (X,T) is a fuzzy simply Lindelof space if $\bigvee_{\alpha \in \Delta}\{\lambda_\alpha\} = 1$, where $intcl[bd(\lambda_\alpha)] = 0$ in (X,T), then $\vee_{n \in N}\{\lambda_{\alpha_n}\} = 1$ in (X,T).

Definition 2.9. [1] A fuzzy set λ in a fuzzy topological space (X,T) is called

(i) a fuzzy semi-open set in (X,T) if $\lambda \leq clint(\lambda)$,

(ii) a fuzzy semi-closed set in (X,T) if $intcl(\lambda) \leq \lambda$.

Definition 2.10. [13] : A fuzzy topological space (X, T) is called a fuzzy globally disconnected space if each fuzzy semi open set in (X, T) is fuzzy open.That is, if $\lambda \leq clint(\lambda)$ for a fuzzy set λ defined on X, then $\lambda \in T$.

Definition 2.11. [5] A fuzzy topological space (X, T) is called a fuzzy first category space if $\vee_{\alpha=1}^{\infty}\{\lambda_\alpha\} = 1_X$, where $\{\lambda_\alpha\}$'s are fuzzy nowhere dense sets in (X,T).A fuzzy topological space which is not of fuzzy first category, is said to be of fuzzy second category.

Theorem 2.1. [11]: If (X, T) is a fuzzy simply Lindelof space, then (X,T) is a fuzzy second category space.

3. Fuzzy simply* Lindelof spaces:

Definition 3.1. A collection $\{\lambda_\alpha : \alpha \in \Delta\}$ of fuzzy simply* open sets of a fuzzy topological space (X,T) is said to be a fuzzy simply* cover of X if $\vee_{\alpha\in\Delta}\{\lambda_\alpha\} = 1$.

Definition 3.2. A fuzzy topological space (X, T) is said to be fuzzy simply* Lindelof if each cover of X by fuzzy simply* open sets has a countable subcover. That is, (X, T) is a fuzzy simply* Lindelof space if $\vee_{\alpha\in\Delta}\{\lambda_\alpha\} = 1$, where $\lambda_\alpha = \mu_\alpha \vee \delta_\alpha$, in which $\mu_\alpha \in T$ and $intcl(\delta_\alpha) = 0$ in(X, T), then there exists a countable subset $\{\alpha_n : n \in N\} \subseteq \Delta$ such that $\vee_{n\in N}\{\lambda_{\alpha_n}\} = 1$ in (X,T).

Example 3.1: Let $X = \{a, b, c\}$. The fuzzy sets λ,μ,δ α, β, γ, η, σ, and τ are defined on X, as follows:

$\lambda : X \to [0, 1]$ defined as $\lambda(a) = 1$; $\lambda(b) = 0.5$; $\lambda(c) = 0.6$;

$\mu : X \to [0, 1]$ defined as $\mu(a) = 0.5$; $\mu(b) = 1$; $\mu(c) = 0.4$;

$\delta : X \to [0, 1]$ defined as $\delta(a) = 0.4$; $\delta(b) = 0.6$; $\delta(c) = 1$;

$\alpha : X \to [0, 1]$ defined as $\alpha(a) = 0.5$; $\alpha(b) = 0.6$; $\alpha(c) = 0.6$;

$\beta : X \to [0, 1]$ defined as $\beta(a) = 0.6$; $\beta(b) = 1$, $\beta(c) = 0.4$;

$\gamma : X \to [0, 1]$ defined as $\gamma(a) = 1$; $\gamma(b) = 0.6$; $\gamma(c) = 0.6$;

$\eta : X \to [0, 1]$ defined as $\eta(a) = 0.6$; $\eta(b) = 1$; $\eta(c) = 1$;

$\sigma : X \to [0, 1]$ defined as $\sigma(a) = 0.6$; $\sigma(b) = 0.6$; $\sigma(c) = 0.4$;

$\tau : X \to [0, 1]$ defined as $\tau(a) = 0.6$; $\tau(b) = 0.6$; $\tau(c) = 0.6$;

Then $T = \{0, \lambda, \mu, \delta, \lambda \vee \mu, \lambda \vee \delta, \mu \vee \delta, \lambda \wedge \mu, \lambda \wedge \delta, \mu \wedge \delta, \lambda \vee (\mu \wedge \delta), \mu \vee (\lambda \wedge \delta), \delta \vee (\lambda \wedge \mu), \lambda \wedge (\mu \vee \delta), \mu \wedge (\lambda \vee \delta), \delta \wedge (\lambda \vee \mu), \lambda \wedge (\mu \wedge \delta), 1\}$ is a fuzzy topology on X. On computation, the fuzzy nowhere dense sets in (X, T), are $1 - \lambda, 1 - \mu, 1 - \delta, 1 - (\lambda \vee \mu), 1 - (\lambda \vee \delta), 1 - (\mu \vee \delta), 1 - (\mu \wedge \delta), 1 - [\lambda \vee (\mu \wedge \delta)], 1 - [\mu \vee (\lambda \wedge \delta)], 1 - [\delta \wedge (\lambda \vee \mu)], 1 - [\delta \vee (\lambda \wedge \mu)], 1 - [\mu \wedge (\lambda \vee \delta)], 1 - \beta, 1 - \gamma, 1 - \eta$, and $1 - \tau$.The fuzzy simply* open sets in (X, T) are $\lambda, \mu, \delta, \lambda \vee \mu, \lambda \vee \delta, \mu \vee \delta, \lambda \wedge \mu, \lambda \wedge \delta, \mu \wedge \delta, \lambda \vee (\mu \wedge \delta), \mu \vee (\lambda \wedge \delta), \delta \vee (\lambda \wedge \mu), \lambda \wedge (\mu \vee \delta), \mu \wedge (\lambda \vee \delta), \delta \wedge (\lambda \vee \mu), \lambda \wedge (\mu \wedge \delta), \alpha, \beta, \eta, \sigma, \tau, \delta \vee (1 - \delta), 1 - (\lambda \wedge \delta), 1 - (\lambda \wedge \mu \wedge \delta)$ and $\mu \vee [1 - (\mu \wedge \delta)]$, . Now $\lambda \vee \mu \vee \delta \vee [\lambda \vee (\mu \wedge \delta)] \vee [\mu \vee \delta] = 1$ and $[\lambda \vee \mu] \vee [\lambda \vee \delta] \vee [\mu \vee (\lambda \wedge \delta)] \vee [\delta \vee (\lambda \wedge \mu)] \vee \beta \vee \eta \vee [\delta \vee (1 - \delta)] \vee \{\mu \vee [1 - (\mu \wedge \delta)]\} = 1$. Thus, for the fuzzy simply* open covers $\{\lambda, \mu, \delta, \lambda \vee (\mu \wedge \delta), \mu \vee \delta\}$, and$\{\lambda \vee \mu, \lambda \vee \delta, \mu \vee (\lambda \wedge \delta), \delta \vee (\lambda \wedge \mu), \beta, \eta, \delta \vee (1 - \delta), \mu \vee [1 - (\mu \wedge \delta)]\}$ of X, there exist countable covers $\{\lambda, \mu \vee \delta\}$ and $\{\lambda \vee \delta, \eta\}$ for X, shows that(X, T) is a fuzzy simply* Lindelof space.

Remarks 3.1: Since each fuzzy open set is a fuzzy simply* open set in a fuzzy topological space, fuzzy Lindelof spaces are fuzzy simply* Lindelof spaces and fuzzy simply* Lindelof spaces need not be fuzzy simply Lindelof spaces. In example 3.1, (X,T) is a fuzzy simply* Lindelof space but not a fuzzy simply Lindelof space. The fuzzy simply open sets in (X,T) are $\lambda,\mu,\delta,\lambda\vee\mu,\lambda\vee\delta,\mu\vee\delta,\mu\wedge\delta,\lambda\vee(\mu\wedge\delta),\mu\vee(\lambda\wedge\delta),\delta\vee(\lambda\wedge\mu),\mu\wedge(\lambda\vee\delta),\delta\wedge(\lambda\vee\mu),\beta,\eta,\gamma$ and τ,. On computation one can see that $\beta\vee\gamma\vee\eta=1and\lambda\vee\beta\vee\eta\vee\tau\vee\mu\vee(\lambda\wedge\delta)\neq 1$.Thus, for the fuzzy simply open cover $\{\beta,\gamma,\eta\}$ of X,$\eta\vee\tau\neq 1$, shows that (X,T) is not a fuzzy simply Lindelof space.

Proposition 3.1: If $\bigwedge_{\alpha\in\Delta}\{\mu_\alpha\}=0$, where $\{\mu_\alpha\}$'s are fuzzy simply* closed sets in a fuzzy simply* Lindelof space (X,T), then there exists a countable subset $\{\alpha_n : n\in N\}\subseteq\Delta$ such that $\bigwedge_{n\in N}\{\mu_{\alpha_n}\}=0$ in (X,T).

Proof : Suppose that $\bigwedge_{\alpha\in\Delta}\{\mu_\alpha\}=0$, where $\{\mu_\alpha\}$'s are fuzzy simply* closed sets in (X,T). Now $1-\bigwedge_{\alpha\in\Delta}\{\mu_\alpha\}=1$, implies that $\bigvee_{\alpha\in\Delta}\{1-\mu_\alpha\}=1$. Since (X,T) is a fuzzy simply* Lindelof space and $\{1-\mu_\alpha\}$'s are fuzzy simply* open sets in (X,T), there exists a countable subcover $\{1-\mu_{\alpha_n}\}_{n\in N}$ of X by fuzzy simply* open sets in (X,T). That is, there exists a countable subset $\{\alpha_n : n\in N\}\subseteq\Delta$ such that $\vee_{n\in N}\{1-\mu_{\alpha_n}\}=1$, in (X,T). This implies that $1-\bigwedge_{n\in N}\{\mu_{\alpha_n}\}=1$ and thus $\bigwedge_{n\in N}\{\mu_{\alpha_n}\}=0$, in (X,T).

Proposition 3.2: If $\{\lambda_\alpha\}_{\alpha\in\Delta}$'s are fuzzy simply* open sets in a fuzzy simply* Lindelof space (X,T) such that $\bigvee_{\alpha\in\Delta}\{\lambda_\alpha\}=1$, then there exists a countable subset $\{\alpha_n : n\in N\}\subseteq\Delta$ such that $\bigwedge_{n\in N}\{1-\lambda_{\alpha_n}\}=0$,where $cl\{1-\lambda_{\alpha_n}\}\neq 1$ in (X,T).

Proof : Let (X,T) be a fuzzy simply* Lindelof space.Then for the cover $\{\lambda_\alpha\}_{\alpha\in\Delta}$ of X by fuzzy simply* open sets in (X,T), there exists a countable subcover $\{\lambda_{\alpha_n}\}_{n\in N}$ of X by fuzzy simply* open sets. That is, there exists a countable subset $\{\alpha_n : n\in N\}\subseteq\Delta$ such that $\vee_{n\in N}\{\lambda_{\alpha_n}\}=1$ and hence $\bigwedge_{n\in N}\{1-\lambda_{\alpha_n}\}=0$ in (X,T). Since $\{\lambda_{\alpha_n}\}$'s are fuzzy simply* open sets in (X,T),$\{\lambda_{\alpha_n}\}=\mu_{\alpha_n}\vee\delta_{\alpha_n}$, where $(\mu_{\alpha_n})\in T$ and $intcl(\delta_{\alpha_n})=0$ in(X,T). Now $int(\lambda_{\alpha_n})=int(\mu_{\alpha_n}\vee\delta_{\alpha_n})\geq int(\mu_{\alpha_n})\vee int(\delta_{\alpha_n})=(\mu_{\alpha_n})\vee 0=(\mu_{\alpha_n})\neq 0$ [since $int(\delta_{\alpha_n})\leq intcl(\delta_{\alpha_n})$ and $(\mu_{\alpha_n})\in T$, $int(\mu_{\alpha_n})=\mu_{\alpha_n}$].Hence $int(\lambda_{\alpha_n})\neq 0$. Then, $cl\{1-\lambda_{\alpha_n}\}=1-int(\lambda_{\alpha_n})\neq 1$,in (X,T). Thus $\bigwedge_{n\in N}(1-\lambda_{\alpha_n})=0$,where $cl(1-\lambda_{\alpha_n})\neq 1$ in (X,T).

Definition 3.3:[2]: A fuzzy topological space (X,T) is called fuzzy sub maximal if for each fuzzy set λ in (X,T) such that $cl(\lambda)=1$, then $\lambda\in T$

Proposition 3.3: If $\vee_{\alpha\in\Delta}\{\mu_\alpha\}=1$, where $\mu_\alpha=(\lambda_\alpha)\vee(1-\lambda_\alpha)$, in which$\{\lambda_\alpha\}$'s are fuzzy nowhere dense sets in a fuzzy sub maximal and fuzzy simply* Lindelof space (X,T), then there exists a fuzzy first category set γ and a fuzzy open set δ in (X,T) such that $\gamma\vee\delta=1$ in (X,T).

Proof: Suppose that $\vee_{\alpha\in\Delta}\{\mu_\alpha\}=1$, where $\mu_\alpha=(\lambda_\alpha)\vee(1-\lambda_\alpha)$, in which $\{\lambda_\alpha\}$'s are fuzzy nowhere dense sets in (X,T). Since (λ_α)'s are fuzzy nowhere dense sets, $intcl(\lambda_\alpha)=0$ in (X,T). But $int(\lambda_\alpha)\leq intcl(\lambda_\alpha)$ implies that $int(\lambda_\alpha)\leq 0$. That is,$int(\lambda_\alpha)=0$ in (X,T) and hence $cl\{1-\lambda_\alpha\}=1-int(\lambda_\alpha)=1-0=1$. Then, $\{1-\lambda_\alpha\}$'s are fuzzy dense sets in (X,T). Since (X,T) is a fuzzy sub maximal space, the fuzzy dense sets $\{1-\lambda_\alpha\}$'s are fuzzy open sets in (X,T). Hence $\mu_\alpha=(\lambda_\alpha)\vee(1-\lambda_\alpha)$, where $\{1-\lambda_\alpha\}\in T$and $\{\lambda_\alpha\}$'s are fuzzy nowhere dense sets in (X,T) implies that $\{\mu_\alpha\}$'s are fuzzy simply* open sets in (X,T). Since (X,T) is a fuzzy simply* Lindelof space and $\vee_{\alpha\in\Delta}\{\mu_\alpha\}=1$, there exists a countable subset $\{\alpha_n : n\in N\}\subseteq\Delta$ such that $\vee_{n\in N}\{\mu_{\alpha_n}\}=1$. Then $\vee_{n\in N}\{(\lambda_{\alpha_n})\vee(1-\lambda_{\alpha_n})\}=1$ in (X,T) and thus $\{\vee_{n\in N}(\lambda_{\alpha_n})\}\vee\{\vee_{n\in N}(1-\lambda_{\alpha_n})\}=1-----(1)$. Since $\{\lambda_{\alpha_n}\}$'s are fuzzy nowhere dense sets in (X,T), $\vee_{n\in N}(\lambda_{\alpha_n})$ is a fuzzy first category set and $\vee_{n\in N}(1-\lambda_{\alpha_n})$ is a fuzzy open set, in (X,T). Let $\gamma=\vee_{n\in N}(\lambda_{\alpha_n})$ and $\delta=\vee_{n\in N}(1-\lambda_{\alpha_n})$. Then, from(1), $\gamma\vee\delta=1$ in (X,T), where γ

is a fuzzy first category set and δ is a fuzzy open set, in (X,T).

Theorem 3.1:[9]: If λ is a fuzzy simply* open set in a fuzzy topological space (X,T), then $1-\lambda = \mu \wedge \delta$, where μ is a closed set and δ is a fuzzy dense set in (X,T).

Proposition 3.4: If (X,T) is a fuzzy simply* Lindelof space, then $\bigwedge_{n\in N}\{\mu_{\alpha_n} \wedge \delta_{\alpha_n}\} = 0$ where$\{\mu_{\alpha_n}\}$'s are fuzzy closed sets and $\{\delta_{\alpha_n}\}$'s are fuzzy dense sets in (X,T).

Proof : Let (X,T) be a fuzzy simply* Lindelof space. Then, for every cover$\{\lambda_\alpha\}_{\alpha\in\Delta}$ of X by fuzzy simply* open sets, there exists a countable subcover $\{\lambda_{\alpha_n}\}_{n\in N}$ of X by fuzzy simply* open sets. That is, $\vee_{\alpha\in\Delta}\{\lambda_\alpha\} = 1$implies that$\vee_{n\in N}\{\lambda_{\alpha_n}\} = 1$,where $\{\lambda_{\alpha_n}\}$'s are fuzzy simply* open sets in (X,T).Now $1-\vee_{n\in N}\{\lambda_{\alpha_n}\} = 0$, implies that $\bigwedge_{n\in N}\{1-\lambda_{\alpha_n}\} = 0$. By theorem 3.1,$1-\{\lambda_{\alpha_n}\} = \{\mu_{\alpha_n}\} \wedge \{\delta_{\alpha_n}\}$,where$\{\mu_{\alpha_n}\}$'s are fuzzy closed sets and $\{\delta_{\alpha_n}\}$'s are fuzzy dense sets in (X,T) and therefore $\bigwedge_{n\in N}\{\mu_{\alpha_n} \wedge \delta_{\alpha_n}\} = 0$, in (X,T).

The following example shows that a fuzzy simply open set need not be a fuzzy simply open set and a fuzzy simply* open set need not be a fuzzy simply open set in a fuzzy topological space.*

Example 3.2: Let $X = \{a,b,c\}$. The fuzzy setsλ,μ,δ and γ are defined on X, as follows:

$\lambda : X \to [0,1]$ defined as $\lambda(a) = 0.5;\quad \lambda(b) = 0.6;\quad \lambda(c) = 1;$

$\mu : X \to [0,1]$ defined as $\mu(a) = 1;\quad \mu(b) = 1;\quad \mu(c) = 0.5;$

$\delta : X \to [0,1]$ defined as $\delta(a) = 0.5;\quad \delta(b) = 0.5;\quad \delta(c) = 0.6;$

$\gamma : X \to [0,1]$ defined as $\gamma(a) = 0.6;\quad \gamma(b) = 0.6;\quad \gamma(c) = 1;$

Then $T = \{0,\lambda,\mu,\delta,\mu\vee\delta,\lambda\wedge\mu,\delta\vee(\lambda\wedge\mu),1\}$ is a fuzzy topology on X. On computation, the fuzzy nowhere dense sets in (X,T), are $1-\lambda, 1-\mu, 1-\delta, 1-\mu\vee\delta, 1-\lambda\wedge\mu, 1-\delta\vee(\lambda\wedge\mu)$ and $1-\gamma$. The fuzzy simply* open sets in (X,T) are $\lambda,\mu,\delta,\mu\vee\delta,\lambda\wedge\mu,\mu\wedge\delta,\delta\vee(\lambda\wedge\mu),\lambda\wedge(\mu\vee\delta),\mu\wedge(\lambda\vee\delta)$. Now, for the fuzzy simply* open cover $\{\lambda,\mu,\delta,\lambda\vee\delta,\mu\vee\delta,\lambda\wedge\mu,\lambda\wedge\delta,\mu\wedge\delta,\lambda\vee(\mu\wedge\delta),\delta\vee(\lambda\wedge\mu),\delta\vee(1-\lambda),\delta\vee[1-(\lambda\wedge\mu)],(\lambda\wedge\mu)\vee(1-\delta),(\mu\wedge\delta)\vee(1-\lambda),1-(\mu\wedge\delta),(\mu\wedge\delta)\vee[1-(\lambda\wedge\mu)]$ and $\{[\delta\vee(\lambda\wedge\mu)]\vee(1-\delta)\}$.

The fuzzy simply open sets in (X,T) are $\lambda,\mu,\delta,\mu\vee\delta,\lambda\wedge\mu,\delta\vee(\lambda\wedge\mu),\gamma$. Note that γ is a fuzzy simply open set but not a fuzzy simply* open set in (X,T) and also is not a fuzzy open set in (X,T). $\mu\wedge\delta$ is a fuzzy simply* open but not a fuzzy simply open set in (X,T).

Theorem 3.2:[8]: If λ is a fuzzy simply open set in a fuzzy topological space (X,T), then $[\lambda\wedge(1-\lambda)]$, is a fuzzy nowhere dense set in (X,T).

Proposition 3.5: If λ is a fuzzy simply open set in a fuzzy topological space (X,T), then $\mu\vee[\lambda\wedge(1-\lambda)]$, is a fuzzy simply* open set in (X,T), where $\mu \in T$.

Proof : Let λ be a fuzzy simply open set in (X,T). Then by theorem 3.2, $[\lambda\wedge(1-\lambda)]$, is a fuzzy nowhere dense set in (X,T). Let $\delta = \mu\vee[\lambda\wedge(1-\lambda)]$,where $\mu\in T$. Then δ is a fuzzy simply* open set in (X,T).

The following proposition gives a condition for obtaining fuzzy simply open set from a fuzzy simply open set in a fuzzy topological space.*

Proposition 3.6: If $\gamma = \mu\vee\lambda$ and $\delta = \mu\vee(1-\lambda)$,where $\mu\in T$ and λ is a fuzzy simply open set in a fuzzy topological space (X,T), then $\alpha\wedge\beta$ is a fuzzy simply* open set in (X,T).

Proof : Let $\gamma = \mu\vee\lambda$ and $\delta = \mu\vee(1-\lambda)$,where $\mu\in T$ and λ is a fuzzy simply open set in (X,T). Now $\gamma\wedge\delta = (\mu\vee\lambda)\wedge[\mu\vee(1-\lambda)] = \mu\vee[\lambda\wedge(1-\lambda)]$. Since λ is a fuzzy simply open set in (X,T), by theorem 3.3, $\lambda\wedge(1-\lambda)$ is a fuzzy nowhere dense set in (X,T). Then $\mu\vee[\lambda\wedge(1-\lambda)]$ is a fuzzy simply* open set in (X,T) and hence $\gamma\wedge\delta$ is a fuzzy simply* open set in (X,T).

Theorem 3.3:[9]: If λ is a fuzzy simply* open set in a fuzzy hyper connected space (X,T), then λ is a fuzzy simply open set in(X, T).

The following proposition gives a condition for fuzzy simply Lindelof spaces to become fuzzy simply Lindelof spaces by means of fuzzy hyper connectedness of fuzzy topological spaces.*

Proposition 3.7: If (X, T) is a fuzzy simply* Lindelof and fuzzy hyper connected space, then (X, T) is a fuzzy simply Lindelof space.

Proof : Let $\{\lambda_\alpha\}_{\alpha\in\Delta}$ be a cover of X by fuzzy simply* open sets in (X, T).Then, $\vee_{\alpha\in\Delta}\{\lambda_\alpha\} = 1$ in (X, T). Since (X,T) is a fuzzy hyper connected space, by theorem 3.3, the fuzzy simply* open sets $\{\lambda_\alpha\}$'s are fuzzy simply open sets in (X, T). Thus $\vee_{\alpha\in\Delta}\{\lambda_\alpha\} = 1$, implies that $\{\lambda_\alpha\}_{\alpha\in\Delta}$ is a cover of X by fuzzy simply open sets in (X, T). Since (X, T) is a fuzzy simply* Lindelof space,the cover $\{\lambda_\alpha\}_{\alpha\in\Delta}$ of X by fuzzy simply* open sets has a countable subcover $\{\lambda_{\alpha_n}\}_{n\in N}$.That is $\vee_{n\in N}\{\lambda_{\alpha_n}\} = 1$ in (X, T). Also since (X, T) is a fuzzy hyper connected space,the fuzzy simply* open sets $\{\lambda_{\alpha_n}\}_{n\in N}$'s are fuzzy simply open sets in (X,T).Thus the cover $\{\lambda_\alpha\}_{\alpha\in\Delta}$ of X by fuzzy simply open sets has a countable subcover $\{\lambda_{\alpha_n}\}_{n\in N}$ implies that (X, T) is a fuzzy simply Lindelof space.

Definition 3.4:[7]: A fuzzy topological space (X, T) is called fuzzy strongly irresolvable space if for each fuzzy dense set λ in (X, T), $clint(\lambda) = 1$,in (X, T)

Theorem 3.4:[9]: If λ is a fuzzy simply* open set in a fuzzy strongly irresolvable space (X, T),then λ is a fuzzy simply open set in(X, T).

The following proposition gives a condition for fuzzy simply Lindelof spaces to become fuzzy simply Lindelof spaces by means of fuzzy strong irresolvability of fuzzy topological spaces.*

Proposition 3.8: If (X, T) is a fuzzy simply* Lindelof and fuzzy strongly irresolvable space, then (X, T) is a fuzzy simply Lindelof space.

Proof : Let $\{\lambda_\alpha\}_{\alpha\in\Delta}$ be a cover of X by fuzzy simply* open sets in (X, T).Then $\vee_{\alpha\in\Delta}\{\lambda_\alpha\} = 1$, in (X, T). Since (X,T) is a fuzzy strongly irresolvable space, by theorem 3.4, the fuzzy simply* open sets $\{\lambda_\alpha\}$'s are fuzzy simply open sets in (X, T). Thus $\vee_{\alpha\in\Delta}\{\lambda_\alpha\} = 1$, implies that $\{\lambda_\alpha\}_{\alpha\in\Delta}$ is a cover of X by fuzzy simply open sets in (X, T). Since (X, T) is a fuzzy simply* Lindelof space,the cover $\{\lambda_\alpha\}_{\alpha\in\Delta}$ of X by fuzzy simply* open sets has a countable subcover $\{\lambda_{\alpha_n}\}_{n\in N}$.That is $\vee_{n\in N}\{\lambda_{\alpha_n}\} = 1$ in (X, T). Again since (X, T) is a fuzzy strongly irresolvable space, by theorem 3.4,the fuzzy simply* open sets $\{\lambda_{\alpha_n}\}$'s are fuzzy simply open sets in (X,T).Thus the cover $\{\lambda_\alpha\}_{\alpha\in\Delta}$ of X by fuzzy simply open sets has a countable subcover $\{\lambda_{\alpha_n}\}_{n\in N}$ implies that (X, T) is a fuzzy simply Lindelof space.

Definition 3.5:[12]:Let (X, T) be a fuzzy topological space. A fuzzy λ is called a fuzzy resolvable set if for each fuzzy closed μ in (X, T), $[c(\lambda \wedge \mu) \wedge cl(\mu \wedge (1 - \lambda))]$, is a fuzzy nowhere dense set in (X, T)

Theorem 3.5:[12]: If λ is a fuzzy simply open set in a fuzzy topological space (X, T), then λ is a fuzzy resolvable set in (X, T).

Proposition 3.9: If $\vee_{\alpha\in\Delta}\{\mu_\alpha\} = 1$, where $\{\mu_\alpha\}$'s are fuzzy simply open sets in a fuzzy hyper connected and fuzzy simply* Lindelof space,then $\vee_{n\in N}\{\mu_{\alpha_n}\} = 1$, where $\{\mu_{\alpha_n}\}$'s are fuzzy resolvable sets in (X, T).

Proof : Suppose that $\vee_{\alpha\in\Delta}\{\mu_\alpha\} = 1$, where $\{\mu_\alpha\}$'s are fuzzy simply open sets in (X, T). Since (X, T) is fuzzy simply*Lindelof and fuzzy hyper connected space, by proposition 3.5,(X, T) is a fuzzy simply Lindelof space and hence for the cover $\{\mu_\alpha\}_{\alpha\in\Delta}$of X by fuzzy simply open sets, there exists a countable subcover $\{\mu_{\alpha_n}\}_{n\in N}$. That is $\vee_{n\in N}\{\mu_{\alpha_n}\} = 1$ in (X, T). Since the fuzzy simply open sets are fuzzy resolvable sets in (X, T),[by theorem 3.5] $\vee_{n\in N}\{\mu_{\alpha_n}\} = 1$, where $\{\mu_{\alpha_n}\}$'s are fuzzy resolvable sets in (X,T).

Theorem 3.6:[13]: If λ is a fuzzy nowhere dense set in a fuzzy globally disconnected space (X, T),then λ is a fuzzy closed set in (X, T).

Proposition 3.10: If $\vee_{\alpha\in\Delta}\{(\lambda_\alpha) \vee (1 - \lambda_\alpha)\} = 1$, where $\{\lambda_\alpha\}$'s are fuzzy nowhere dense sets in a fuzzy globally disconnected and fuzzy simply* Lindelof space, then there exists a countable subset $\{\alpha_n : n \in N\} \subseteq \Delta$ such that $\vee_{n\in N}\{(\lambda_{\alpha_n}) \vee (1 - \lambda_{\alpha_n})\} = 1$ in (X, T).

Proof : Let $\{\lambda_\alpha\}$'s be fuzzy nowhere dense sets in (X,T). Since (X,T) is a fuzzy globally disconnected space, by theorem 3.7, $\{\lambda_\alpha\}$'s are fuzzy closed sets and hence $\{1-\lambda_\alpha\}$'s are fuzzy open sets in (X,T). Then $\{(\lambda_\alpha)\vee(1-\lambda_\alpha)\}$'s are fuzzy simply* open sets in (X,T).Since (X,T) is a fuzzy simply* Lindelof space and $\vee_{\alpha\in\Delta}\{(\lambda_\alpha)\vee(1-\lambda_\alpha)\}=1$,there exists a countable subset $\{\alpha_n : n\in N\}\subseteq\Delta$ such that $\vee_{n\in N}\{(\lambda_{\alpha_n})\vee(1-\lambda_{\alpha_n})\}=1$ in (X,T).

Proposition 3.11: If λ is a fuzzy simply* open set in a fuzzy topological space (X,T), then $\lambda=\mu\vee\delta$, where μ is a fuzzy semi open set and δ is a fuzzy semi closed set in (X,T).

Proof : If λ is a fuzzy simply* open set in (X,T), then $\lambda=\mu\vee\delta$, where μ is a fuzzy open set and δ is a fuzzy nowhere dense set in (X,T). Since every fuzzy open set is a fuzzy semi open set in a fuzzy topological space, μ is a fuzzy semi open set in (X,T). Also $intcl(\delta)=0$, implies that $intcl(\delta))\leq\delta$, and hence δ is a fuzzy semi closed set in (X,T). Hence $\lambda=\mu\vee\delta$, where μ is a fuzzy semi open set and δ is a fuzzy semi closed set in (X,T).

Proposition 3.12: If $\vee_{\alpha\in\Delta}\{\lambda_\alpha\}=1$, where $\{\lambda_\alpha\}$'s are fuzzy simply* open sets in a fuzzy simply* Lindelof space, then $\vee_{n\in N}\{(\mu_{\alpha n})\vee(\delta_{\alpha_n})\}$, where $\{\mu_{\alpha_n}\}$'s are fuzzy semi open sets and $\{\delta_{\alpha_n}\}$'s are fuzzy semi closed sets in (X,T).

Proof : Suppose that $\vee_{\alpha\in\Delta}\{\mu_\alpha\}=1$, where $\{\lambda_\alpha\}$'s are fuzzy simply* open sets in a fuzzy simply* Lindelof space (X,T). Then, there exists a countable subset $\{\alpha_n : n\in N\}\subseteq\Delta$ such that $\vee_{n\in N}\{\lambda_{\alpha_n}\}=1$. Since $\{\lambda_\alpha\}$'s are fuzzy simply* open sets (X,T), by proposition 3.11,$\lambda_{\alpha_n}=\mu_{\alpha_n}\vee\delta_{\alpha_n}$,where $\{\mu_{\alpha_n}\}$'s are fuzzy semi open sets and $\{\delta_{\alpha_n}\}$'s are fuzzy semi closed sets, in (X,T) and hence $\vee_{n\in N}\{\mu_{\alpha_n}\vee\delta_{\alpha_n}\}=1$ in (X,T).

Theorem 3.7:[10]: If a fuzzy simply open set λ is a fuzzy dense set in a fuzzy topological space (X,T),then λ is a fuzzy semi open set in (X,T).

Proposition 3.13: If $\vee_{\alpha\in\Delta}\{\lambda_\alpha\}=1$, where $\{\lambda_\alpha\}$'s are fuzzy simply open sets in a fuzzy simply* Lindelof and fuzzy strongly irresolvable space (X,T) and if the fuzzy simply open sets are fuzzy dense sets in (X,T), then there exists a countable subset $\{\alpha_n : n\in N\}\subseteq\Delta$ such that $\vee_{n\in N}\{\lambda_{\alpha n}\}=1$, where $\{\lambda_{\alpha_n}\}$'s are fuzzy semi open sets in (X,T)

Proof : Suppose that $\vee_{\alpha\in\Delta}\{\lambda_\alpha\}=1$, where $\{\lambda_\alpha\}$'s are fuzzy simply open sets in (X,T). Since (X,T) is a fuzzy simply* Lindelof and fuzzy strongly irresolvable space, there exists a countable subcover $\{\lambda_{\alpha n}\}_{n\in N}$ for X. That is $\vee_{n\in N}(\lambda_{\alpha_n})=1$. Since (X,T) is a fuzzy strongly irresolvable space, by theorem 3.4, $\{\lambda_{\alpha_n}\}$'s are fuzzy simply open sets in (X,T).By hypothesis, the fuzzy simply open sets $\{\lambda_{\alpha_n}\}$'s are fuzzy dense sets in (X,T).Then by theorem 3.7,$\{\lambda_{\alpha_n}\}$'s are fuzzy semi open sets in (X,T). Thus $\vee_{n\in N}(\lambda_{\alpha_n})=1$, where $\{\lambda_{\alpha_n}\}$'s are fuzzy semi open sets (X,T).

Proposition 3.14: If (X,T) is a fuzzy simply* Lindelof and fuzzy strongly irresolvable space, then (X,T) is a fuzzy second category space.

Proof : Let (X,T) be a fuzzy simply* Lindelof and fuzzy strongly irresolvable space.Then, by proposition 3.8 (X,T) is a fuzzy simply Lindelof space and hence by theorem 2.1,(X,T) is a fuzzy second category space.

References

[1] K. K. Azad, *On fuzzy semi-continuity,fuzzy almost continuity and fuzzy weakly continuity*, J. Math. Anal. Appl. 52 (1981), 14-32.

[2] G.Balasubramanian, *Maximal Fuzzy Topologies*, Kybernetika,31(5)(1995), 459 - 464.

[3] A.S.Bin shahna, *On fuzzy compactness and fuzzy Lindelofness* ,Bull. Calcutta Math.Soc. 83(1991),146 - 150.

[4] C. L. Chang, *Fuzzy topological spaces*, J. Math. Anal. Appl, 24 (1968), 182-190.

[5] G.Thangaraj and G.Balasubramanian, *On Somewhat Fuzzy Continuous Functions*, J. Fuzzy. Math, 11(2)(2003), 725-736.

[6] G.Thangaraj and G.Balasubramanian, *On Some Generalizations of Fuzzy Lindelof Spaces*, J. Fuzzy. Math, Vol.15,No. 3, (2007),1-7.

[7] G.Thangaraj and V.Seenivasan, *On Fuzzy Strongly Irresolvable Spaces*, Proc. Nat. Conf. on Fuzzy Math. and Graph Theory, Jamal Mohamed College, Trichy, Tamilnadu, India (2008), 1-7.

[8] G.Thangaraj and K.Dinakaran, *On Fuzzy Simply Continuous Functions*, J. Fuzzy. Math, Vol 25, No.4, (2016), 99-124.

[9] G.Thangaraj and K.Dinakaran, *On Fuzzy Simply* Continuous Functions*, Adv. Fuzzy. Math, Vol 11, No.2, (2016), 245-264.

[10] G.Thangaraj and K.Dinakaran , *on Fuzzy Weakly Simply Continuous Functions*,J.Tri.Math.,17(2016),1-22.

[11] G. Thangaraj and S.Dharmasaraswathi, *On Fuzzy simply Lindelof Spaces*, J. Fuzzy. Math, Vol.15,No. 3,(2017),1-7.

[12] G. Thangaraj, B.Mathivathani and P.Sathya , *On Fuzzy Resolvable sets and Fuzzy Resolvable Functions*, Adv. Fuzzy. Math,Vol 12,No.6,(2017),1171-1181 .

[13] G. Thangaraj and S.Muruganantham, *On Fuzzy Globally disconnected space*,(Communicated to J.Tripura Math. Soc., India).

[14] L. A. Zadeh, *Fuzzy sets*, Inform. and Control, 8 (1965), 338-353.