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Generalized fuzzy soft semi open sets in generalized fuzzy soft topological spaces

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ABSTRACT. In the present paper, we continue the study on generalized fuzzy soft topological spaces and investigate the properties of generalized fuzzy soft semi open (closed) sets and generalized fuzzy soft semi interior (closure) sets. Also, we introduce generalized fuzzy soft semi continuous mappings and generalized fuzzy soft semi open (closed) mappings which are important for further research on generalized fuzzy soft topology.

1 Introduction

Many Mathematical concepts can be represented by the notion of set theory, which dichotomize the situation into the conditions: either yes or no. Till 1965, Mathematicians were concerned only about well-defined things, and smartly avoided any other possibility which are more realistic in nature. For instance the set of tall persons in a room, the set of hot days in a year etc. In the year 1965, Prof. L.A. Zadeh [19] introduced fuzzy set to accommodate real life situations by giving partial membership to each element of a situation under consideration. Keeping in view that fuzzy set theory lacks the parametrization tool. Chang [4] introduced the concept of fuzzy topology on a set X by axiomatizing a collection T of fuzzy subsets of X. The concept of soft sets was first introduced by Molodtsov [13] in 1999 as a general mathematical tool for dealing with uncertain objects. In [13, 14], Molodtsov

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successfully applied the soft theory in several directions, such as smoothness of functions, game theory, operations research, Riemann integration, Perron integration, probability, theory of measurement, and so on. After presentation of the operations of soft sets [11], the properties and applications of soft set theory have been studied increasingly [1, 9, 14]. C. agman et al. [2] and Shabir et al. [17] introduced soft topological space independently in 2011. Maji et al. [10] introduced the concept of fuzzy soft set and some properties regarding fuzzy soft union, intersection, complement of a fuzzy soft set, De Morgan Law etc. Tanay et al.[18] introduced the defnition of fuzzy soft topology over a subset of the initial universe set. Later, Roy and Samanta [16] gave the denition of fuzzy soft topology over the initial universe set. In [5], Karal and Ahmed defined the notion of a mapping on classes of fuzzy soft sets, which is fundamental important in fuzzy soft set theory, to improve this work and they studied properties of fuzzy soft images and fuzzy soft inverse image s of fuzzy soft sets. Kandil et al. [6] introduce the notion of fuzzy semi open soft sets, fuzzy soft semi closed sets, fuzzy soft semi interior, fuzzy soft semi closure and fuzzy soft semi separation axioms. Majumdar and Samanta [12] introduced generalized fuzzy soft sets and successfully applied their notion in a decision making problem. Chakraborty and Mukherjee [3] introduced generalized fuzzy soft union, intersection, De Morgan Law and several other properties of generalized fuzzy soft sets. Also, they introduced the topological structure, closure, interior of generalized fuzzy soft sets. and studied some of their properties. Khedr et al. [7] introduced the concept of a generalized fuzzy soft point, generalized fuzzy soft nbd of a point, generalized fuzzy soft base (subbase), generalized fuzzy soft subspace. Khedr et al. [8] introduced the concept of a generalized fuzzy soft mapping on families of generalized fuzzy soft sets. Also they introduced a generalized fuzzy soft continuous mapping and generalized fuzzy soft open(closed, homeomorphism) for generalized fuzzy soft topological spaces, and studied its basic properties. In the present paper, we introduce some new concepts in generalized fuzzy soft topological spaces such as generalized fuzzy semi soft open(closed) sets and generalized fuzzy soft semi interior(closure). In particular, we study the relationship between generalized fuzzy soft semi interior and generalized fuzzy soft semi closure. Also, we introduce the notion of generalized fuzzy soft semi function in generalized fuzzy soft topological spaces and study its basic properties.

2 Preliminaries

First we recall basic definitions and results.

Definition 2.1. *[19] Let X be a non-empty set. A fuzzy set A in X is defined by a membership function* $\mu_A : X \to [0, 1]$ *whose value* $\mu_A(x)$ *represents the 'grade of membership' of x in A for* $x \in X$*. The set of all fuzzy sets in a set X is denoted by* I^X*, where I is the closed unit interval* $[0, 1]$.

Definition 2.2. *[13] Let X be an initial universe set and E be a set of parameters. Let* $P(X)$ *denotes the power set of X and* $A \subseteq E$*. A pair* (f, A) *is called a soft set over X, if f is a mapping from A into* $P(X)$*, i.e.,* $f : A \to P(X)$*. In other words, a soft set is a parameterized family of subsets of the set X. For* $e \in A$*,* $f(e)$ *may be considered as the set of* $e-$*approximate elements of the soft set* (f, A).

Definition 2.3. *[16] Let X be an initial universe set and E be a set of parameters. Let $A \subseteq E$. A fuzzy soft set f_A over X is a mapping from E to I^X, i.e., $f_A : E \longrightarrow I^X$, where $f_A(e) \neq \bar{0}$ if $e \in A \subset E$, and $f_A(e) = \bar{0}$ if $e \notin A$, where $\bar{0}$ is denotes empty fuzzy set in X.*

Definition 2.4. *[12] Let X be a universal set of elements and E be a universal set of parameters for X. Let $F : E \longrightarrow I^X$ and μ be a fuzzy subset of E, i.e., $\mu : E \longrightarrow I$. Let F_μ be the mapping $F_\mu : E \longrightarrow I^X \times I$ defined as follows: $F_\mu(e) = (F(e), \mu(e))$, where $F(e) \in I^X$ and $\mu(e) \in I$. Then F_μ is called a generalised fuzzy soft set (GFSS in short) over (X, E). The family of all generalized fuzzy soft sets (GFSSs in short) over (X, E) is denotes by $GFSS(X, E)$.*

Definition 2.5. *[12] Let F_μ and G_δ be two GFSSs over (X, E). F_μ is said to be a GFS subset of G_δ, denoted by $F_\mu \sqsubseteq G_\delta$, if*

(1) μ is a fuzzy subset of δ,

(2) $F(e)$ is also a fuzzy subset of $G(e), \forall e \in E$.

Definition 2.6. *[12] Let F_μ be a GFSS over (X, E). The generalized fuzzy soft complement of F_μ, denoted by F_μ^c, is defined by F_μ^c=G_δ, where $\delta(e) = \mu^c(e)$ and $G(e) = F^c(e)\, \forall e \in E$. Obviously $(F_\mu^c)^c$=F_μ.*

Definition 2.7. *[3] Let F_μ and G_δ be two GFSSs over (X, E). The union of F_μ and G_δ, denoted by $F_\mu \sqcup G_\delta$, is The GFSSH_ν, defined as $H_\nu : E \longrightarrow I^X \times I$ such that $H_\nu(e) = (H(e), \nu(e))$, where $H(e) = F(e) \vee G(e)$ and $\nu(e) = \mu(e) \vee \delta(e)$, $\forall e \in E$.*

Definition 2.8. *[3] Let F_μ and G_δ be two GFSSs over (X, E). The Intersection of F_μ and G_δ, denoted by $F_\mu \sqcap G_\delta$, is the GFSS M_σ, defined as $M_\sigma : E \longrightarrow I^X \times I$ such that $M_\sigma(e) = (M(e), \sigma(e))$, where $M(e) = F(e) \wedge G(e)$ and $\sigma(e) = \mu(e) \wedge \delta(e), \forall e \in E$.*

Theorem 2.9. *[3] Let $\{(F_\mu)_\lambda, \lambda \in \Lambda\} \sqsubseteq GFSS(X, E)$. Then the following statements hold:*

(1) $[\sqcup_{\lambda\in\Lambda}(F_\mu)_\lambda]^c = \sqcap_{\lambda\in\Lambda}(F_\mu)_\lambda^c$,

(2) $[\sqcap_{\lambda\in\Lambda}(F_\mu)_\lambda]^c = \sqcup_{\lambda\in\Lambda}(F_\mu)_\lambda^c$.

Definition 2.10. *[12] AGFSS is said to be a generalized null fuzzy soft set, denoted by $\widetilde{0}_\theta$, if $\widetilde{0}_\theta : E \longrightarrow I^X \times I$ such that $\widetilde{0}_\theta(e) = (\widetilde{0}(e), \theta(e))$ where $\widetilde{0}(e) = \overline{0}\ \forall e \in E$ and $\theta(e) = 0\, \forall e \in E$ (Where $\overline{0}(x) = 0,\ \forall x \in X$).*

Definition 2.11. *[12] A GFSS is said to be a generalized absolute fuzzy soft set, denoted by $\widetilde{1}_\triangle$, if $\widetilde{1}_\triangle : E \longrightarrow I^X \times I$, where $\widetilde{1}_\triangle(e) = (\widetilde{1}(e), \triangle(e))$ is defined by $\widetilde{1}(e) = \overline{1}$, $\forall e \in E$ and $\triangle(e) = 1$, $\forall e \in E$ (Where $\overline{1}(x) = 1, \forall x \in X$).*

Definition 2.12. *[3] Let T be a collection of generalized fuzzy soft sets over (X, E). Then T is said to be a generalized fuzzy soft topology ($GFST$, in short) over (X, E) if the following conditions are satisfied:*

(1) $\widetilde{0}_\theta$ and $\widetilde{1}_\triangle$ are in T.

(2) Arbitrary unions of members of T belong to T.

(3) Finite intersections of members of T belong to T.

The triple (X, T, E) is called a generalized fuzzy soft topological space (GFST- space, in short) over (X, E).

The member of T are called generalized fuzzy soft open set [GFS open for short] in (X, T, E) and their generalized fuzzy soft complements are called GFS closed sets in (X, T, E). The family of all GFSclosed sets in (X, T, E) is denotes by T^c.

Definition 2.13. *[3] Let (X, T, E) be a $GFST-$space and $F\mu \in GFSS(X, E)$. The generalized fuzzy soft closure of $F\mu$, denoted by $cl(F\mu)$, is the intersection of all GFS closed supper sets of $F\mu$. i.e., $cl(F\mu) = \sqcap\{H_\nu : H_\nu \in T^c, F\mu \sqsubseteq H_\nu\}$. Clearly, $cl(F\mu)$ is the smallest GFS closed set over (X, E) which contains $F\mu$.*

Definition 2.14. *[7] The generalized fuzzy soft set $F_\mu \in GFSS(X, E)$ is called a generalized fuzzy soft point (GFS point for short) over (X, E) if there exist $e \in E$ and $x \in X$ such that*

(1) $F(e)(x) = \alpha(0 < \alpha \leq 1)$ and $F(e)(y) = 0$ for all $y \in X - \{x\}$,

(2) $\mu(e) = \lambda(0 < \lambda \leq 1)$ and $\mu(e') = 0$ for all $e' \in E - \{e\}$. We denote this generalized fuzzy soft point $F_\mu = (e_\lambda, x_\alpha)$. (e, x) and (λ, α) are called respectively, the support and the value of (e_λ, x_α). The class of all GFS points in (X, E), denoted by $GFSP(X, E)$.

Definition 2.15. *[7] Let F_μ be a GFSS over (X, E). We say that $(e_\lambda, x_\alpha)\widetilde{\in}F_\mu$ read as (e_λ, x_α) belongs to the $GFSSF_\mu$ if for the element $e \in E$, $\alpha \leq F(e)(x)$ and $\lambda \leq \mu(e)$.*

Definition 2.16. *[7] Let (X, T, E) be a $GFST-$space. A GFSS F_μ in $GFS(X, E)$ is called a generalized fuzzy soft neighborhood(briefly, $GFS-$nbd) of H_ν [resp. (e_λ, x_α)] if there exists $G_\delta \in T$ such that $H_\nu \sqsubseteq G_\delta \sqsubseteq F_\mu$ [resp.$(e_\lambda, x_\alpha)\widetilde{\in} \sqsubseteq G_\delta \sqsubseteq F_\mu$].*

Definition 2.17. *[8] Let $GFSS(X, E)$ and $GFSS(Y, K)$ be the families of all GFSSs over (X, E) and (Y, K), respectively. Let $u : X \longrightarrow Y$ and $p : E \longrightarrow K$ be two mappings. Then a mapping $f_{up} : GFSS(X, E) \longrightarrow GFSS(Y, K)$ is defined as follows: for a GFSS $F_\mu \in GFSS(X, E)$, $\forall e' \in p(E) \sqsubseteq K$ and $y \in Y$,we have*

$$f_{up}(F_\mu)(e')(y) = \begin{cases} (\bigvee_{x\in u^{-1}(y)} \bigvee_{e\in p^{-1}(e')} F(e)(x), \bigvee_{e\in p^{-1}(e')} \mu(e)), \\ \qquad \text{if } u^{-1}(y) \neq \phi, p^{-1}(k) \neq \phi, \\ (0, 0), \qquad \text{otherwise.} \end{cases}$$

f_{up} is called the generalized fuzzy soft mapping [GFS mapping for short] and $f_{up}(F_\mu)$ is called the generalized fuzzy soft image (GFS image for short) of a $GFSS$ F_μ.

Definition 2.18. *[8] Let $u : X \longrightarrow Y$ and $p : E \longrightarrow K$ be mappings.*

Let $f_{up} : GFSS(X,E) \longrightarrow GFSS(Y,K)$ be a GFS mapping and $G_\delta \in GFSS(Y,K)$. Then $f^{-1}_{up}(G_\delta) \in GFSS(X,E)$ is defined as follows:

$$f^{-1}_{up}(G_\delta)(e)(x) = (G(p(e)(u(x)), \delta(p(e)))\,, \text{for } e \in E, x \in X.$$

$f^{-1}_{up}(G_\delta)$ is called the GFS inverse image of G_δ.

If u and p are injective, then the GFS mapping f_{up} is said to be generalized fuzzy soft injective (GFS injective for short). If u and p are surjective, then the GFS mapping f_{up} is said to be generalized fuzzy soft surjective (GFS surjective for short). The GFS mapping f_{up} is called generalized fuzzy soft constant (GFS constant for short), if u and p are constant. f_{up} is said to be generalized fuzzy soft bijective (GFS bijective for short), if f_{up} is GFS injective and GFS surjective mapping.

Definition 2.19. *[8] Let (X, T_1, E) and (Y, T_2, K) be two GFST-spaces, and $f_{up} : (X, T_1, E) \longrightarrow (Y, T_2, K)$ be a GFS mapping. Then f_{up} is called:*

(i) Generalized fuzzy soft continuous (GFS-continuous, in short), if $f^{-1}_{up}(G_\delta) \in T_1$, for all $G_\delta \in T_2$,

(ii) Generalized fuzzy soft open (GFS open, in short), if $f_{up}(F_\mu) \in T_2$, for each $F_\mu \in T_1$.

Theorem 2.20. *[8] Let $GFSS(X,E)$ and $GFSS(Y,K)$ be two families of GFSSs. For the GFS mapping $f_{up} : GFSS(X,E) \longrightarrow GFSS(Y,K)$, we have the following properties.*

(1)$f^{-1}_{up}(G_\delta)^c = (f^{-1}_{up}(G_\delta))^c, \forall G_\delta \in GFSS(Y,K)$.

(2) $f_{up}(f^{-1}_{up}(G_\delta)) \sqsubseteq G_\delta, \forall G_\delta \in GFSS(Y,K)$. If f_{up} is GFS surjective, then the equality holds.

(3) $F_\mu \sqsubseteq f^{-1}_{up}(f_{up}(F_\mu)), \forall F_\mu \in GFSS(X,E)$. If f_{up} is GFS injective, then the equality holds.

(4) $f_{up}(\widetilde{0}_{\theta_X}) = \widetilde{0}_{\theta_Y}$ and $f_{up}(\widetilde{1}_{\triangle X}) \sqsubseteq \widetilde{1}_{\triangle_Y}$. If f_{up} is GFS injective, then the equality holds.

(5) $f^{-1}_{up}(\widetilde{0}_{\theta_Y}) = \widetilde{0}_{\theta_X}$ and $f^{-1}_{up}(\widetilde{1}_{\triangle_Y}) = \widetilde{1}_{\triangle_X}$.

(6) If $F_\mu \sqsubseteq H_\nu$, then $f_{up}(F_\mu) \sqsubseteq f_{up}(H_\nu), \forall F_\mu, H_\nu \in GFSS(X,E)$.

(7) If $G_\delta \sqsubseteq J_\sigma$, then $f^{-1}_{up}(G_\delta) \sqsubseteq f^{-1}_{up}(J_\sigma), \forall G_\delta, J_\sigma \in GFSS(Y,K)$.

(8) $f^{-1}_{up}(\sqcup_{i\in J}(G_\delta)_i) = \sqcup_{i\in J} f^{-1}_{up}(G_\delta)_i$ and $f^{-1}_{up}(\sqcap_{i\in J} G_\delta)_i) = \sqcap_{i\in J} f^{-1}_{up}(G_\delta)_i, \forall\ (G_\delta)_i \in GFSS(Y,K)$.

(9) $f_{pu}(\sqcup_{i\in J}(F_\mu)_i) = \sqcup_{i\in J} f_{up}(F_\mu)_i$ and $f_{up}(\sqcap_{i\in J}(F_\mu)_i) \sqsubseteq \sqcap_{i\in J} f_{up}(F_\mu)_i, \forall (F_\mu)_i \in GFSS(X,E)$. If f_{up} is GFS injective, then the equality holds.

Definition 2.21. *[6] Let (X, τ, E) be a fuzzy soft topological space and $f_A \in FSS(X,E)$. If $f_A \sqsubseteq cl(int(f_A))$, then f_A is called fuzzy soft semi open set. We denote the set of all fuzzy soft semi open sets by $FSSOS(X, \tau, E)$, or $FSSOS(X)$ and the set of all fuzzy soft semi closed sets by $FSSCS(X, \tau, E)$, or $FSSCS(X)$.*

3 Generalized Fuzzy Soft Semi Open (Closed) Sets

Generalization of closed and open sets in topological spaces are of resent a advances. Here, we introduce generalized fuzzy soft semi open and generalized fuzzy soft semi closed sets and study various properties and notions related to these structures. The concepts of closure and interior are generalized via generalized fuzzy soft semi open and generalized fuzzy soft semi closed.

Definition 3.1. *Let (X, T, E) be a GFST−space and $F_\mu \in GFSS(X, E)$. If $F_\mu \sqsubseteq cl(int(F_\mu))$, then F_μ is called a generalized fuzzy soft semi open set [GFS semi open set in short]. We denote the set of all GFS semi open sets by $GFSSOS(X, T, E)$ or $GFSSOS(X)$.*

Definition 3.2. *Let (X, T, E) be a GFST−space and $F_\mu \in GFSS(X, E)$. If $int(cl(F_\mu)) \sqsubseteq F_\mu$, then F_μ is called a generalized fuzzy soft semi closed set [GFS semi closed set in short].*
We denote the set of all GFS semi open sets by $GFSSCS(X, T, E)$ or $GFSSCS(X)$.

Example 3.3. *Let $X = \{x^1, x^2, x^3\}$ and $E = \{e^1, e^2, e^3\}$.*
Then $T = \left\{\widetilde{0}_\theta, \widetilde{1}_\triangle, (F_\mu)_1, (F_\mu)_2, (F_\mu)_3, (F_\mu)_4, (F_\mu)_5, (F_\mu)_6, (F_\mu)_7, (F_\mu)_8, (F_\mu)_9, (F_\mu)_{10}\right\}$ is a GFS topology over (X, E) where

$(F_\mu)_1 = \{(e^1 = \{\frac{x^1}{0.2}, \frac{x^2}{0.4}, \frac{x^3}{0.1}\}, 0.1), (e^2 = \{\frac{x^1}{0}, \frac{x^2}{0}, \frac{x^3}{0}\}, 0), (e^3 = \{\frac{x^1}{0}, \frac{x^2}{0}, \frac{x^3}{0}\}, 0)\}$
$(F_\mu)_2 = \{(e^1 = \{\frac{x^1}{0.2}, \frac{x^2}{0.8}, \frac{x^3}{0.5}\}, 0.7), (e^2 = \{\frac{x^1}{0.8}, \frac{x^2}{0}, \frac{x^3}{0.1}\}, 0.1), (e^3 = \{\frac{x^1}{0.4}, \frac{x^2}{0.3}, \frac{x^3}{0}\}, 0.3)\}$
$(F_\mu)_3 = \{(e^1 = \{\frac{x^1}{0.2}, \frac{x^2}{0.8}, \frac{x^3}{0.5}\}, 0.7), (e^2 = \{\frac{x^1}{0.8}, \frac{x^2}{0}, \frac{x^3}{0.7}\}, 0.6), (e^3 = \{\frac{x^1}{0.6}, \frac{x^2}{0.3}, \frac{x^3}{0.1}\}, 0.3)\}$
$(F_\mu)_4 = \{(e^1 = \{\frac{x^1}{0.2}, \frac{x^2}{0.8}, \frac{x^3}{0.5}\}, 0.7), (e^2 = \{\frac{x^1}{0.8}, \frac{x^2}{0}, \frac{x^3}{0.7}\}, 0.6), (e^3 = \{\frac{x^1}{0.6}, \frac{x^2}{0.3}, \frac{x^3}{0.1}\}, 0.3)\}$
$(F_\mu)_5 = \{(e^1 = \{\frac{x^1}{0.2}, \frac{x^2}{0.5}, \frac{x^3}{0.5}\}, 0.4), (e^2 = \{\frac{x^1}{0.7}, \frac{x^2}{0}, \frac{x^3}{0.7}\}, 0.6), (e^3 = \{\frac{x^1}{0.6}, \frac{x^2}{0.1}, \frac{x^3}{0.1}\}, 0.1)\}$
$(F_\mu)_6 = \{(e^1 = \{\frac{x^1}{0.2}, \frac{x^2}{0.8}, \frac{x^3}{0.5}\}, 0.7), (e^2 = \{\frac{x^1}{0.8}, \frac{x^2}{0}, \frac{x^3}{0.1}\}, 0.1), (e^3 = \{\frac{x^1}{0.4}, \frac{x^2}{0.3}, \frac{x^3}{0}\}, 0.3)\}$
$(F_\mu)_7 = \{(e^1 = \{\frac{x^1}{0.2}, \frac{x^2}{0.6}, \frac{x^3}{0.5}\}, 0.5), (e^2 = \{\frac{x^1}{0.7}, \frac{x^2}{0.1}, \frac{x^3}{0.9}\}, 0.8), (e^3 = \{\frac{x^1}{0.6}, \frac{x^2}{0.5}, \frac{x^3}{0.1}\}, 0.5)\}$
$(F_\mu)_8 = \{(e^1 = \{\frac{x^1}{0.2}, \frac{x^2}{0.5}, \frac{x^3}{0.5}\}, 0.4), (e^2 = \{\frac{x^1}{0.7}, \frac{x^2}{0}, \frac{x^3}{0.1}\}, 0.1), (e^3 = \{\frac{x^1}{0.4}, \frac{x^2}{0.1}, \frac{x^3}{0}\}, 0.1)\}$
$(F_\mu)_9 = \{(e^1 = \{\frac{x^1}{0.2}, \frac{x^2}{0.6}, \frac{x^3}{0.5}\}, 0.5), (e^2 = \{\frac{x^1}{0.7}, \frac{x^2}{0}, \frac{x^3}{0.1}\}, 0.1), (e^3 = \{\frac{x^1}{0.4}, \frac{x^2}{0.3}, \frac{x^3}{0}\}, 0.3)\}$
$(F_\mu)_{10} = \{(e^1 = \{\frac{x^1}{0.2}, \frac{x^2}{0.6}, \frac{x^3}{0.5}\}, 0.5), (e^2 = \{\frac{x^1}{0.7}, \frac{x^2}{0}, \frac{x^3}{0.7}\}, 0.6), (e^3 = \{\frac{x^1}{0.6}, \frac{x^2}{0.3}, \frac{x^3}{0.1}\}, 0.3)\}$

Then. The GFSS

$H_\nu = \{(e^1 = \{\frac{x^1}{0.6}, \frac{x^2}{0.4}, \frac{x^3}{0.4}\}, 0.2), (e^2 = \{\frac{x^1}{0.2}, \frac{x^2}{0}, \frac{x^3}{0.1}\}, 0.1), (e^3 = \{\frac{x^1}{0.4}, \frac{x^2}{0.3}, \frac{x^3}{0.1}\}, 0.4)\}$

is GFS semi open set of (X, T, E) where

$int(H_\nu) = \{(e^1 = \{\frac{x^1}{0.2}, \frac{x^2}{0.4}, \frac{x^3}{0.1}\}, 0.1), (e^2 = \{\frac{x^1}{0}, \frac{x^2}{0}, \frac{x^3}{0}\}, 0),$
$(e^3 = \{\frac{x^1}{0}, \frac{x^2}{0}, \frac{x^3}{0}\}, 0)\},$
$cl(int(H_\nu)) = \{(e^1 = \{\frac{x^1}{0.8}, \frac{x^2}{0.4}, \frac{x^3}{0.5}\}, 0.4), (e^2 = \{\frac{x^1}{0.3}, \frac{x^2}{0.9}, \frac{x^3}{0.1}\}, 0.1),$
$(e^3 = \{\frac{x^1}{0.4}, \frac{x^2}{0.5}, \frac{x^3}{0.9}\}, 0.5)\},$

and $H_\nu \sqsubseteq cl(int(H_\nu)$. The GFSS

$K_\gamma = \{(e^1 = \{\frac{x^1}{0.2}, \frac{x^2}{0.5}, \frac{x^3}{0.6}\}, 0.5), (e^2 = \{\frac{x^1}{0.7}, \frac{x^2}{0.1}, \frac{x^3}{0.9}\}, 0.9), (e^3 = \{\frac{x^1}{0.6}, \frac{x^2}{0.5}, \frac{x^3}{0.1}\}, 0.5)\}$

is GFS semi closed set of (X, T, E) where $int(cl(K_\gamma)) \sqsubseteq K_\gamma$.

Theorem 3.4. *Let (X, T, E) be a $GFST-$space. Then*

(1) Arbitrary union of GFS semi open sets is GFS semi open set.

(2) Arbitrary intersection of GFS semi closed sets is GFS semi closed set.

Proof. (1) Let $\{(F_\mu)_i), i \in J\} \subseteq GFSSOS(X)$. Then $\forall i \in J$, $(F_\mu)_i \sqsubseteq cl(int(F_\mu)_i$. It follows that $\sqcup_{i\in J}(F_\mu)_i \sqsubseteq \sqcup_{i\in J} cl(int(F_\mu)_i) = cl(\sqcup_{i\in J} int(F_\mu)_i) \sqsubseteq cl(int(\sqcup_{i\in J}(F_\mu)_i))$.
Hence, $\sqcup_{i\in J}(F_\mu)_i \in GFSSOS(X) \forall i \in J$.

(2) By similarly way.

$\square$

Theorem 3.5. *Let (X, T, E) be a $GFST-$space and*
$F_\mu \in GFSS(X, E)$. Then

(1) $F_\mu \in GFSSOS(X)$ if and only if $cl(F_\mu) = cl(int(F_\mu))$.

(2) If $H_\nu \in T$, then $H_\nu \sqcap cl(F_\mu) \sqsubseteq cl(H_\nu \sqcap F_\mu)$.

Proof. Immediate.

$\square$

Theorem 3.6. *Let (X, T, E) be a $GFST-$space and $F_\mu \in GFSS(X, E)$. Then*

(1) $F_\mu \in GFSSOS(X)$ if and only if there exists GFS open set H_ν such that $H_\nu \sqsubseteq F_\mu \sqsubseteq cl(H_\nu)$

(2) If $F_\mu \in GFSSOS(X)$ and $F_\mu \sqsubseteq G_\delta \sqsubseteq cl(F_\mu)$. Then $G_\delta \in GFSSOS(X)$.

Proof. (1) Let $F_\mu \in GFSSOS(X)$. Then $F_\mu \sqsubseteq cl(int(F_\mu))$. Take $H_\nu = int(F_\mu) \in T$. So, we have $H_\nu \sqsubseteq F_\mu \sqsubseteq cl(H_\nu)$. Sufficiency, let $H_\nu \sqsubseteq F_\mu \sqsubseteq cl(H_\nu)$ for some $H_\nu \in T$. Then $H_\nu \sqsubseteq int(F_\mu)$. It follows that, $cl(H_\nu) \sqsubseteq cl(int(F_\mu))$. Thus, $F_\mu \sqsubseteq cl(H_\nu) \sqsubseteq cl(int(F_\mu))$. Therefore, $F_\mu \in GFSSOS(X)$.

(2) $F_\mu \in GFSSOS(X)$. Then $H_\nu \sqsubseteq F_\mu \sqsubseteq cl(H_\nu)$ for some $H_\nu \in T$. It follows that, $H_\nu \sqsubseteq F_\mu \sqsubseteq G_\delta$. Thus, $H_\nu \sqsubseteq G_\delta \sqsubseteq cl(F_\mu) \sqsubseteq cl(H_\nu)$. Hence $H_\nu \sqsubseteq G_\delta \sqsubseteq cl(F_\mu) \sqsubseteq cl(H_\nu)$ for some $H_\nu \in T$. Therefore, $G_\delta \in GFSSOS(X)$.

$\square$

Theorem 3.7. *Let (X, T, E) be a $GFST-$space and $M_\eta \in GFSS(X, E)$. Then*

(1) $M_\eta \in GFSSCS(X)$ if and only if there exists GFS closed set J_σ such that $int(J_\sigma) \sqsubseteq M_\eta \sqsubseteq J_\sigma$

(2) If $M_\eta \in GFSSCS(X)$ and $int(M_\eta) \sqsubseteq K_\gamma \sqsubseteq M_\eta$. Then $K_\gamma \in GFSSCS(X)$.

Proof. (1) Let $M_\eta \in GFSSCS(X)$. Then $int(cl(M_\eta)) \sqsubseteq M_\eta$. Take $J_\sigma = cl(M_\eta) \in T^c$. So, we have $int(J_\sigma) \sqsubseteq M_\eta \sqsubseteq J_\sigma$. Sufficiency, let $int(J_\sigma) \sqsubseteq M_\eta \sqsubseteq J_\sigma$ for some $J_\sigma \in T^c$. Then $cl(M_\eta) \sqsubseteq J_\sigma$. It follows that, $int(cl(M_\eta)) \sqsubseteq int(J_\sigma) \sqsubseteq M_\eta$. Therefore, $M_\eta \in GFSSCS(X)$.

(2) $M_\eta \in GFSSCS(X)$. Then $int(J_\sigma) \sqsubseteq M_\eta \sqsubseteq J_\sigma$ for some $J_\sigma \in T^c$. It follows that, $K_\gamma \sqsubseteq M_\eta \sqsubseteq J_\sigma$. Thus, $int(J_\sigma) \sqsubseteq int(M_\eta) \sqsubseteq k_\gamma \sqsubseteq J_\sigma$. Hence $int(J_\sigma) \sqsubseteq k_\gamma \sqsubseteq J_\sigma$ for some $J_\sigma \in T^c$. Therefore, $K_\gamma \in GFSSCS(X)$.

$\square$

Corollary 3.8. *Let (X, T, E) be a $GFST-$space and $F_\mu \in GFSS(X, E)$. Then $F_\mu \in GFSSCS(X)$ if and only if $F_\mu = F_\mu \sqcup int(cl(F_\mu))$*

.

Theorem 3.9. *If F_μ is a GFSS in $GFST-$space (X, T, E). Then the following are equivalent:*

(1) F_μ is GFS semi closed set.

(2) F_μ^c is GFS semi open set.

Proof. $1 \Longrightarrow 2$ If F_μ is GFS semi closed set.Then $int(cl(F_\mu)) \sqsubseteq F_\mu \Longrightarrow F_\mu^c \sqsubseteq (int(cl(F_\mu)))^c = cl(int(F_\mu^c))$. Thus F_μ^c is GFS semi open set.

$2 \Longrightarrow 1$ Obvious.

□

Theorem 3.10. *A GFSS $F_\mu \in GFSSOS(X)$ if and only if for every GFS point $(e_\lambda, x_\alpha)\widetilde{\in} F_\mu$, there exists a GFSS $H_\nu \in GFSSOS(X)$ such that $(e_\lambda, x_\alpha)\widetilde{\in} H_\nu \sqsubseteq F_\mu$.*

Proof. Take $H_\nu = F_\mu$, this shows that the conduction is necessary.

For sufficiency, we have $F_\mu = \sqcup_{(e_\lambda,x_\alpha)\widetilde{\in} F_\mu}(e_\lambda, x_\alpha) \sqsubseteq H_\nu \sqsubseteq F_\mu$.

□

Definition 3.11. *Let (X, T, E) be a $GFST-$space and $F_\mu \in GFSS(X, E)$ and $(e_\lambda, x_\alpha) \in GFSP(X, E)$. Then*

(1) (e_λ, x_α) is called generalized fuzzy soft semi cluster point of F_μ if $F_\mu \sqcap G_\delta \neq \widetilde{0}_\theta$ $\forall G_\delta \in GFSSOS(X)$, $(e_\lambda, x_\alpha)\widetilde{\in} G_\delta$. The set of all generalized fuzzy soft semi cluster points of F_μ is called the generalized fuzzy soft semi closure of F_μ and is denoted by $cl^s(F_\mu)$ or [$gfssclF_\mu$] consequently, $gfssclF_\mu = \sqcap\{H_\nu : F_\mu \sqsubseteq H_\nu, H_\nu \in GFSSCS(X)\}$.

(2) (e_λ, x_α) is called generalized fuzzy soft semi interior point of F_μ if there exists a GFSS $H_\nu \in GFSSOS(X)$ such that $(e_\lambda, x_\alpha)\widetilde{\in} H_\nu \sqsubseteq F_\mu$. The set of all generalized fuzzy soft semi interior point of F_μ is called the generalized fuzzy soft semi interior of F_μ and is denoted by $int^s(F_\mu)$ or [$gfssint(F_\mu)$] consequently, $gfssintF_\mu = \sqcup\{G_\delta : G_\delta \sqsubseteq F_\mu, G_\delta \in GFSSOS(X)\}$.

$gfsscl(F_\mu)$ is the smallest GFS semi closed set containing in F_μ and $gfssint(F_\mu)$ is the largest GFS semi open set contained F_μ.

Theorem 3.12. *Let (X, T, E) be a $GFST-$space. Let F_μ and H_ν be two GFSSs over (X, E), then*

(1) $F_\mu \in GFSSCS(X) \Longleftrightarrow F_\mu = gfsscl(F_\mu)$,

(2) $F_\mu \in GFSSOS(X) \Longleftrightarrow F_\mu = gfssint(F_\mu)$.

(3) $(gfsscl(F_\mu))^c = gfssint(F_\mu^c)$.

(4) $(gfssint(F_\mu))^c = gfsscl(F_\mu^c)$.

(5) $F_\mu \sqsubseteq H_\nu \Longrightarrow gfsscl(F_\mu) \sqsubseteq gfsscl(H_\nu)$,

(6) $F_\mu \sqsubseteq H_\nu \Longrightarrow gfssint(F_\mu) \sqsubseteq gfssint(H_\nu)$,

(7) $gfsscl(\widetilde{0}_\theta) = \widetilde{0}_\theta$ and $gfsscl(\widetilde{1}_\triangle) = \widetilde{1}_\triangle$,

(8) $gfssint(\widetilde{0}_\theta) = \widetilde{0}_\theta$ and $gfssint(\widetilde{1}_\triangle) = \widetilde{1}_\triangle$,

(9) $gfsscl(F_\mu \sqcup H_\nu) = gfsscl(F_\mu) \sqcup gfsscl(H_\nu)$,

(10) $gfssint(F_\mu \sqcap H_\nu) = gfssint(F_\mu) \sqcap gfssint(H_\nu)$,

(11) $gfsscl(F_\mu \sqcap H_\nu) \sqsubseteq gfsscl(F_\mu) \sqcap gfsscl(H_\nu)$,

(12) $gfssint(F_\mu \sqcup H_\nu) \sqsubseteq gfssint(F_\mu) \sqcup gfssint(H_\nu)$,

(13) $gfsscl(gfsscl(F_\mu)) = gfsscl(F_\mu)$,

(14) $gfssint(gfssint(F_\mu)) = gfssint(F_\mu)$.

Proof. (1) Let F_μ be a GFS semi closed set. Then it is the smallest GFS semi closed set containing itself and hence $F_\mu = gfsscl(F_\mu)$. On other hand, let $F_\mu = gfsscl(F_\mu)$ and $gfsscl(F_\mu) \in GFSSCS(X) \Longrightarrow F_\mu \in GFSSCS(X)$.

(2) Similar to (1).

(3) Since $gfsscl(F_\mu) = \sqcap\{H_\nu : F_\mu \sqsubseteq H_\nu, H_\nu \in GFSSCS(X)\}$. Then $(gfsscl(F_\mu))^c = \sqcup\{H_\nu^c : H_\nu^c \sqsubseteq F_\mu, H_\nu^c \in GFSSOS(X)\} = gfssint(F_\mu^c)$.

(4) Similar to (3).

(5) Follows from definition.

(6) Follows from definition.

(7) Since $\widetilde{0}_\theta$ and $\widetilde{1}_\triangle$ are GFS semi closed sets so $gfsscl(\widetilde{0}_\theta) = \widetilde{0}_\theta$ and $gfsscl(\widetilde{1}_\triangle) = \widetilde{1}_\triangle$.

(8) Since $\widetilde{0}_\theta$ and $\widetilde{1}_\triangle$ are GFS semi open sets so $gfssint(\widetilde{0}_\theta) = \widetilde{0}_\theta$ and $gfssint(\widetilde{1}_\triangle) = \widetilde{1}_\triangle$.

(9) We have $F_\mu \sqsubseteq F_\mu \sqcup H_\nu$ and $H_\nu \sqsubseteq F_\mu \sqcup H_\nu$ Then by (5), $gfsscl(F_\mu) \sqsubseteq gfsscl(F_\mu \sqcup H_\nu)$ and $gfsscl(H_\nu) \sqsubseteq gfsscl(F_\mu \sqcup H_\nu) \Longrightarrow gfsscl(F_\mu) \sqcup gfsscl(H_\nu) \sqsubseteq gfsscl(F_\mu \sqcup H_\nu)$.

Now, $gsfscl(F_\mu), gfsscl(H_\nu) \in GFSSCS(X) \Longrightarrow gfsscl(F_\mu) \sqcup gfsscl(H_\nu) \in GFSSCS(X)$. Then $F_\mu \sqsubseteq gfsscl(F_\mu)$ and $H_\nu \sqsubseteq gfsscl(H_\nu)$ imply $F_\mu \sqcup H_\nu \sqsubseteq gfsscl(F_\mu) \sqcup gfsscl(H_\nu)$. i.e., $gfsscl(F_\mu) \sqcup gfsscl(H_\nu)$ is GFS semi closed set containing $F_\mu \sqcup H_\nu$. But $gfsscl(F_\mu \sqcup H_\nu)$ is the smallest GFS semi closed set containing $F_\mu \sqcup H_\nu$. Hence $gfsscl(F_\mu \sqcup H_\nu) \sqsubseteq gfsscl(F_\mu) \sqcup gfsscl(H_\nu)$. So, $gfsscl(F_\mu \sqcup H_\nu) = gfsscl(F_\mu) \sqcup gfsscl(H_\nu)$.

(10) Similar to (9).

(11) We have $F_\mu \sqcap H_\nu \sqsubseteq F_\mu$ and $F_\mu \sqcap H_\nu \sqsubseteq H_\nu \Longrightarrow gfsscl(F_\mu \sqcap H_\nu) \sqsubseteq gfsscl(F_\mu)$ and $gfsscl(F_\mu \sqcap H_\nu) \sqsubseteq gfsscl(H_\nu) \Longrightarrow gfsscl(F_\mu \sqcap H_\nu) \sqsubseteq gfsscl(F_\mu) \sqcap gfsscl(H_\nu)$.

(12) Similar to (11).

(13) Since $gfsscl(F_\mu) \in GFSSCS(X)$ so by (1), $gfsscl(gfsscl(F_\mu)) = gfsscl(F_\mu)$.

(14) Since $gfssint(F_\mu) \in GFSSOS(X)$ so by (2), $gfssint(gfint(F_\mu)) = gfssint(F_\mu)$.

□

Remark 3.13. *If F_μ is GFS semi open (GFS semi closed) set, then $int(F_\mu)$, $gfssint(F_\mu)[cl(F_\mu)$ and $gfsscl(F_\mu)]$ are GFS semi open (GFS semi closed).*

Lemma 3.14. *Every GFS open (resp. closed) set in a GFST−space (X, T, E) is GFS semi open (resp. closed) set.*

Proof. Let F_μ be GFS open set. Then $int(F_\mu) = F_\mu$. Since $F_\mu \sqsubseteq cl(F_\mu)$, then $F_\mu \sqsubseteq cl(int(F_\mu))$. Thus, $F_\mu \in GFSSOS(X)$.

□

Remark 3.15. *The converse of Lemma* **??**, *is not true in general as shown in the following example.*

Example 3.16. *Let $X = \{x^1, x^2, x^3\}$ and $E = \{e^1, e^2, e^3\}$.*
Then $T = \left\{\widetilde{0}_\theta, \widetilde{1}_\triangle, (F_\mu)_1, (F_\mu)_2, (F_\mu)_3, (F_\mu)_4, (F_\mu)_5, (F_\mu)_6, \right\}$ is a GFS topology over (X, E) where

$(F_\mu)_1 = \{(e^1 = \{\frac{x^1}{0.5}, \frac{x^2}{0.75}, \frac{x^3}{0.4}\}, 0.3), (e^2 = \{\frac{x^1}{0.3}, \frac{x^2}{0.8}, \frac{x^3}{0.7}\}, 0.4)\}$

$(F_\mu)_2 = \{(e^2 = \{\frac{x^1}{0.4}, \frac{x^2}{0.6}, \frac{x^3}{0.3}\}, 0.2), (e^3 = \{\frac{x^1}{0.2}, \frac{x^2}{0.4}, \frac{x^3}{0.45}\}, 0.4)\}$

$(F_\mu)_3 = \{(e^2 = \{\frac{x^1}{0.3}, \frac{x^2}{0.6}, \frac{x^3}{0.3}\}, 0.2)\}$

$(F_\mu)_4 = \{(e^1 = \{\frac{x^1}{0.5}, \frac{x^2}{0.75}, \frac{x^3}{0.4}\}, 0.3), (e^2 = \{\frac{x^1}{0.4}, \frac{x^2}{0.8}, \frac{x^3}{0.7}\}, 0.4), (e^3 = \{\frac{x^1}{0.2}, \frac{x^2}{0.4}, \frac{x^3}{0.45}\}, 0.4)\}$

$(F_\mu)_5 = \{(e^2 = \{\frac{x^1}{0.4}, \frac{x^2}{0.8}, \frac{x^3}{0.7}\}, 0.4), (e^3 = \{\frac{x^1}{0.2}, \frac{x^2}{0.4}, \frac{x^3}{0.45}\}, 0.4)\}$

$(F_\mu)_6 = \{(e^2 = \{\frac{x^1}{0.3}, \frac{x^2}{0.8}, \frac{x^3}{0.7}\}, 0.4)\}$

Then the GFSS K_γ where

$K_\gamma = \{(e^1 = \{\frac{x^1}{0.4}, \frac{x^2}{0.3}, \frac{x^3}{0.2}\}, 0.2), (e^2 = \{\frac{x^1}{0.6}, \frac{x^2}{0.9}, \frac{x^3}{0.7}\}, 0.4), (e^3 = \{\frac{x^1}{0.2}, \frac{x^2}{0.3}, \frac{x^3}{0.1}\}, 0.1)\}$

is GFS semi open set of (X, T, E), but it is not GFS open.

Remark 3.17. *$\widetilde{0}_\theta$ and $\widetilde{1}_\triangle$ are always GFS semi closed and GFS semi open.*

Remark 3.18. *Every GFS clopen set is both GFS semi closed and GFS semi open.*

Theorem 3.19. *Let (X, T, E) be a GFST−space and $F_\mu, H_\nu \in GFSS(X, E)$. If either $F_\mu \in GFSSOS(X)$ or $H_\nu \in GFSSOS(X)$. Then $int(cl(F_\mu \sqcap H_\nu)) = int(cl(F_\mu)) \sqcap int(cl(H_\nu))$.*

Proof. Let $F_\mu, H_\nu \in GFSS(X,E)$. Then we generally have $int(cl(F_\mu \sqcap H_\nu)) \sqsubseteq int(cl(F_\mu)) \sqcap int(cl(H_\nu))$. Suppose that $F_\mu \in GFSSOS(X)$. Then $cl(F_\mu) = cl(int(F_\mu))$ from Theorem 3.5, (1).

Therefore,
$$\begin{aligned} int(cl(F_\mu)) \sqcap int(cl(H_\nu)) &\sqsubseteq int[cl(F_\mu) \sqcap int(cl(H_\nu))] \\ &= int[cl(int(F_\mu)) \sqcap int(cl(H_\nu))] \\ &\sqsubseteq int(cl[(int(F_\mu) \sqcap int(cl(H_\nu))]) \\ &\sqsubseteq int(cl(int[(int(F_\mu) \sqcap cl(H_\nu)])) \\ &\sqsubseteq int(cl(int(cl[(int(F_\mu) \sqcap (H_\nu)]))) \\ &\sqsubseteq int(cl(int(cl[(F_\mu) \sqcap (H_\nu)]))) \\ &\sqsubseteq int(cl[(F_\mu) \sqcap (H_\nu)]) \end{aligned}$$

from Theorem 3.5 (2). If $H_\nu \in GFSSOS(X)$, the proof is similar. This completes the proof. □

Theorem 3.20. *Let (X, T, E) be a $GFST-$space. Let F_μ be GFS open set and $H_\nu \in GFSSOS(X)$. Then $F_\mu \sqcap H_\nu \in GFSSOS(X)$.*

Proof. Let $F_\mu \in T$ and $H_\nu \in GFSSOS(X)$. Then
$$\begin{aligned} F_\mu \sqcap H_\nu &\sqsubseteq int(F_\mu) \sqcap cl(int(H_\nu)) \\ &\sqsubseteq cl(int(F_\mu) \sqcap int(H_\nu)) \\ &= cl(int(F_\mu) \sqcap (H_\nu))) \end{aligned}$$
from Theorem 3.5 (2). Hence $F_\mu \sqcap H_\nu \in GFSSOS(X)$. □

4 Generalized Fuzzy Soft Semi Continuous Mappings

In [5], Karal et al. defined the notion of a mapping on classes of fuzzy soft sets, which is fundamental important in fuzzy soft set theory, to improve this work and they studied properties of fuzzy soft images and fuzzy soft inverse images of fuzzy soft sets. Khedr et al. [8] introduced the concept of a generalized fuzzy soft mapping on families of generalized fuzzy soft sets. Also they introduced a generalized fuzzy soft continuous mapping, generalized fuzzy soft open, generalized fuzzy soft closed mappings, and generalized fuzzy soft homeomorphism for generalized fuzzy soft topological spaces, and studied its basic properties. Here, we introduce the notions of generalized fuzzy soft semi mapping in generalized fuzzy soft topological spaces and study its basic properties.

Definition 4.1. *Let (X, T_1, E) and (Y, T_2, K) be tow $GFST-$ spaces and $f_{up} : (X, T_1, E) \longrightarrow (Y, T_2, K)$ be a GFS mapping. Then, the mapping f_{up} is called :*

(1) Generalized fuzzy soft semi continuous [in short $GFS-$semi continuous] if $f_{up}^{-1}(G_\delta) \in GFSSOS(X)$ for all $G_\delta \in T_2$.

(2) Generalized fuzzy soft semi open [in short $GFS-$semi open] if $f_{up}(F_\mu) \in GFSSOS(Y)$ for all $F_\mu \in T_1$.

(3) Generalized fuzzy soft semi closed [in short $GFS-$semi closed] if $f_{up}(F_\mu) \in GFSSCS(Y)$ for all $F_\mu \in T_1^c$.

(4) Generalized fuzzy soft irresolute [in short $GFS-$irresolute] if $f_{up}^{-1}(G_\delta) \in GFSSOS(X)$ for all $G_\delta \in GFSSOS(Y)$.

(5) Generalized fuzzy soft irresolute open [in short GFS−irresolute open] if $f_{up}(F_\mu) \in GFSSOS(Y)$ for all $F_\mu \in GFSSOS(X)$.

(6) Generalized fuzzy soft irresolute closed [in short GFS− irresolute closed] if $f_{up}(F_\mu) \in GFSSCS(Y)$ for all $F_\mu \in GFSSCS(X)$.

Example 4.2. *Let $X = \{x^1, x^2, x^3\}$, $E = \{e^1, e^2, e^3\}$ and $f_{up} : (X, T_1, E) \longrightarrow (Y, T_2, K)$ be the constant GFS mapping where T_1 is the indiscrete GFS topology and T_2 is the discrete GFS topology such that $u(x) = x_1 \forall x \in X$ and $p(e) = e_1$ $\forall e \in E$. Let G_δ be GFSS over (X, E) defined as follows:*

$G_\delta = \{(e^1 = \{\frac{x^1}{0.1}, \frac{x^2}{0.5}, \frac{x^3}{0.6}\}, 0.2), (e^2 = \{\frac{x^1}{0.6}, \frac{x^2}{0.2}, \frac{x^3}{0.5}\}, 0.4)\}$.

Then $G_\delta \in T_2$. Now, we find $f^{-1}_{up}(G_\delta)$ as follows:

$$
\begin{aligned}
f^{-1}_{up}(G_\delta)(e_1)(x_1) &= (G(p(e_1))(u(x_1)), \delta(p(e_1))) \\
&= (G(e_1)(x_1), \delta(e_1)) \\
&= (0.1, 0.2) \\
f^{-1}_{up}(G_\delta)(e_1)(x_2) &= (G(p(e_1))(u(x_2)), \delta(e_1)) \\
&= (G(e_1)(x_1), \delta(e_1)) = (0.1, 0.2) \\
f^{-1}_{up}(G_\delta)(e_1)(x_3) &= (G(p(e_1))(u(x_3)), \delta(e_1)) \\
&= (G(e_1)(x_1), \delta(e_1)) \\
&= (0.1, 0.2) \\
f^{-1}_{up}(G_\delta)(e_2)(x_1) &= (G(p(e_2))(u(x_1)), \delta(p(e_2))) \\
&= (G(e_1)(x_1), \delta(e_1)) \\
&= (0.1, 0.2) \\
f^{-1}_{up}(G_\delta)(e_2)(x_2) &= (0.1, 0.2) \\
f^{-1}_{up}(G_\delta)(e_2)(x_3) &= (0.1, 0.2) \\
f^{-1}_{up}(G_\delta)(e_3)(x_1) &= (G(p(e_3))(u(x_1)), \delta(p(e_3))) \\
&= (G(e_1)(x_1), \delta(e_1)) \\
&= (0.1, 0.2) \\
f^{-1}_{up}(G_\delta)(e_3)(x_2) &= (0.1, 0.2) \\
f^{-1}_{up}(G_\delta)(e_3)(x_3) &= (0.1, 0.2).
\end{aligned}
$$

Hence $f^{-1}_{up}(G_\delta) \notin GFSSOS(X)$. Therefore, f_{up} is not GFS− semi continuous mapping.

Theorem 4.3. *Every GFS−continuous is GFS−semi continuous mapping.*

Proof. Immediate from Lemma 3.14. □

Theorem 4.4. *Let (X, T_1, E) and (Y, T_2, K) be a GFST−spaces and f_{up} be a GFS mapping such that $f_{up} : (X, T_1, E) \longrightarrow (Y, T_2, K)$. Then the following are equivalent:*

(1) f_{up} is a GFS−semi continuous mapping.

(2) $f_{up}^{-1}(G_\delta) \in GFSSCS(X) \forall G_\delta \in T^2$,

(3) $f_{up}(gfsscl(F_\mu)) \sqsubseteq cl_{T_2}(f_{up}(F_\mu)) \forall F_\mu \in GFSS(X,E)$,

(4) $gfsscl(f_{up}^{-1}(G_\delta)) \sqsubseteq f_{up}^{-1}(cl_{T_2}(G_\delta)) \forall G_\delta \in GFSS(Y,K)$,

(5) $f_{up}^{-1}(int_{T_2}(G_\delta)) \sqsubseteq gfssint(f_{up}^{-1}(G_\delta)) \forall G_\delta \in GFSS(Y,K)$.

Proof. $1 \Longrightarrow 2$. Let G_δ GFS closed set over (Y,K). Then $G_\delta^c \in T_2$ and $f_{up}^{-1}(G_\delta^c) \in GFSSOS(X)$ from definition 4.1. Since $f_{up}^{-1}(G_\delta^c) = (f_{up}^{-1})(G_\delta))^C$ from theorem 2.20. Thus, $f_{up}^{-1}(G_\delta) \in GFSSCS(X)$.

$2 \Longrightarrow 3$. Let $F_\mu \in GFSS(X,E)$. Since $F_\mu \sqsubseteq f_{up}^{-1}(f_{up}(F_\mu)) \sqsubseteq f_{up}^{-1}(cl_{T_2}(f_{up}(F_\mu))) \in GFSSCS(X)$ from (2) and theorem 2.20. Then $F_\mu \sqsubseteq gfsscl(F_\mu) \sqsubseteq f_{up}^{-1}(cl_{T_2}(f_{up}(F_\mu)))$. Hence, $f_{up}(gfsscl(F_\mu)) \sqsubseteq f_{up}(f_{up}^{-1}(cl_{T_2}(f_{up}(F_\mu)))) \sqsubseteq cl_{T_2}(f_{up}(F_\mu))$ from theorem 2.20. Thus, $f_{up}(gfsscl(F_\mu)) \sqsubseteq cl_{T_2}(f_{up}(F_\mu))$.

$4 \Longrightarrow 2$. Let G_δ GFS closed set over (Y,K). Then $gfsscl(f_{up}^{-1}(G_\delta)) \sqsubseteq f_{up}^{-1}(cl_{T_2}(G_\delta))$ $\forall G_\delta \in GFSS(Y,K)$ from (4). But clearly $f_{up}^{-1}(G_\delta) \sqsubseteq gfsscl(f_{up}^{-1}(G_\delta))$. This means that, $f_{up}^{-1}(G_\delta) = gfsscl(f_{up}^{-1}(G_\delta))$, and consequently $f_{up}^{-1}(G_\delta) \in GFSSCS(X)$.

$1 \Longrightarrow 5$. Let $G_\delta \in GFSS(Y,K)$. Then $f_{up}^{-1}(int_{T_2}(G_\delta)) \in GFSSOS(X)$ from (1). Hence,

$$f_{up}^{-1}(int_{T_2}(G_\delta)) = gfssint(f_{up}^{-1}(int_{T_2}(G_\delta))) \\ \sqsubseteq gfssint(f_{up}^{-1}(G_\delta)).$$

Thus $f_{up}^{-1}(int_{T_2}(G_\delta)) \sqsubseteq gfssint(f_{up}^{-1}(G_\delta))$.

$5 \Longrightarrow 1$. Let G_δ GFS open set over (Y,K).
Then $int_{T_2}(G_\delta) = G_\delta$ and $f_{up}^{-1}(int_{T_2}(G_\delta)) f_{up}^{-1}(G_\delta) \sqsubseteq gfssint(f_{up}^{-1}(G_\delta))$ from (5). But we have $gfssint(f_{up}^{-1}(G_\delta)) \sqsubseteq f_{up}^{-1}(G_\delta)$. This mean that,
$gfssint(f_{up}^{-1}(G_\delta)) = f_{up}^{-1}(G_\delta)$. Thus f_{up} is a $GFS-$semi continuous mapping.

□

Theorem 4.5. *Let (X,T_1,E) and (Y,T_2,K) be two $GFST-$ spaces and f_{up} be a GFS mapping such that $f_{up} : (X,T_1,E) \longrightarrow (Y,T_2,K)$ Then the following are equivalent:*

(1) f_{up} is a $GFS-$semi open mapping.

(2) $f_{up}(int_{T_1}(F_\mu)) \sqsubseteq gfssint(f_{up}(F_\mu)) \forall F_\mu \in GFSS(X,E)$.

Proof. $1 \Longrightarrow 2$. Let $F_\mu \in GFSS(X,E)$. Since $int_{T_1}(F_\mu) \in T_1$. Then $f_{up}(int_{T_1}(F_\mu)) \in GFSSOS(Y) \forall F_\mu \in T_1$ by (1). It follows that,
$f_{up}(int_{T_1}(F_\mu)) = gfssint(f_{up}(int_{T_1}(F_\mu)) \sqsubseteq gfssint(f_{up}(F_\mu))$.
Therefore, $f_{up}(int_{T_1}(F_\mu)) \sqsubseteq gfssint(f_{up}(F_\mu)) \forall F_\mu GFSS(X,E)$.

$2 \Longrightarrow 1$. Let $F_\mu \in T_1$. By hypothesis, $f_{up}(int_{T_1}(F_\mu)) = f_{up}(F_\mu) \sqsubseteq gfssint(f_{up}(F_\mu)) \in GFSSOS(Y)$, but $gfssint(f_{up}(F_\mu)) \sqsubseteq f_{up}(F_\mu)$. So $gfssint(f_{up}(F_\mu)) = f_{up}(F_\mu) \in GFSSOS(Y) \forall F_\mu \in T_1$. Hence, f_{up} is a $GFS-$semi open mapping.

□

Theorem 4.6. *Let $f_{up} : GFSS(X,E) \longrightarrow GFSS(Y,K)$ be a $GFS-$semi open mapping. If $G_\delta \in GFSS(Y,K)$ and $H_\nu \in T_1^c$ such that $f_{up}^{-1}\ (G_\delta) \sqsubseteq H_\nu$. Then there exist $K_\gamma \in GFSSCS(Y)$ such that $G_\delta \sqsubseteq K_\gamma$ and $f_{up}^{-1}\ (K_\gamma) \sqsubseteq H_\nu$.*

Proof. Let $G_\delta \in GFSS(Y,K)$ and $H_\nu \in T_1^c$ such that $f_{up}^c(G_\delta) \sqsubseteq H_\nu$. Then $f_{up}(H_\nu^c) \sqsubseteq G_\delta^c$ from Theorems 2.20, where $H_\nu^c \in T_1$. Since f_{up} is $GFS-$semi open mapping. Then $f_{up}(H_\nu^c) \in GFSSOS(Y)$. Take $K_\gamma = [f_{up}(H_\nu^c)]^c$. Hence $K_\gamma \in GFSSCS(Y)$ such that $G_\delta \sqsubseteq K_\gamma$ and $f_{up}^{-1}(K_\gamma) = f_{up}^{-1}([f_{up}(H_\nu^c)]^c) \sqsubseteq f_{up}^{-1}(G_\delta^c)^c = f_{up}^{-1}(G_\delta) \sqsubseteq H_\nu$. This completes the proof.

□

Theorem 4.7. *Let (X,T_1,E) and (Y,T_2,K) be two $GFST-$spaces and f_{up} be a GFS mapping such that $f_{up} : (X,T_1,E) \longrightarrow (Y,T_2,K)$ Then the following are equivalent:*

(1) f_{up} is a $GFS-$semi closed mapping.

(2) $gfsscl(f_{up}(F_\mu)) \sqsubseteq f_{up}(cl_{T_2}(F_\mu)) \forall F_\mu \in GFSS(X,E)$.

Proof. It follows immediately from Theorem 4.6. □

5 Conclusion

Recently, many scientists have studied the soft set theory, which is initiated by Molodtsov [13] and easily applied to many problems having uncertainties from social life. In this work, we introduce the some new concepts in generalized fuzzy soft topological spaces such as generalized fuzzy semi open soft sets, generalized fuzzy semi closed soft sets, generalized fuzzy semi soft interior, generalized fuzzy semi soft closure and generalized fuzzy semi continuous functions and have established several interesting properties. We hope that the discussions in this paper will help researchers to enhance and promote the study on generalized fuzzy soft topology for its applications in practical life.

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