International Journal of Advances in Mathematics
Volume 2018, Number 4, Pages 34-43, 2018
eISSN 2456-6098
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Existence of a Non-Oscillating solution for a System of Nonlinear ODEs

B. V. K. Bharadwaj* and Pallav Kumar Baruah[1]
*,1Department of Mathematics and Computer Science
Sri Sathya Sai Institute of Higher Learning
Prasanthinilayam - 515134, INDIA.
Email:bvkbharadwaj@sssihl.edu.in

ABSTRACT. In this paper we have considered a systems of ODEs of second order and have shown the existence of a non oscillating solution for such system. We have applied the fixed point technique to show that under certian conditions there exists at least one solution to the system which is not only non-oscillating, but also asymptotically constant.

1 Introduction

In iterature we find many differential equations whose oscillating or non oscillating solutions are of particular interest. The study of oscillations of second order differential equations finds its roots in the famous paper published by F V Atkinson [1]. After this paper, lot of researchers studied the existence of such solutions under different conditions in [4, 5, 6, 7] e.t.c. A simple and unified presentation of these results that eliminates the need for specific methodologies in each of the classical cases (linear, sublinear and superlinear) is presented by Dube in [4]. In [4] general criteria for the existence of a non-oscillating solution for the equation

$$y''(x) + F(x, y(x)) = 0 \tag{1.1}$$

where $F(t, x(t))$ is real valued continuous function on $[0, \infty) \times R$, $x \geq 0$, are found using the fixed point theorem technique. The following is the main result proved in [4].

* Corresponding Author.
Received April 17, 2018; revised June 21, 2018; accepted June 28, 2018.
2010 Mathematics Subject Classification: 34A99, 34C11.
Key words and phrases: Nonlinear Coupled Ordinary Differential Equations, Fixed-point Theorem, Non-Oscillation.

Theorem 1.1. *Let* $X = \{u \in C[0,\infty)| : 0 \leq u(t) \leq M, \text{ for all } t \geq 0\}$*, where* $M > 0$ *is given but fixed. Assume that* $F : \mathbb{R}^+ \times \mathbb{R}^+ \to \mathbb{R}^+$ *and that for any* $u \in X$*,*

$$\int_0^\infty t\,F(t,u(t))\,dt \leq M \tag{1.2}$$

and that there exists a function $k : \mathbb{R}^+ \to \mathbb{R}^+$ *such that* k *is continuous and*

$$\int_0^\infty t\,k(t)\,dt < 1, \tag{1.3}$$

such that for any $u, v \in \mathbb{R}^+$*, we also have*

$$|F(t,u) - F(t,v)| \leq k(t)|u - v|, \quad t \geq 0. \tag{1.4}$$

Then (1.1) *has a positive (and so non-oscillatory) monotone solution on* $(0,\infty)$ *such that* $y(x) \to M$ *as* $x \to \infty$*.*

The goal of [4] is to provide a condition for the existence of asymptotically constant solution to the differential equation (1.1). To this end, conditions (1.2), (1.3), (1.4), though strong, guarantee much more than non-oscillation of solutions of (1.1).

In [9], Hempel has given necessary and sufficient conditions for the solutions of (1.1) to be positive asymptotically. These conditions are not of the Lipschitzian type but are of a more general nature than those given by Dube in [4]. In [13], the authors present a theorem which replaces the Lipschitzian condition (1.4) with another condition to guarantee the existence of a non-oscillatory solution for (1.1), which is also asymptotically constant. The application of these results on a type of first order system of Nonlinear ODEs is also shown. However, presenting a proof for more generalized and weaker conditions like the one given by Hempel in [9] is yet to be explored. Asymptotic representation of solutions to (1.1) is studied recently in [11], [10] and necessary and sufficient conditions for the solutions of (1.1) to asymptotically looking like first degree polynomials are presented.

Systems of differential equations arise in many areas of science. Particularly systems of ODEs of second order are encountered while solving elliptic systems. Interested reader may look in to [15, 16] and references therein. Higher order differential systems and asymptotic representation of their solutions are studied in [14].

In this paper we have considered the following system of nonlinear ordinary differential equations.

$$\begin{aligned} y_1''(t) + f_1(t, y_2(t)) &= 0 \\ y_2''(t) + f_2(t, y_1(t)) &= 0 \end{aligned} \tag{1.5}$$

where $f_i(t, y_{3-i}(t))$s $(i = 1, 2)$ are positive real valued continuous functions on $[t_0,\infty) \times R$, $t_0 \geq 0$. We have shown the existence of a non oscillating solution for equation (1.5). We have given both Lipschitzian and non-Lipschitzian type conditions on the nonlinear functions.

2 Mathematical Preliminaries

In this section we present some mathematical preliminaries required to prove the main result. The main results and applications are presented in the next two sections.

Definition 2.1. A pair of real valued continuous functions $y_1(t), y_2(t)$ are called solutions of (1.5) if they satisfy both the equations in (1.5) simultaneously on the interval $[t_0, \infty)$.

Definition 2.2. A continuous real valued function defined on $[t_0, \infty]$ is said to be oscillatory if it has arbitrarily large zeros and otherwise it is said to be non oscilaltory. A solution (y_1, y_2) of the system (1.5) is called oscillatory if both y_1 and y_2 are oscillatory

Schauder's theorem:

Let E be a Banach space and X any nonempty convex and closed subset of E. If S is a continuous mapping of X into itself and SX is relatively compact, then the mapping S has at least one fixed point (i.e. there exists an $x \in X$ with $x = Sx$).

Let $E = B\left([0, \infty)\right)$, where $B\left([0, \infty)\right)$ is the Banach space of all continuous and bounded real valued functions on the interval $[0, \infty]$, endowed with the sup-norm $\|.\|$:

$\|h\| = sup_{t \geq 0} |h(t)|$ for $h \in B\left([0, \infty)\right)$

We need the following compactness criterion for subsets of $B\left([0, \infty)\right)$ which is a corollary of the Arzela-Ascoli theorem. This compactness criterion is an adaptation of a lemma due to Avramescu [2].

Compactness criterion:

Let H be an equicontinuous and uniformly bounded subset of the Banach space $B\left([0, \infty)\right)$. If H is equiconvergent at ∞, it is also relatively compact.

Note that a set H of real-valued functions defined on the interval $[0, \infty)$ is called equiconvergent at ∞ if all functions in H are convergent in R at the point ∞ and, in addition, for every $\epsilon > 0$ there exists a $T \geq 0$ such that, for all functions $h \in H$, it holds $|h(t) - lim_{s \to \infty} h(s)| < \epsilon$ for all $t \geq T$.

Banach's theorem:

Let E be a Banach Space and let S be a contraction mapping that maps E to itself. Then S has a unique fixed point in E

3 Results

Theorem 3.1. *Let*

$$|f_i(t, z)| \leq p_i(t) g_i\left(|z|\right) + q_i(t) \tag{3.1}$$

for all $(t, z) \in [t_0, \infty) \times R$ *and* $i = 1, 2$,

where p_i *and* q_i, $i = 1, 2$ *are nonnegative continuous real-valued functions on* $[t_0, \infty)$ *such that*

$$\int_{t_0}^{\infty} t p_i(t) dt < \infty$$
$$\int_{t_0}^{\infty} t q_i(t) dt < \infty \tag{3.2}$$

for $i = 1, 2$ and g_i, $i = 1, 2$ are nonnegative continuous real-valued functions on $[0, \infty)$ which are not identically zero. Let there exists positive constants $K_1 > 0, K_2 > 0$ and $T > t_0$ such that

$$\left[\int_T^\infty (s-T)p_1(s)ds\right]\Theta_1 + \int_T^\infty (s-T)q_1(s)ds \leq K_1 \tag{3.3}$$

and

$$\left[\int_T^\infty (s-T)p_2(s)ds\right]\Theta_2 + \int_T^\infty (s-T)q_2(s)ds \leq K_2 \tag{3.4}$$

where Θ_1, Θ_2 are defined as

$$\Theta_1 = sup\,\{g_1(z) : 0 \leq z \leq K_2\}$$

$$\Theta_2 = sup\,\{g_2(z)) : 0 \leq z \leq K_1\}$$

Then the system (1.5) *has a solution pair $\{y_1, y_2\}$ on the interval $[T, \infty]$ such that $y_1 \to K_1$ and $y_2 \to K_2$ assymptotically and therefore non-oscillatory.*

Proof. Consider the Banach Space $E = B\left([T, \infty]\right)$ with the sup-norm $\|.\|$, and define

$$Y_1 = \{y_1 \in E : \|y_1\| \leq K_1\}$$

$$Y_2 = \{y_2 \in E : \|y_2\| \leq K_2\}$$

Clearly Y_1, Y_2 are non-empty closed convex subsets of E.
Let y_1 and y_2 be two arbitrary functions in Y_1 and Y_2 respectively. Then

$$g_1\left(|y_2(t)|\right) \leq \Theta_1$$

$$g_2\left(|y_1(t)|\right) \leq \Theta_2$$

2 From (3.1) we get

$$|f_1\left(t, y_2(t)\right)| \leq \Theta_1 p_1(t) + q_1(t)$$

$$|f_2\left(t, y_1(t)\right)| \leq \Theta_2 p_2(t) + q_2(t) \tag{3.5}$$

for every $t \geq T$

Thus, from (3.2) we can conclude that

$$\int_T^\infty (s-T)f_1\left(s, y_2(s)\right)ds$$

$$\int_T^\infty (s-T)f_2\left(s, y_1(s)\right)ds$$

exist in R.

Now, by using (3.5) for every $t \geq T$ we get that

$\left|\int_t^\infty (s-T)f_1\left(s, y_2(s)\right)ds\right|$

$$\leq \Theta_1 \int_T^\infty (s-T)p_1(s)ds + \int_T^\infty (s-T)q_1(s)ds$$

$\left|\int_t^\infty (s-T)f_2\left(s, y_1(s)\right)ds\right|$

$$\leq \Theta_2 \int_T^\infty (s-T)p_2(s)ds + \int_T^\infty (s-T)q_2(s)ds$$

From (3.3) and (3.4) we get

$$\left| \int_t^{\infty} (s-T) f_1\left(s, y_2(s)\right) ds \right| \leq K_1$$
$$\left| \int_t^{\infty} (s-T) f_2\left(s, y_1(s)\right) ds \right| \leq K_2 \tag{3.6}$$

for every $t \geq T$. As this is true for any pair y_1, y_2, we now define mappings S_1 and S_2 on Y_1 and Y_2 respectively as

$$\left(S_1 y_1\right)(t) = K_1 - \int_t^{\infty} (s-T) f_1\left(s, y_2(s)\right) ds$$

with

$$y_2(s) = \int_s^{\infty} (r-T) f_2\left(r, y_1(r)\right) dr$$

and

$$\left(S_2 y_2\right)(t) = K_2 - \int_t^{\infty} (s-T) f_2\left(s, y_1(s)\right) ds$$

with

$$y_1(s) = \int_s^{\infty} (r-T) f_1\left(r, y_2(r)\right) dr$$

for every $t \geq T$.

Clearly we can see that S_i maps Y_i, $i = 1, 2$, into itself and is valid. Now we shall show that these mappings have fixed points using the Schauder's fixed point theorem. We will do this for S_1 and similar proof follows for S_2 also, which we exclude.

Since $S_1 Y_1 \subseteq Y_1$ and Y_1 is closed, convex, $S_1 Y_1$ is uniformly bounded. Moreover for some $t \geq T$, we have

$$\begin{aligned}
|S_1 y_1(t) - K_1| &= \left| \int_t^{\infty} (s-T) f_1\left(s, y_2(s)\right) ds \right| \\
&\leq \int_t^{\infty} (s-T) \left| f_1\left(s, y_2(s)\right) \right| ds \\
&\leq \Theta_1 \int_t^{\infty} (s-t) p_1(s) ds + \int_t^{\infty} (s-t) q_1(s) ds
\end{aligned} \tag{3.7}$$

So, by using (3.2) and suitably choosing T, we can easily see that $S_1 Y_1$ is equiconvergent at ∞.

Now by using (3.5) for any $y_1 \in Y_1$ and for every t_1, t_2 with $T \leq t_1 < t_2$, we get

$$\left| K_1 - \int_{t_2}^{\infty} (s-t_2) f_1\left(s, y_2(s)\right) ds - K_1 + \int_{t_1}^{\infty} (s-t_1) f_1\left(s, y_2(s)\right) ds \right|$$

$$\begin{aligned}
&= \left| \int_{t_1}^{\infty} \left[\int_r^{\infty} f_1\left(s, y_2(s)\right) ds \right] dr - \int_{t_2}^{\infty} \left[\int_r^{\infty} f_1\left(s, y_2(s)\right) ds \right] dr \right| \\
&= \left| \int_{t_1}^{t_2} \left[\int_r^{\infty} f_1\left(s, y_2(s)\right) ds \right] dr \right| \\
&\leq \int_{t_1}^{t_2} \left[\int_r^{\infty} \left| f_1\left(s, y_2(s)\right) \right| ds \right] dr
\end{aligned}$$

$$\leq \Theta_1 \int_{t_1}^{t_2} \left[\int_r^\infty p_1(s) ds \right] dr + \int_{t_1}^{t_2} \left[\int_r^\infty q_1(s) ds \right] dr$$

By using condition (3.2) we can always have a bound on the right hand side of the inequality, so $S_1 Y_1$ is equicontinuous. So by the given compactness criterion, $S_1 Y_1$ is relatively compact.

Now we will show that the mapping S_1 is continuous. Let y_{1v} be an abitrary sequence in Y_1, converging to y_1 under the norm defined before. From (3.5) we have

$$|f_1 (t, y_{1v}(t))| \leq \Theta_1 p_1(t) + q_1(t)$$

for every $t \geq T$ and for all v

Now, by applying the Lebesgue dominated convergence theorem we get

$$\begin{aligned} lim_{v \to \infty} \int_t^\infty (s - t) f_1 (s, y_{1v}(s)) \, ds \\ = \int_t^\infty (s - t) f_1 (s, y_1(s)) \, ds \end{aligned}$$

So we have shown the pointwise convergence i.e

$$\lim_{v \to \infty} (S_1 y_{1v}) (t) = (S_1 y_1) (t)$$

Now, consider an arbitrary subsequence u_μ of $S_1 y_{1v}$. Since $S_1 Y$ is relatively compact, there exists a subsequence v_λ of u_μ and a v in E such that v_λ converges uniformly to v. So

$$\lim_{v \to \infty} (S_1 y_{1v}) (t) = (S_1 y_1) (t) = v$$

for all $t \geq T$ under the sup-norm. So S_1 is continuous.

Thus we have shown that S_1 satisfies all the assumptions of Schauder's theorem, So S_1 has a fixed point $y_1 \in Y_1$ such that $S_1 y_1 = y_1$. That implies

$$y_1(t) = K_1 - \int_t^\infty (s - t) f_1 (s, y_2(s)) \, ds$$

So we can see that

$$y_1''(t) = f_1 (t, y_2(t))$$

for all $t \geq T$.

and also $y_1 \to K_1$ as $t \to \infty$. similarly we can show for the other function y_2 also that $y_2 \to K_2$. For arbitrary large values of t we see that the solutions $y_i \to K_i$, $i = 1, 2$. This also means that the solutions will not have large zeros. so the solution is non-oscillatory. □

Remark 3.2. Conditions (3.3) and (3.4) can also be rephrased as

$$\begin{aligned} \left[\int_T^\infty (s - T) p_i(s) ds \right] \{ sup \{ g_i(z) : 0 \leq z \leq max \{K_1, K_2\} \} \} \\ + \int_T^\infty (s - T) q_i(s) ds \leq max \{K_1, K_2\} \end{aligned}$$

for $i = 1, 2$. These conditions can be used when we work with product topology $Y_1 \times Y_2$. In this case, the space

$$Y_1 \times Y_2 = \left\{ (y_1, y_2) \in E \times E : \| (y_1, y_2) \|_{Y_1 \times Y_2} \leq K \right\} \tag{3.8}$$

is defined, where $\|y_1, y_2\|_{Y_1\times Y_2} = max\{\|y_1\|, \|y_2\|\}$ and $K = max\{K_1, K_2\}$
Now the operator S can be defined on this space $Y_1 \times Y_2$ as

$$S(y_1, y_2)(t) = \left(K_1 - \int_t^\infty (s-T) f_1(s, y_2(s))\, ds, K_2 - \int_t^\infty (s-T) f_2(s, y_1(s))\, ds\right) \tag{3.9}$$

where

$$\begin{aligned} y_2(s) &= \int_s^\infty (r-T) f_2(r, y_1(r))\, dr \\ y_1(s) &= \int_s^\infty (r-T) f_1(r, y_2(r))\, dr \end{aligned} \tag{3.10}$$

Now, to show that the space $Y_1 \times Y_2$ is a non empty closed convex subset of $E \times E$, S maps $Y_1 \times Y_2$ in to itself and S satisfies the assumptions of Schauder's theorem, we can follow the same arguement as in the earlier proof.

Theorem 3.3. *Let $K_1 > 0, K_2 > 0$ be given and fixed. Assume that f_1 and f_2 are functions from R^+XR to R^+ and satisfy*

$$\int_{t_0}^\infty t f_i(t, y_{3-i}) dt \le K_i \tag{3.11}$$

for $i = 1, 2$ and

$$\begin{aligned} \left|f_1(t,z) - f_1(t,z')\right| &\le a(t)\left|z - z'\right| \\ \left|f_2(t,z) - f_2(t,z')\right| &\le b(t)\left|z - z'\right| \end{aligned} \tag{3.12}$$

where $a(t)$, $b(t)$ be continuous functions from R^+ to R^+ such that

$$\begin{aligned} \int_{t_0}^\infty tb(t)\left[\int_{t_0}^\infty sa(s)ds\right] dt &< 1 \\ \int_{t_0}^\infty ta(t)\left[\int_{t_0}^\infty sb(s)ds\right] dt &< 1 \end{aligned} \tag{3.13}$$

Then the system (1.5) *has a unique solution pair $\{y_1, y_2\}$ on the interval (t_0, ∞) such that $y_1 \to K_1$ and $y_2 \to K_2$ assymptotically and therefore non-oscillatory.*

Proof. Let $X = \{y \in B([t_0, \infty]) : 0 \le y(t) \le K\}$
where $K = min\{K_1, K_2\}$
We can clearly see that the set X defined in the statement is a closed subset of $B([t_0, \infty))$. It follows that $(X, \|.\|)$, where $\|.\|$ is the usual supremum norm, is a Banach Space.
Let us now define the operator S on the set X as follows

$$Sy_1(t) = K_1 - \int_t^\infty (s-t) f_1\left(s, \int_s^\infty (\tau - s) f_2(\tau, y_1) d\tau\right) ds \tag{3.14}$$

The integral part in the right hand side is definitely convergent due to (3.11) Indeed for any given $y_1 \in X$ and $t \ge 0$,

$$\begin{aligned} 0 &\le \int_t^\infty (s-t) f_1\left(s, \int_s^\infty (\tau - s) f_2(\tau, y_1) d\tau\right) ds \\ &\le \int_{t_0}^\infty s f_1\left(s, \int_s^\infty (\tau - s) f_2(\tau, y_1) d\tau\right) ds \le K \end{aligned} \tag{3.15}$$

Thus, we can see that $SX \subseteq X$

We will show that this operator has a unique fixed point using the Banach's fixed point theorem. Clearly such a fixed point will be a solution that we are looking for. So, it suffices to show that S is a contraction.

Consider now,

$$\begin{aligned} \left\|Sy_1 - Sy_1^{'}\right\| = \| &\int_t^{\infty} (s-t) f_1 \left(s, \int_s^{\infty} (\tau - s) f_2(\tau, y_1) d\tau\right) ds \\ &- \int_t^{\infty} (s-t) f_1 \left(s, \int_s^{\infty} (\tau - s) f_2(\tau, y_1^{'}) d\tau\right) ds\| \end{aligned} \tag{3.16}$$

$$\begin{aligned} < \int_t^{\infty} (s-t) a(s) \left| \int_s^{\infty} (\tau - s) f_2(\tau, y_1) d\tau - \int_s^{\infty} (\tau - s) f_2(\tau, y_1^{'}) d\tau \right| ds \\ < \int_t^{\infty} (s-t) a(s) \left[\int_s^{\infty} (\tau - s) \left| f_2(\tau, y_1) - f_2(\tau, y_1^{'}) \right| d\tau \right] ds \\ < \int_t^{\infty} (s-t) a(s) \left[\int_s^{\infty} (\tau - s) b(\tau) \left| y_1 - y_1^{'} \right| d\tau \right] ds \\ < \left\| y_1 - y_1^{'} \right\| \int_t^{\infty} (s-t) a(s) \left[\int_s^{\infty} (\tau - s) b(\tau) d\tau \right] ds \end{aligned} \tag{3.17}$$

Now taking (3.13) in to consideration, we can easily show that S is a contraction, hence by contraction principle S will have a unique fixed point. In a similar way we can also show the existence of a y_2

These fixed points y_1 and y_2 definitely satisfiy the differential system (1.5) and $y_i \to K_i$ as $t \to \infty$ for $i = 1, 2$. □

Corollary 3.4. *Assume that f_1 and f_2 are are functions from $R^+ X R^+$ to R^+ and satisfy*

$$f_i(t, y) \leq h_i(t) g_i(y) \tag{3.18}$$

for $i = 1, 2$, where $h_i : R^+ \to R^+$ are continuous functions such that

$$\int_{t_0}^{\infty} s h_i(s) ds < 1 \tag{3.19}$$

for $i = 1, 2$ and $g_i : [t_0, K_i] \to [t_0, K_i]$ are continuous functiuons for $i = 1, 2$. Also assume that

$$\begin{aligned} \left| f_1(t, z) - f_1(t, z^{'}) \right| \leq a\,(t) \left| z - z^{'} \right| \\ \left| f_2(t, z) - f_2(t, z^{'}) \right| \leq b\,(t) \left| z - z^{'} \right| \end{aligned} \tag{3.20}$$

where $a(t)$, $b(t)$ be continuous functions from R^+ to R^+ such that

$$\begin{aligned} \int_{t_0}^{\infty} t b(t) \left[\int_{t_0}^{\infty} s a(s) ds \right] dt < 1 \\ \int_{t_0}^{\infty} t a(t) \left[\int_{t_0}^{\infty} s b(s) ds \right] dt < 1 \end{aligned} \tag{3.21}$$

Then the system (1.5) *has a unique solution pair $\{y_1, y_2\}$ on the interval $(0, \infty)$ such that $y_1 \to K_1$ and $y_2 \to K_2$ assymptotically and therefore non-oscillatory.*

Proof. we will proceed as in the previous theorem till (3.15). Now to show this estimate use (3.18) instead of (3.11). Rest of the arguement in the proof is similar to the previous theorem proof. □

4 Examples

Example 4.1. consider the system of differential equations

$$\begin{aligned} y_1''(t) + p(t)|y_2(t)|^{\gamma_1} &= 0 \\ y_2''(t) + q(t)|y_1(t)|^{\gamma_2} &= 0 \end{aligned} \tag{4.1}$$

Where $p(t), q(t)$ are continuous real valued functions such that

$$\int_{t_0}^{\infty} tp(t) < \infty$$
$$\int_{t_0}^{\infty} tq(t) < \infty$$

and γ_1, γ_2 are positve constants. Let T be a point with $T \geq t_0$ and suppose there exists positive constants K_1 and K_2 such that

$$\left[\int_T^{\infty} (s-T)p(t)ds\right](K_2)^{\gamma_1} \leq K_1$$
$$\left[\int_T^{\infty} (s-T)q(t)ds\right](K_1)^{\gamma_2} \leq K_2$$

Note 4.2. By applying Theorem 3.1, existence of a non-oscillating solution to (4.1) such that $y_i \to K_i$ for $i = 1,2$ as $t \to \infty$ is guaranteed.

Following Remark 3.2, existence of a non-oscillating, asymptotically constant solution can be shown for the above example if

$$\left[\int_T^{\infty} (s-T)p(t)ds\right](max\{K_1, K_2\})^{\gamma_1} \leq max\{K_1, K_2\}$$
$$\left[\int_T^{\infty} (s-T)q(t)ds\right](max\{K_1, K_2\})^{\gamma_2} \leq max\{K_1, K_2\}$$

Note 4.3. Consider the case where

$$\gamma_i = 2n_i - 1$$

where n_i is an integer greater than 1 and let $t_0 = 0$.

We see that $g_i(y) = |y|^{\gamma_i}$ are self maps on the interval $[0,1]$. So it can be shown that

$$\|g_i(y) - g_i\widehat{y})\| \leq (2n_i - 1)\|y - \widehat{y}\| \tag{4.2}$$

for $i = 1,2$. Therefore, $g_i s$ are Lipschitzian with the Lipschitz constant $2n_i - 1$ for $i = 1,2$. It then follows from the Corollary 3.4 that if p and q are chosen in such a way that

$$\int_0^{\infty} tp(t) < 1/(2n_1 - 1)$$
$$\int_0^{\infty} tq(t) < 1/(2n_2 - 1)$$

then (4.1) will have a unique non oscillating solution on $[0, \infty)$ which assymptotically tends to a positive constant.

5 Acknowledgement and Dedication

We dedicate this work to the Founder Chancellor of Sri Sathya Sai Institute of Higher Learning, Bhagawan Sri Sathya Sai Baba.
This work is funded by National Board of Higher Mathematics, INDIA (Grant No: 2/48(12)/2009-R&D II/2808).

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