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Some new types of soft b-ordered mappings

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ABSTRACT. In [14] and [15], we introduced and studied the concepts of soft topological ordered spaces and some soft ordered maps, respectively. To contribute on these topics, we employ a notion of soft b-open sets to propose newly soft ordered maps, that consequently generalize existing comparable notions, namely soft xb-continuous, soft xb-open, soft xb-closed and soft xb-homeomorphism maps, where $x \in \{I, D, B\}$. We construct some examples to elucidate the relationships among them and discuss the necessary and sufficient conditions for each one of them. Additionally, we present the counterparts of these soft ordered maps on topological ordered spaces and investigate the links between them and the soft ordered maps initiated herein.

1. Introduction

A topological ordered space is a triple $(X, \tau, \preceq)$ consisting of two independent mathematical structures on a non-empty set X, namely a topology τ and a partial order relation $\preceq$. This concept was first introduced and studied by Nachbin [36] in 1965. He introduced new definitions, characterizations and many properties concerning the normal, regular and completely regular spaces via topological ordered spaces. Then McCartan [32], in 1968, studied T_i-ordered and strong T_i-ordered spaces $(i = 0, 1, 2, 3, 4)$ and gave complete descriptions for T_i-ordered spaces. Some authors generalized topological ordered spaces by replacing a partial order relation with some binary relations such as [28, 33, 39]; and the other authors generalized topological ordered spaces by replacing a topology with a supra topology such as [2, 5, 8, 12, 18, 20, 21, 22].

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In 1999, Molotdsov [35] introduced a new mathematical tool for dealing and approaching uncertainties and vagueness, namely soft set. He investigated the advantages of soft set theory compared with the probability and fuzzy set theory. Also, he studied its applications in several directions such as smoothness of functions, operations research, game theory, Riemann-integration and other disciplines. The research of the soft sets theory is becoming more and more active as a result of the ease of soft set's definition and its wide applications. Maji et al. [31] formulated various operators on soft sets such as soft subset and equality relations and soft union and intersection of two soft sets. To develop the soft set theory and make it more flexible, these operators were redefined in different manners (see, for example, [17, 30, 40]).

Shabir and Naz [42] came up the idea of soft topological spaces and studied their main properties. In particular, they introduced soft separation axioms and got some significant results related to them. Deep investigation for these soft axioms were given by Min [34]. To study and probe topological notions on soft topologies, Aygünoğlu and Aygün [16] formulated a concept of soft compact spaces, Rong [41] established the concepts of soft countability axioms and Lin [29] initiated the concepts of soft connected and paracompact spaces. Akdag and Ozkan [3] presented and investigated the notions of soft b-open sets and soft b-continuous maps. Then El-Sheikh et al. [26] introduced soft bT_i-spaces $(i = 0,1,2,3,4)$ and characterize each one of them. Abd El-latif [1] established the concepts of soft b-connected and soft b-hyperconnected spaces and discussed their main properties. Al-shami [9] introduced and studied a concept of soft somewhere dense sets. Al-shami [6, 7, 10] and El-Shafei et al. [24] pointed out some alleged results by presenting interesting counterexamples. In 2018, we [23] defined two new soft relations and investigated main properties. Recently, we [11] formulated the concepts of supra soft T_i-spaces and supra p-soft T_i-spaces $(i = 0,1,2,3,4)$.

The main goal of this work is to give other applications of soft b-open sets in defining some ordered maps on soft topological ordered spaces. We called them soft xb-continuous, soft xb-open, soft xb-closed and soft xb-homeomorphism maps, where $x \in \{I, D, B\}$. We describe each one of these maps and construct some examples in order to show the relationships among them. Also, we study the interrelations between the soft maps introduced herein and their counterparts of maps introduced in topological ordered spaces.

2. Preliminaries

This section is allocated to recall the definitions and results that will be needed in this manuscript.

Definition 2.1. [35] A notation G_E is said to be a soft set over X if G is a mapping of a set of parameters E into 2^X and it is written as a set of ordered pairs $G_E = \{(e, G(e)) : e \in E \text{ and } G(e) \in 2^X\}$.

Definition 2.2. [31] A soft set G_E over X is called a null soft set, denoting by $\widetilde{\Phi}$, if $G(e) = \emptyset$ for each $e \in E$; and it is called an absolute soft set, denoting by $\widetilde{X}$, if $G(e) = X$ for each $e \in E$.

Definition 2.3. [4] The relative complement of a soft set G_E is denoted by G^c_E, where $G^c : E \to 2^X$ is a mapping defined by $G^c(e) = X \setminus G(e)$ for each $e \in E$.

In this connection, it is worth noting that $x \notin G_E$ does not imply that $x \in G^c_E$.

Definition 2.4. [42] For $x \in X$ and a soft set G_E over X, we say that $x \in G_E$ if $x \in G(e)$ for each $e \in E$; and $x \notin G_E$ if $x \notin G(e)$ for some $e \in E$.

Definition 2.5.[42] A soft topology on a non-empty set X is a collection τ of soft sets over X under a fixed parameters set E such that τ contains absolute soft and null soft sets; and it is closed under finite soft intersection and closed under arbitrary soft union.
We stand for a triple (X, τ, E) as a soft topological space. Every member of τ is called soft open and its relative complement is called soft closed.

Proposition 2.6.[42] *Let (X, τ, E) be a soft topological space. Then $\tau_e = \{G(e) : G_E \in \tau\}$ defines a topology on X for each $e \in E$.*

Definition 2.7.[42] Let $x \in X$. A soft set (x, E) over X is defined as $x(e) = x$, for each $e \in E$.

Definition 2.8.[37] Consider (X, τ, E) is a soft topological space and τ_e is a topology on X as in the above proposition. Then $\tau^\star = \{G_E : G(e) \in \tau_e \text{ for each } e \in E\}$ is a soft topology on X finer than τ.
In this work, we term $\tau^\star$ an extended soft topology.

Definition 2.9.[43] Consider $f : X \to Y$ and $\phi : A \to B$ are two maps and let $f_\phi : S(X_A) \to S(Y_B)$ be a soft map. Let G_K and H_L be soft subsets of $S(X_A)$ and $S(Y_B)$, respectively. Then

(i) $f_\phi(G_K) = (f_\phi(G))_B$ is a soft subset of $S(Y_B)$ such that

$$f_\phi(G)(b) = \begin{cases} \bigcup_{a\in\phi^{-1}(b)\bigcap K} f(G(a)) & : \quad \phi^{-1}(b)\bigcap K \neq \emptyset \\ \emptyset & : \quad \phi^{-1}(b)\bigcap K = \emptyset \end{cases}$$

for each $b \in B$.

(ii) $f_\phi^{-1}(H_L) = (f_\phi^{-1}(H))_A$ is a soft subset of $S(X_A)$ such that

$$f_\phi^{-1}(H)(a) = \begin{cases} f^{-1}(H(\phi(a))) & : \quad \phi(a) \in L \\ \emptyset & : \quad \phi(a) \notin L \end{cases}$$

for each $a \in A$.

Remark 2.10. *Henceforth, a soft map $f_\phi : S(X_A) \to S(Y_B)$ implies that a map f of the universe set X into the universe set Y and a map ϕ of the set of parameters A into the set of parameters B.*

Definition 2.11.[43] A soft map $f_\phi : S(X_A) \to S(Y_B)$ is said to be injective (resp. surjective, bijective) if f and ϕ are injective (resp. surjective, bijective).

Proposition 2.12.[43] *Consider $f_\phi : S(X_A) \to S(Y_B)$ is a soft map and let G_A and H_B be two soft subsets of $S(X_A)$ and $S(Y_B)$, respectively. Then we have the following results:*

(i) $G_A \widetilde{\subseteq} f_\phi^{-1} f_\phi(G_A)$ and the equality relation holds if f_ϕ is injective.

(ii) $f_\phi f_\phi^{-1}(H_B) \widetilde{\subseteq} H_B$ and the equality relation holds if f_ϕ is surjective.

Definition 2.13. [3] A soft subset H_E of (X,τ,E) is said to be soft b-open if $H_E \widetilde{\subseteq} int(cl(H_E)) \widetilde{\bigcup} cl(int(H_E))$. Its relative complement is said to be soft b-closed.

Definition 2.14.[3, 42] For a soft subset H_E of (X,τ,E), we define the following four operators:

(i) $int(H_E)$ (resp. $int_b(H_E)$) is the largest soft open (resp. soft b-open) set contained in H_E.

(ii) $cl(H_E)$ (resp. $cl_b(H_E)$) is the smallest soft closed (resp. soft b-closed) set containing H_E.

Definition 2.15.[3] A soft map $f_\phi : (X,\tau,A) \to (Y,\theta,B)$ is said to be:

(i) Soft b-continuous if the inverse image of each soft open subset of (Y,θ,B) is a soft b-open subset of (X,τ,A).

(ii) Soft b-open (resp. Soft b-closed) if the image of each soft open (resp. Soft closed) subset of (X,τ,A) is a soft b-open (resp. soft b-closed) subset of (Y,θ,B).

(iii) Soft b-homeomorphism if it is bijective, soft b-continuous and soft b-open.

Definition 2.16.[19, 37] A soft subset P_E over X is called soft point if there exists $e \in E$ and there exists $x \in X$ such that $P(e) = \{x\}$ and $P(a) = \emptyset$ for each $a \in E \setminus \{e\}$. A soft point will be shortly denoted by P_e^x and we say that $P_e^x \in G_E$ if $x \in G(e)$.

Definition 2.17.[14] Let $\preceq$ be a partial order relation on a non-empty set X and let E be a set of parameters. A triple $(X,E,\preceq)$ is said to be a partially ordered soft set.

Definition 2.18.[14] We define an increasing soft operator $i : (SS(X_E),\preceq) \to (SS(X_E),\preceq)$ and a decreasing soft operator $d : (SS(X_E),\preceq) \to (SS(X_E),\preceq)$ as follows: For each soft subset G_E of $SS(X_E)$

(i) $i(G_E) = (iG)_E$, where iG is a mapping of E into X given by $iG(e) = i(G(e)) = \{x \in X : y \preceq x$ for some $y \in G(e)\}$.

(ii) $d(G_E) = (dG)_E$, where dG is a mapping of E into X given by $dG(e) = d(G(e)) = \{x \in X : x \preceq y$ for some $y \in G(e)\}$.

Definition 2.19.[14] Let $\preceq$ be a partial order relation on a non-empty set X and let E be a set of parameters. A triple $(X,E,\preceq)$ is said to be a partially ordered soft set.

Definition 2.20.[14] A soft subset G_E of a partially ordered soft set $(X,E,\preceq)$ is said to be increasing (resp. decreasing) if $G_E = i(G_E)$(resp. $G_E = d(G_E)$).

Theorem 2.21.[14] *If a soft map $f_\phi : (S(X_A),\preceq_1) \to (S(Y_B),\preceq_2)$ is increasing, then the inverse image of each increasing (resp. decreasing) soft subset of $\widetilde{Y}$ is an increasing (resp. a decreasing) soft subset of $\widetilde{X}$.*

Definition 2.22.[14] A quadrable system $(X,\tau,E,\preceq)$ is said to be a soft topological ordered space, where (X,τ,E) is a soft topological space and $(X,E,\preceq)$ is a partially ordered soft set. Henceforth, the two notations $(X,\tau,E,\preceq_1)$ and $(Y,\theta,F,\preceq_2)$ stand for soft topological ordered spaces.

Definition 2.23.[15] The composition of two soft maps $f_\phi : (X,\tau,E,\preceq_1) \to (Y,\theta,F,\preceq_2)$ and $g_\lambda : (Y,\theta,F,\preceq_2) \to (Z,\upsilon,K,\preceq_3)$ is a soft map $f_\phi \circ g_\lambda : (X,\tau,E,\preceq_1) \to (Z,\upsilon,K,\preceq_3)$ and is given by $(f_\phi \circ g_\lambda)(P_e^x) = f_\phi(g_\lambda(P_e^x))$.

3. Soft $I(D,B)b$-continuity

This section introduces $I(D,B)b$-continuity concepts at soft point, ordinary point and the universe set. The results related to the equivalent conditions for each one of these concepts at the ordinary points are proved and some illustrative examples are given.

Definition 3.1. A soft subset H_E of $(X, \tau, E, \preceq_1)$ is said to be:

(i) Soft I (resp. soft D, soft B) b-open if it is soft b-open and increasing (resp. decreasing, balancing).

(ii) Soft I (resp. soft D, soft B) b-closed if it is soft b-closed and increasing (resp. decreasing, balancing).

Definition 3.2. A soft map $f_\phi : (X, \tau, E, \preceq_1) \to (Y, \theta, F, \preceq_2)$ is called:

(i) Soft I (resp. soft D, soft B) b-continuous at $P_e^x \in \widetilde{X}$ if for each soft open set H_F containing $f_\phi(P_e^x)$, there exists a soft I (resp. soft D, soft B) b-open set G_E containing P_e^x such that $f_\phi(G_E)\widetilde{\subseteq} H_F$.

(ii) Soft I (resp. soft D, soft B) b-continuous at $x \in X$ if it is soft I (resp. soft D, soft B) b-continuous at each P_e^x.

(iii) Soft I (resp. soft D, soft B) b-continuous if it is soft I (resp. soft D, soft B) b-continuous at each $x \in X$.

Theorem 3.3. *A soft map $f_\phi : (X, \tau, E, \preceq_1) \to (Y, \theta, F, \preceq_2)$ is soft I (resp. soft D, soft B) b-continuous if and only if the inverse image of each soft open subset of $\widetilde{Y}$ is a soft I (resp. soft D, soft B) b-open subset of $\widetilde{X}$.* *Proof.* We prove the theorem in case of f_ϕ is soft Db-continuous and the other cases can be achieved similarly.
Necessity: Let G_F be a soft open subset of $\widetilde{Y}$, Then we have the following two cases:

(i) Either $f_\phi^{-1}(G_F) = \widetilde{\emptyset}$.

(ii) Or $f^{-1}(G_F) \neq \widetilde{\emptyset}$. By choosing $P_e^x \in X$ such that $P_e^x \in f_\phi^{-1}(G_F)$, we obtain that $f_\phi(P_e^x) \in G_F$. So there exists a soft Db-open set H_E containing P_e^x such that $f_\phi(H_E)\widetilde{\subseteq} G_F$. Since P_e^x is chosen arbitrary, then $f_\phi^{-1}(G_F) = \widetilde{\bigcup}_{P_e^x \in f_\phi^{-1}(G_F)} H_E$.

From the two cases above, we conclude that $f_\phi^{-1}(G_F)$ is a soft Db-open subset of $\widetilde{X}$. *Sufficiency:* Let G_F be a soft open subset of $\widetilde{Y}$ containing $f_\phi(P_e^x)$. Then $P_e^x \in f_\phi^{-1}(G_F)$. By hypothesis, $f_\phi^{-1}(G_F)$ is a soft Db-open set. Since $f_\phi(f_\phi^{-1}(G_F))\widetilde{\subseteq} G_F$, then f_ϕ is a soft Db-continuous map at $P_e^x \in X$ and since P_e^x is chosen arbitrary, then f_ϕ is a soft Db-continuous map. □

Remark 3.4. *From Definition 3.2, we can note the following:*

(i) Every soft $I(D, B)$ b-continuous map is always soft b-continuous.

(ii) Every soft Bb-continuous map is soft Ib-continuous or soft Db-continuous.

The two examples below elucidate that the converse of the two results of the remark above need not be true in general.

Example 3.5. *Let $E = \{0.1, 0.2\}$ be a parameters set and $X = \{i, j, k, l\}$ be a universe set and consider $\phi : E \to E$ and $f : X \to X$ are two identity maps. Let $\preceq = \triangle \bigcup \{(i, k)\}$ be a partial order relation on X and consider $\tau = \{\widetilde{\emptyset}, \widetilde{X}, F_E, G_E\}$*

and $\theta = \{\widetilde{\emptyset}, \widetilde{Y}, H_E\}$ are two soft topologies on X, where $F_E = \{(0.1, \{i\}), (0.2, \{k, l\})\}$, $G_E = \{(0.1, \emptyset), (0.2, \{k\})\}$ and $H_E = \{(0.1, \{i\}), (0.2, \{j, k\})\}$. For a soft map $f_\phi : (X, \tau, E, \preceq) \to (X, \theta, E, \preceq)$, we find that $f_\phi^{-1}(H_E) = H_E$ is a soft b-open set. So f_ϕ is a soft b-continuous map. On the other hand, $f_\phi^{-1}(H_E)$ is neither a soft D b-open set nor a soft I b-open set. Hence f_ϕ is not a soft I(soft D, soft B) b-continuous map.

Example 3.6. *In Example above, if we replace only the partial order relation by $\preceq = \triangle \bigcup \{(j, l)\}$ (resp. $\preceq = \triangle \bigcup \{(l, i)\}$), then the soft map f_ϕ is soft D-continuous (resp. soft I-continuous), but is not soft B-continuous.*

Definition 3.7. For a soft subset H_E of $(X, \tau, E, \preceq)$, we define the following six operators:

(i) H_E^{ibo}(resp. H_E^{dbo}, H_E^{bbo}) is the largest soft I (resp. soft D, soft B) b-open set contained in H_E.

(ii) H_E^{ibcl}(resp. H_E^{dbcl}, H_E^{bbcl}) is the smallest soft I (resp. soft D, soft B) b-closed set containing H_E.

Lemma 3.8. *For any soft subset H_E of $(X, \tau, E, \preceq)$, the following statements hold:*

(i) $(H_E^{dbcl})^c = (H_E^c)^{ibo}$.

(ii) $(H_E^{ibcl})^c = (H_E^c)^{dbo}$.

(iii) $(H_E^{bbcl})^c = (H_E^c)^{bbo}$.

Proof.

(i) $(H_E^{dbcl})^c = \{\widetilde{\bigcup} F_E : F_E$ is a soft Db-closed set containing $H_E\}^c$

$= \widetilde{\bigcap}\{F_E^c : F_E^c$ is a soft Ib-open set contained in $H_E^c\} = (H_E^c)^{ibo}$.

By analogy with **(i)**, one can prove **(ii)** and **(iii)**. □

Theorem 3.9. *The following five properties of a soft map $f_\phi : (X, \tau, E \preceq_1) \to (Y, \theta, F, \preceq_2)$ are equivalent:*

(i) f_ϕ is soft I b-continuous;

(ii) $f_\phi^{-1}(L_F)$ is a soft D b-closed subset of $\widetilde{X}$ for each soft closed subset L_F of $\widetilde{Y}$;

(iii) $(f_\phi^{-1}(M_F))^{dbcl} \widetilde{\subseteq} f_\phi^{-1}(cl(M_F))$ for every $M_F \widetilde{\subseteq} \widetilde{Y}$;

(iv) $f_\phi(N_E^{dbcl}) \widetilde{\subseteq} cl(f_\phi(N_E))$ for every $N_E \widetilde{\subseteq} \widetilde{X}$;

(v) $f_\phi^{-1}(int(M_F)) \widetilde{\subseteq} (f_\phi^{-1}(M_F))^{ibo}$ for every $M_F \widetilde{\subseteq} \widetilde{Y}$.

Proof. **(i) ⇒ (ii)** : Consider L_F is a soft closed subset of $\widetilde{Y}$. By hypothesis, $f_\phi^{-1}(L_F^c)$ is a soft Ib-open subset of $\widetilde{X}$ and by the fact that $f_\phi^{-1}(L_F^c) = (f_\phi^{-1}(L_F))^c$, we obtain that $f_\phi^{-1}(L_F)$ is soft Db-closed as required.
(ii) ⇒ (iii) : It follows from statement **(ii)** that $f_\phi^{-1}(cl(M_E))$ is a soft Db-closed subset of $\widetilde{X}$ for every $M_F \widetilde{\subseteq} \widetilde{Y}$. So $(f_\phi^{-1}(M_F))^{dbcl} \widetilde{\subseteq} (f_\phi^{-1}(cl(M_F))^{dbcl} = f_\phi^{-1}(cl(M_F))$.
(iii) ⇒ (iv) : From the fact that $N_E^{dbcl} \widetilde{\subseteq} (f_\phi^{-1}(f_\phi(N_E))^{dbcl}$ and from **(iii)**, we have $(f_\phi^{-1}(f_\phi(N_E))^{dbcl} \widetilde{\subseteq} f_\phi^{-1}(cl(f_\phi(N_E))$. This implies that $f_\phi(N_E^{dbcl}) \widetilde{\subseteq} cl(f_\phi(N_E))$.
(iv) ⇒ (v) : For any soft subset M_F of $\widetilde{Y}$, we obtain from Lemma 3.8 that $f_\phi(\widetilde{X} - (f_\phi^{-1}(N_E))^{ibo}) = f_\phi(((f_\phi^{-1}(N_E))^c)^{dbcl})$. It follows from statement **(iv)**, that $f_\phi(((f_\phi^{-1}(N_E))^c)^{dbcl}) \widetilde{\subseteq} cl(f_\phi(f_\phi^{-1}(N_E))^c) = cl(f_\phi(f_\phi^{-1}(N_E^c))) \widetilde{\subseteq} cl(\widetilde{Y} - N_E) = \widetilde{Y} - int(N_E)$. Therefore $(\widetilde{X} - (f_\phi^{-1}(N_E))^{ibo}) \widetilde{\subseteq} f_\phi^{-1}(\widetilde{Y} - int(N_E)) = \widetilde{X} - f_\phi^{-1}(int(N_E))$. Thus $f_\phi^{-1}(int(N_E)) \widetilde{\subseteq} (f_\phi^{-1}(N_E))^{ibo}$.

(v) $\Rightarrow$ (i): Consider M_F is a soft open subset of $\widetilde{Y}$. Then $f_{\phi}^{-1}(M_F) = f_{\phi}^{-1}(int(M_F))\widetilde{\subseteq}(f_{\phi}^{-1}(M_F))^{ibo}$. So $(f_{\phi}^{-1}(M_F))^{ibo} = f_{\phi}^{-1}(M_F)$ and this means that $f_{\phi}^{-1}(M_F)$ is a soft Ib-open subset of $\widetilde{X}$. Hence the desired result is proved. □

Theorem 3.10. *The following five properties of a soft map $f_{\phi} : (X, \tau, E \preceq_1) \to (Y, \theta, F, \preceq_2)$ are equivalent:*

(i) f_{ϕ} is soft D b-continuous (resp. soft B b-continuous);

(ii) $f_{\phi}^{-1}(L_F)$ is a soft I b-closed (resp. soft B b-closed) subset of $\widetilde{X}$ for each soft closed subset L_F of $\widetilde{Y}$;

(iii) $(f_{\phi}^{-1}(M_F))^{ibcl}\widetilde{\subseteq}f_{\phi}^{-1}(cl(M_F))$(resp. $(f_{\phi}^{-1}(M_F))^{bbcl}\widetilde{\subseteq}f_{\phi}^{-1}(cl(M_F))$ for every $M_F\widetilde{\subseteq}\widetilde{Y}$;

(iv) $f_{\phi}(N_E^{ibcl})\widetilde{\subseteq}cl(f_{\phi}(N_E))$(resp. $f_{\phi}(N_E^{bbcl})\widetilde{\subseteq}cl(f_{\phi}(N_E))$ for every $N_E\widetilde{\subseteq}\widetilde{X}$;

(v) $f_{\phi}^{-1}(int(M_F))\widetilde{\subseteq}(f_{\phi}^{-1}(M_F))^{dbo}$(resp. $f_{\phi}^{-1}(int(M_F))\widetilde{\subseteq}(f_{\phi}^{-1}(M_F))^{bbo}$ for every $M_F\widetilde{\subseteq}\widetilde{Y}$.

Proof. The proof is similar to that of Theorem 4.9. □

Definition 3.11. A map $(X, \tau, \preceq_1) \to (Y, \theta, \preceq_2)$ is said to be I (resp. D, B) b-continuous if the inverse image of each open set is I (resp. D, B) b-open.

Theorem 3.12. *Let $\tau^{\star}$ be an extended soft topology on X. Then a soft map $g_{\phi} : (X, \tau^{\star}, E, \preceq_1) \to (Y, \theta, F, \preceq_2)$ is soft I (resp. soft D, soft B) b-continuous If and only if a map $g : (X, \tau_e^{\star}, \preceq_1) \to (Y, \theta_{\phi(e)}, \preceq_2)$ is I (resp. D, B) b-continuous.*

Proof. Necessity: Let U be an open subset of $(Y, \theta_{\phi(e)}, \preceq_2)$. Then there exists a soft open subset G_F of $(Y, \theta, F, \preceq_2)$ such that $G(\phi(e)) = U$. Since g_{ϕ} is a soft I (resp. soft D, soft B) b-continuous map, then $g_{\phi}^{-1}(G_F)$ is a soft I (resp. soft D, soft B) b-open set. From Definition 2.9, it follows that a soft subset $g_{\phi}^{-1}(G_F) = (g_{\phi}^{-1}(G))_E$ of $(X, \tau, E, \preceq_1)$ is given by $g_{\phi}^{-1}(G)(e) = g^{-1}(G(\phi(e)))$ for each $e \in E$. By hypothesis, $\tau^{\star}$ is an extended soft topology on X, we obtain that a subset $g^{-1}(G(\phi(e))) = g^{-1}(U)$ of $(X, \tau_e, \preceq_1)$ is I (resp. D, B) b-open. Hence a map g is I (resp. D, B) b-continuous.

Sufficiency: Let G_F be a soft open subset of $(Y, \theta, F, \preceq_2)$. Then from Definition 2.9, it follows that a soft subset $g_{\phi}^{-1}(G_F) = (g_{\phi}^{-1}(G))_E$ of $(X, \tau^{\star}, E, \preceq_1)$ is given by $g_{\phi}^{-1}(G)(e) = g^{-1}(G(\phi(e)))$ for each $e \in E$. Since a map g is I (resp. D, B) b-continuous, then a subset $g^{-1}(G(\phi(e)))$ of $(X, \tau_e^{\star}, \preceq_1)$ is I (resp. D, B) b-open. By hypothesis, $\tau^{\star}$ is an extended soft topology on X, we obtain that $g_{\phi}^{-1}(G_F)$ is a soft I (resp. soft D, soft B) b-open subset of $(X, \tau^{\star}, E, \preceq_1)$. Hence a soft map g_{ϕ} is soft I (resp. soft D, soft B) b-continuous. □

Proposition 3.13. *Let $f_{\phi} : (X, \tau, E \preceq_1) \to (Y, \theta, F, \preceq_2)$ be a bijective soft B b-continuous map. If $\preceq_1$ is linearly ordered, then θ is the indiscrete soft topology.*

4. Soft $I(D, B)b$-openness and soft $I(D, B)b$-closedness

In the following part, we present the notions of soft $I(D, B)b$-open and soft $I(D, B)b$-closed maps and elucidate the relationships among them with the help of examples. Then we characterize each one of these concepts and study some properties.

Definition 4.1. A soft map $f_\phi : (X, \tau, E, \preceq_1) \to (Y, \tau, F, \preceq_2)$ is called:

(i) Soft I (resp. soft D, soft B) b-open if the image of every soft open subset of $\widetilde{X}$ is a soft I (resp. soft D, soft B) b-open subset of $\widetilde{Y}$.

(ii) Soft I (resp. soft D, soft B) b-closed if the image of every soft closed subset of $\widetilde{X}$ is a soft I (resp. soft D, soft B) b-closed subset of $\widetilde{Y}$.

Remark 4.2. *From Definition 4.1, we can note the following:*

(i) Every soft $I(D, B)b$-open map is soft b-open.

(ii) Every soft $I(D, B)b$-closed map is soft b-closed.

(iii) Every soft Bb-open (resp. soft Bb-closed) map is soft Ib-open or soft Db-open (resp. soft Ib-closed or soft Db-closed).

We construct the following two examples to show that the converse of the three statements of remark above fails.

Example 4.3. *Assume that a soft map $f_\phi : (X, \tau, E, \preceq) \to (X, \theta, E, \preceq)$ is the same as in Example 3.5. Then it is soft b-open and soft b-closed map. Because $f_\phi(F_E) = F_E$ is neither a soft Db-open set nor a soft Ib-open set, then f_ϕ is not a soft I(soft D, soft B) b-open map and because $f_\phi(F_E^c) = F_E^c$ is neither a soft Db-closed set nor a soft Ib-closed set, then f_ϕ is not a soft I(soft D, soft B) b-closed map.*

Example 4.4. *Assume that a soft map $f_\phi : (X, \tau, E, \preceq) \to (X, \theta, E, \preceq)$ is the same as in Example 3.5. Then if we replace the partial order relation by $\preceq = \triangle \bigcup \{(l, k)\}$, then the soft map f_ϕ is soft Ib-open, but it is not soft Bb-open. Also, if we replace the partial order relation by $\preceq = \triangle \bigcup \{(l, j)\}$), then the soft map f_ϕ is soft Db-closed, but it is not soft Bb-closed.*

Theorem 4.5. *The following three properties of a soft map $f_\phi : (X, \tau, E \preceq_1) \to (Y, \theta, F, \preceq_2)$ are equivalent:*

(i) f_ϕ is soft Ib-open;

(ii) $int(f_\phi^{-1}(M_F)) \widetilde{\subseteq} f_\phi^{-1}(M_F^{ibo})$ for every $M_F \widetilde{\subseteq} \widetilde{Y}$;

(iii) $f_\phi(int(N_E)) \widetilde{\subseteq} (f_\phi(N_E))^{ibo}$ for every $N_E \widetilde{\subseteq} \widetilde{X}$.

Proof. **(i)** $\Rightarrow$ **(ii)**: Given a soft subset M_F of $\widetilde{Y}$, it is obvious that $int(f_\phi^{-1}(M_F))$ is a soft open subset of $\widetilde{X}$. Then, by hypothesis, it follows that $f_\phi(int(f_\phi^{-1}(M_F)))$ is a soft Ib-open subset of $\widetilde{Y}$. Since $f_\phi(int(f_\phi^{-1}(M_F))) \widetilde{\subseteq} f_\phi(f_\phi^{-1}(M_F)) \widetilde{\subseteq} M_F$, then $int(f_\phi^{-1}(M_F)) \widetilde{\subseteq} f_\phi^{-1}(M_F^{ibo})$.

(ii) $\Rightarrow$ **(iii)**: Given a soft subset N_E of $\widetilde{X}$, from **(ii)**, we obtain that $int(f_\phi^{-1}(f_\phi(N_E))) \widetilde{\subseteq} f_\phi^{-1}((f_\phi(N_E))^{ibo})$. Since $int(N_E) \widetilde{\subseteq} f_\phi^{-1}(f_\phi(int(f_\phi^{-1}(f_\phi(N_E))))) \widetilde{\subseteq} f_\phi^{-1}((f_\phi(N_E))^{ibo})$, then $f_\phi(int(N_E)) \widetilde{\subseteq} (f_\phi(N_E))^{ibo}$ as required.

(iii) $\Rightarrow$ **(i)**: Let G_E be a soft open subset of $\widetilde{X}$. Then $f_\phi(int(G_E)) = f_\phi(G_E) \widetilde{\subseteq} (f_\phi(G_E))^{ibo}$. Hence f_ϕ is a soft Ib-open map. □

In a similar manner, one can prove the following theorem.

Theorem 4.6. *The following three properties of a soft map* $f_\phi : (X, \tau, E \preceq_1) \to (Y, \theta, F, \preceq_2)$ *are equivalent:*

(i) f_ϕ is soft Db-open (resp. soft Bb-open);

(ii) $int(f_\phi^{-1}(M_F))\widetilde{\subseteq} f_\phi^{-1}(M_F^{dbo})$(resp. $int(f_\phi^{-1}(M_F))\widetilde{\subseteq} f_\phi^{-1}(M_F^{bbo})$) for every $M_F\widetilde{\subseteq}\widetilde{Y}$;

(iii) $f_\phi(int(N_E))\widetilde{\subseteq}(f_\phi(N_E))^{dbo}$(resp. $f_\phi(int(N_E))\widetilde{\subseteq}(f_\phi(N_E))^{bbo}$) for every $N_E\widetilde{\subseteq}\widetilde{X}$.

Theorem 4.7. *The following three statements hold for a soft map* $f_\phi : (X, \tau, E \preceq_1) \to (Y, \theta, F, \preceq_2)$*:*

(i) f_ϕ is soft Ib-closed if and only if $(f_\phi(G_E))^{ibcl}\widetilde{\subseteq} f_\phi(cl(G_E))$ for every $G_E\widetilde{\subseteq}\widetilde{X}$.

(ii) f_ϕ is soft Db-closed if and only if $(f_\phi(G_E))^{dbcl}\widetilde{\subseteq} f_\phi(cl(G_E))$ for every $G_E\widetilde{\subseteq}\widetilde{X}$.

(iii) f_ϕ is soft Bb-closed if and only if $(f_\phi(G_E))^{bbcl}\widetilde{\subseteq} f_\phi(cl(G_E))$ for every $G_E\widetilde{\subseteq}\widetilde{X}$.

Proof. We only prove the first statement and the others follow similar lines.
Necessity: Since f_ϕ is soft Ib-closed, then $f_\phi(cl(G_E))$ is a soft Ib-closed subset of $\widetilde{Y}$ and since $f_\phi(G_E)\widetilde{\subseteq} f_\phi(cl(G_E))$, then $(f_\phi(G_E))^{ibcl}\widetilde{\subseteq} f_\phi(cl(G_E))$.
Sufficiency: Consider H_E is a soft closed subset of $\widetilde{X}$. Then $f_\phi(H_E)\widetilde{\subseteq}(f_\phi(H_E))^{ibcl}\widetilde{\subseteq} f_\phi(cl(H_E)) = f_\phi(H_E)$. Therefore $f_\phi(H_E) = (f_\phi(H_E))^{ibcl}$. This means that $f_\phi(H_E)$ is a soft Ib-closed set. Hence the proof is complete. □

Theorem 4.8. *The following three statements hold for a bijective soft map* $f_\phi : (X, \tau, E \preceq_1) \to (Y, \theta, F, \preceq_2)$*:*

(i) f_ϕ is soft I (resp. soft D, soft B) b-open if and only if f_ϕ is soft D(resp. soft D, soft B) b-closed.

(ii) f_ϕ is soft I (resp. soft D, soft B) b-open if and only if f_ϕ^{-1} is soft I (resp. soft D, soft B) b-continuous.

(iii) f_ϕ is soft D(resp. soft I, soft B) b-closed if and only if f_ϕ^{-1} is soft I (resp. soft D, soft B) b-continuous.

Proof. For the sake of brevity, we only give proofs for the cases outside the parenthesis and the cases between parenthesis can be made similarly.

(i) To prove the necessary condition, let H_E be a soft closed subset of $\widetilde{X}$ and consider f_ϕ is a soft Ib-open map. Then H_E^c is soft open and $f_\phi(H_E^c)$ is soft Ib-open. It follows from the bijectiveness of f_ϕ, that $f_\phi(H_E^c) = [f_\phi(H_E)]^c$. This automatically implies that $f_\phi(H_E)$ is soft Db-closed. Thus f_ϕ is a soft Db-closed map. In a similar manner, we can prove the sufficient condition.

(ii) *Necessity:* Let G_E be a soft open subset of $\widetilde{X}$ and consider f_ϕ is a soft Ib-open map. Then $f_\phi(G_E)$ is soft Ib-open. It follows from the bijectiveness of f_ϕ, that $f_\phi(G_E) = (f_\phi^{-1})^{-1}(G_E)$. This automatically implies that $(f_\phi^{-1})^{-1}(G_E)$ is soft Ib-open. Thus f_ϕ^{-1} is a soft Ib-continuous map. In a similar manner, we can prove the sufficient condition.

(iii) The proof of this statement comes immediately from **(i)** and **(ii)** above.

□

Definition 4.9. A map $(X, \tau, \preceq_1) \to (Y, \theta, \preceq_2)$ is said to be:

(i) I (resp. D, B) b-open if the image of each open set is I (resp. D, B) b-open.

(ii) I (resp. D, B) b-closed if the image of each open set is I (resp. D, B) b-closed.

Theorem 4.10. *Let $\theta^\star$ be an extended soft topology on Y and ϕ is an injective map. Then a soft map $g_\phi : (X, \tau, E, \preceq_1) \to (Y, \theta^\star, F, \preceq_2)$ is soft I (resp. soft D, soft B) b-open If and only if a map $g : (X, \tau_e, \preceq_1) \to (Y, \theta^\star_{\phi(e)}, \preceq_2)$ is I (resp. D, B) b-open.* *Proof.* To prove the necessary part, let U be an open subset of $(X, \tau_e, \preceq_1)$ and $\phi(e) = f$. Then there exists a soft open subset G_E of $(X, \tau, E, \preceq_1)$ such that $G(e) = U$. Since g_ϕ is a soft I (resp. soft D, soft B) b-open map, then $g_\phi(G_E)$ is a soft I (resp. soft D, soft B) b-open set. From Definition 2.9, it follows that a soft subset $g_\phi(G_E) = (g_\phi(G))_F$ of $(Y, \theta, F, \preceq_2)$ is given by $g_\phi(G)(f) = \bigcup_{e \in \phi^{-1}(f)} g(G(e))$ for each $f \in F$. By hypothesis, $\theta^\star$ is an extended soft topology on Y, a subset $\bigcup_{e \in \phi^{-1}(f)} g(G(e)) = g(U)$ of $(Y, \theta_{\phi(e)}, \preceq_2)$ is I (resp. D, B) b-open. Hence a map g is I (resp. D, B) b-open.

To prove the sufficient part, let G_E be a soft open subset of $(X, \tau, E, \preceq_1)$. Then from Definition 2.9, it follows that a soft subset $g_\phi(G_E) = (g_\phi(G))_F$ of $(Y, \theta^\star, F, \preceq_2)$ is given by $g_\phi(G)(f) = \bigcup_{e \in \phi^{-1}(f)} g(G(e))$ for each $f \in F$. Since a map g is I (resp. D, B) b-open, then a subset $\bigcup_{e \in \phi^{-1}(f)} g(G(e))$ of $(Y, \theta^\star_{\phi(e)}, \preceq_2)$ is I (resp. D, B) b-open. By hypothesis, $\theta^\star$ is an extended soft topology on Y, $g_\phi(G_E)$ is a soft I (resp. soft D, soft B) b-open subset of $(Y, \theta^\star, F, \preceq_2)$. Hence a soft map g_ϕ is soft I (resp. soft D, soft B) b-open. □

The result above is restated in case of a soft I (resp. soft D, soft B) b-closed map and one can prove them similarly. So the proof will be omitted.

Theorem 4.11. *Let $\theta^\star$ be an extended soft topology on Y and ϕ is an injective map. Then a soft map $g_\phi : (X, \tau, E, \preceq_1) \to (Y, \theta^\star, F, \preceq_2)$ is soft I (resp. soft D, soft B) b-closed if and only if a map $g : (X, \tau_e, \preceq_1) \to (Y, \theta^\star_{\phi(e)}, \preceq_2)$ is I (resp. D, B) b-closed.*

Proposition 4.12. *Let an bijective soft map $f_\phi : (X, \tau, E \preceq_1) \to (Y, \theta, F, \preceq_2)$ be soft Bb-open or soft Bb-closed. Then, if $\preceq_2$ is linearly ordered, then τ is the indiscrete soft topology.*

Proposition 4.13. *Let $f_\phi : (X, \tau, E, \preceq_1) \to (Y, \theta, F, \preceq_2)$ and $g_\lambda : (Y, \theta, F, \preceq_2) \to (Z, \upsilon, K, \preceq_3)$ be two soft maps. Then for $x \in \{I, D, B\}$, then following properties hold.*

(i) If f_ϕ is a soft xb-continuous map and g_λ is a soft continuous map, then $g_\lambda \circ f_\phi$ is a soft x-continuous map.

(ii) If f_ϕ is a soft open (resp. soft closed) map and g_λ is a soft xb-open (resp. xb-closed) map, then $g_\lambda \circ f_\phi$ is a soft x-open (resp. xb-closed) map.

(iii) If $g_\lambda \circ f_\phi$ is a soft x-open map and f_ϕ is surjective soft continuous, then g_λ is a soft x-open map.

(iv) If $g_\lambda \circ f_\phi$ is a soft closed map and g_λ is an injective soft x-continuous map, then f_ϕ is a soft y-closed map, where $(x, y) \in \{(I, D), (D, I), (B, B)\}$.

5. Soft $I(D, B)b$-homeomorphism

We define and investigate in this section, the concepts of soft $I(D, B)$-homeomorphism maps. We discussed their main features and verify some findings related to them.

Definition 5.1. A bijective soft map $g_\phi : (X, \tau, E, \preceq_1) \to (Y, \theta, F, \preceq_2)$ is called soft I (resp. soft D, soft B) b-homeomorphism if it is soft Ib-continuous and soft Ib-open (resp. soft Db-continuous and soft Db-open, soft Bb-continuous and soft Bb-open).

Remark 5.2. *From Definition 5.1, we can note the following:*

(i) Every soft $I(softD, softB)b$-homeomorphism map is soft b-homeomorphism.

(ii) Every soft Bb-homeomorphism map is soft Ib-homeomorphism or soft Db-homeomorphism.

The two items of the remark above are not conversely as the following examples show.

Example 5.3. *Assume that a soft map $f_\phi : (X, \tau, E, \preceq) \to (X, \theta, E, \preceq)$ is the same as in Example 3.5. Then we obtain from the discussion of Example 3.5 and Example 4.3 that f_ϕ is soft b-homeomorphism, but it is not soft I (soft D, soft B) b-homeomorphism.*

Example 5.4. *Assume that a soft map $f_\phi : (X, \tau, E, \preceq_1) \to (X, \theta, E, \preceq_2)$ is the same as in Example 3.5, where the two partial order relations $\preceq_1 = \triangle \bigcup \{(j, l)\}$ and $\preceq_2$ is an equality relation, then the soft map f_ϕ is soft Db-homeomorphism, but is not soft Bb-homeomorphism. Also, if we define the two partial order relations $\preceq_1 = \triangle \bigcup \{(l, i)\}$ and $\preceq_2$ is an equality relation, then the soft map f_ϕ is soft Ib-homeomorphism, but is not soft Bb-homeomorphism.*

Theorem 5.5. *Consider $f_\phi : (X, \tau, E, \preceq_1) \to (Y, \theta, F, \preceq_2)$ is a bijective soft map and let $(\gamma, \lambda) \in \{(Ib, dbcl), (Db, ibcl), (Bb, bbcl)\}$. If $(f_\phi(G_E))^\lambda = f_\phi(cl_b(G_E)) = cl_b(f_\phi(G_E)) = f_\phi(G_E^\lambda)$ for every $G_E \widetilde{\subseteq} \widetilde{X}$,* then f_ϕ is soft γ-homeomorphism.

Proof. We make a proof for the theorem in case of $(\gamma, \lambda) = (Ib, dbcl)$ and the other follow similar line. The equality relation $(f_\phi(G_E))^{dbcl} = f_\phi(cl_b(G_E)) = cl_b(f_\phi(G_E)) = f_\phi(G_E^{dbcl})$ implies that $f_\phi(G_E^{dbcl}) \widetilde{\subseteq} cl_b(f_\phi(G_E)) \widetilde{\subseteq} cl(f_\phi(G_E))$ and $(f_\phi(G_E))^{dbcl} \widetilde{\subseteq} f_\phi(cl_b(G_E)) \widetilde{\subseteq} f_\phi(cl(G_E))$. So f_ϕ is soft Ib-continuous and soft Db-closed map. Hence the desired result is proved. □

Theorem 5.6. *If a bijective soft map $f_\phi : (X, \tau, E, \preceq_1) \to (Y, \theta, F, \preceq_2)$ is soft Ib-continuous (resp. soft Db-continuous, soft Bb-continuous), Then the following three statements are equivalent:*

(i) f_ϕ is soft Ib-homeomorphism (resp. soft Db-homeomorphism, soft Bb-homeomorphism);

(ii) f_ϕ^{-1} is soft Ib-continuous (resp. soft Db-continuous, soft Bb-continuous);

(iii) f_ϕ is soft Db-closed (resp. soft Ib-closed, soft Bb-closed).

Proof. **(i)** ⇒ **(ii)** Since f_ϕ is a soft Ib-homeomorphism (resp. soft Db-homeomorphism, soft Bb-homeomorphism) map, then f_ϕ is soft Ib-open (resp. soft Db-open , soft Bb-open). It follows from item **(ii)** of Theorem 4.8, that f_ϕ^{-1} is soft Ib-continuous (resp. soft Db-continuous, soft Bb-continuous).
(ii) ⇒ **(iii)** The proof follows from item **(iii)** of Theorem 4.8.
(iii) ⇒ **(i)** It sufficient to prove that f_ϕ is a soft Ib-open (resp. soft Db-open, soft Bb-open) map. This follows from item **(i)** of Theorem 4.8. □

Definition 5.7. A map $(X, \tau, \preceq_1) \to (Y, \theta, \preceq_2)$ is said to be I (resp. D, B) b-homeomorphism if it is bijective, I (resp. D, B) b-continuous and I (resp. D, B) b-open.

Theorem 5.8. *Let $\tau^\star$ and $\theta^\star$ be extended soft topologies on X and Y, respectively. Then a soft map $g_\phi : (X, \tau^\star, E, \preceq_1) \to (Y, \theta^\star, F, \preceq_2)$ is soft I (resp. soft D, soft B) b-homeomorphism if and only if a map $g : (X, \tau^\star_e, \preceq_1) \to (Y, \theta^\star_{\phi(e)}, \preceq_2)$ is I (resp. D, B) b-homeomorphism.*

Proof. The proof is obtained immediately from Theorem 3.12 and Theorem 4.10. □

Proposition 5.9. *Let the two soft topologies τ and θ on X and Y, respectively, do not belong to $\{$discrete soft topology, indiscrete soft topology$\}$. If a soft map $f_\phi : (X, \tau, E \preceq_1) \to (Y, \theta, F, \preceq_2)$ is soft Bb-homeomorphism, then $\preceq_1$ and $\preceq_2$ is not linearly ordered.*

6. Conclusion

In 2018, we [14] have formulated the concept of soft topological ordered spaces as an extended of the soft topological spaces notion. Then we [15] have utilized monotone soft sets to define some soft ordered maps and have investigated main properties. In the present work, we have used a soft b-open set notion to propose the concepts of soft xb-continuous, soft xb-open, soft xb-closed and soft xb-homeomorphism maps, where $x \in \{I, D, B\}$. We have given various characterizations for these concepts and have showed the relationships among them with the help of examples. In addition, we have proved many interesting results related to them. It can be seen that our results are certainly more general than many results in [15]. Our next step is to examine the results obtained herein by using another form of generalized soft open sets.

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