International Journal of Advances in Mathematics

Volume 2018, Number 4, Pages 57-66, 2018

eISSN 2456-6098

Supra g^*-Closed Sets in Supra Bitopological Spaces

R.Gowri[1], A.K.R.Rajayal*

[1] Department of Mathematics, Government College for Women(A), Kumbakonam, India

[2] Department of Mathematics, Government College for Women(A), Kumbakonam, India

* E-Mail: rajayalk@gmail.com

ABSTRACT. The objective of this paper is to introduce a new class of sets $S_{\tau_{ij}}$-g^*-closed sets and $S_{\tau_{ij}}$-g^*-open sets in supra bitopological spaces and some properties of these sets are investigated. Also, two supra bitopological spaces $S_{\tau_{ij}}$-$T^*_{1/2}$ and $S_{\tau_{ij}}$-$T^{**}_{1/2}$-spaces are introduced.

---

1 Introduction

The study of bitopological spaces was initiated by Kelly[7]. Sheik John and Sundaram [14] introduced the concept of generalized star closed sets in bitopological spaces. The supra topological spaces have been introduced by Mashhour[10] in 1983. In topological space the arbitrary union condition is enough to have a supra topological space. Here every topological space is a supra topologicl space but the converse is not always true. A triplet $(X, S_{\tau_1}, S_{\tau_2})$ where X is a non empty set S_{τ_1} and S_{τ_1} are supra topologies on X is called a supra bitopological spaces by Gowri and Rajayal[2]. Ramkumar, Ravi and Joseph[12] introduced supra g^*-closed sets and we obtain some properties of supra g^*-closed sets. The purpose of this paper is to introduce the concept of $S_{\tau_{ij}}$-g^*-closed sets and $S_{\tau_{ij}}$-g^*-open sets in supra bitopological spaces. Also, we introduce $S_{\tau_{ij}}$-$T^*_{1/2}$ and $S_{\tau_{ij}}$-$T^{**}_{1/2}$-spaces.

---

*corresponding author

Received March 18, 2018; revised May 13, 2018; accepted May 26, 2018.

2010 Mathematics Subject Classification: 54D05, 54D10, 54D08, 54D20.

Key words and phrases: Supra bitopology, $S_{\tau_{ij}}$-g^*-closed sets, $S_{\tau_{ij}}$-g^*-open sets, $S_{\tau_{ij}}$-$T^*_{1/2}$-space and $S_{\tau_{ij}}$-$T^{**}_{1/2}$-space.

2 Preliminaries

In this section we recall the basic definitions of supra bitopological spaces.

Definition 2.1[10] (X, S_τ) is said to be a supra topological space if it is satisfying these conditions:
(1) $X, \emptyset \in S_\tau$
(2) The union of any number of sets in S_τ belongs to S_τ.

Definition 2.2[10] Each element $A \in S_\tau$ is called a supra open set in (X, S_τ), and its compliment is called a supra closed set in (X, S_τ).

Definition 2.3[10] If (X, S_τ) is a supra topological spaces, $A \subseteq X$, $A \neq \emptyset$, S_{τ_A} is the class of all intersection of A with each element in S_τ, then (A, S_{τ_A}) is called a supra subspace of (X, S_τ).

Definition 2.4[10] The supra closure of the set A is denoted by S_τ-cl(A) and is defined as
S_τ-cl(A) = $\cap\{B$: B is a supra closed and $A \subseteq B\}$.

Definition 2.5[10] The supra interior of the set A is denoted by S_τ-int(A) and is defined as
S_τ-int(A) = $\cup\{B$: B is a supra open and $B \subseteq A\}$.

Definition 2.6[2] If S_{τ_1} and S_{τ_2} are two supra topologies on a non-empty set X, then the triplet $(X, S_{\tau_1}, S_{\tau_2})$ is said to be a supra bitopological space.

Definition 2.7[2] Each element of S_{τ_i} is called a supra τ_i-open sets(briefly S_{τ_i}-open sets) in $(X, S_{\tau_1}, S_{\tau_2})$. Then the complement of S_{τ_i}-open sets are called a supra τ_i-closed sets(breifly S_{τ_i}-closed sets), for i = 1,2.

Definition 2.8[2] If $(X, S_{\tau_1}, S_{\tau_2})$ is a supra bitopological space, $Y \subseteq X$, $Y \neq \emptyset$ then $(Y, S_{\tau_1^*}, S_{\tau_2^*})$ is a supra bitopological subspace of $(X, S_{\tau_1}, S_{\tau_2})$ if
$S_{\tau_1^*} = \{U \cap Y;$ U is a $S_{\tau_1} -$ open in X$\}$ and
$S_{\tau_2^*} = \{V \cap Y;$ V is a $S_{\tau_2} -$ open in X$\}$.

Definition 2.9[2] The S_{τ_i}-closure of the set A is denoted by S_{τ_i}-cl(A) and is defined as
S_{τ_i}-cl(A) = $\cap\{B$: B is a $S_{\tau_i} -$ closed and $A \subseteq B, for\ i = 1, 2\}$.

Definition 2.10[2] The S_{τ_i}-interior of the set A is denoted by S_{τ_i}-int(A) and is defined as
S_{τ_i}-int(A)= $\cup\{B$: B is a $S_{\tau_i} -$ open and $B \subseteq A, for\ i = 1, 2\}$.

Definition 2.11[13] Let (X, μ) be a supra topological space. A subset A of X is supra g-closed if $cl^\mu(A) \subseteq U$ whenever $A \subseteq U$ and U is supra open.

Definition 2.12[12] Let (X, μ) be a supra topological space. A subset A of X is supra g^*-closed if $cl^\mu(A) \subseteq U$ whenever $A \subseteq U$ and U is supra g-open.

3 Supra generalized star closed sets in supra bitopological spaces

In this section we introduce the concept of $S_{\tau_{ij}}$-g^*-closed sets in supra bitopological spaces.

Definition 3.1 A subset A of a supra bitopological space $(X, S_{\tau_1}, S_{\tau_2})$ is said to be supra τ_{ij}-generalized star-closed(briefly $S_{\tau_{ij}}$-g^*-closed) if S_{τ_j}-cl(A) $\subseteq$ U whenever A $\subseteq$ U and U $\in S_{\tau_i}$-g-open, where i,j = 1,2 and i $\neq$ j.
The complement of $S_{\tau_{ij}}$-g^*-closed set is said to be $S_{\tau_{ij}}$-g^*-open.
The family of all $S_{\tau_{ij}}$-g^*-closed(resp. $S_{\tau_{ij}}$-g^*-open)sets of $(X, S_{\tau_1}, S_{\tau_2})$ is denoted by $S_{\tau_{ij}}$-g^*C(X)(resp. $S_{\tau_{ij}}$-g^*O(X)), where i,j = 1,2 and i $\neq$ j.

Example 3.2
Let X = $\{a, b, c\}$,
$S_{\tau_1} = \{\emptyset, X, \{a\}, \{a, b\}, \{b, c\}\}$,
$S_{\tau_2} = \{\emptyset, X, \{a, c\}, \{b, c\}\}$,
$S_{\tau_{12}}$-g^*C(X) = $\{\emptyset, X, \{a\}, \{b\}, \{a, c\}\}$.

Definition 3.3 A subset A of supra bitopological space $(X, S_{\tau_1}, S_{\tau_2})$ is said to be pairwise S_τ-g^*-closed(briefly p-S_τ-g^*-closed) if A is $S_{\tau_{12}}$-g^*-closed and $S_{\tau_{21}}$-g^*-closed. The complement of a pairwise S_τ-g^*-closed set is said to be pairwise S_τ-g^*-open(briefly p-S_τ-g^*-open).

Remark 3.4 By setting $S_{\tau_1} = S_{\tau_2}$ in Definition 3.1, a $S_{\tau_{ij}}$-g^*-closed becomes S_τ-g^*-closed.

Proposition 3.5 *If A is S_{τ_j}-closed subset of $(X, S_{\tau_1}, S_{\tau_2})$ then A is $S_{\tau_{ij}}$-g^*-closed.*

Proof. It is obvious that every S_{τ_j}-closed set is $S_{\tau_{ij}}$-g^*-closed.
The converse of the above Proposition 3.5, is not true as seen from the following example. □

Example 3.6
Let X = $\{a, b, c\}$,
$S_{\tau_1} = \{\emptyset, X, \{a, b\}, \{b, c\}\}$,
$S_{\tau_2} = \{\emptyset, X, \{a\}\, \{a, c\}, \{b, c\}\}$,
$S_{\tau_{12}}$-g^*C(X) = $\{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a, c\}, \{b, c\}\}$.
Here $\{c\}$ is $S_{\tau_{12}}$-g^*-closed but it is not S_{τ_2}-closed.

Proposition 3.7 *If A is both S_{τ_i}-g-open and $S_{\tau_{ij}}$-g^*-closed, then A is S_{τ_j}-closed.*

Proof. Suppose that A is S_{τ_i}-g-open and $S_{\tau_{ij}}$-g^*-closed. Then $A \subseteq A$ implies that S_{τ_j}-cl(A) $\subseteq$ A. Obviously, $A \subseteq$ S_{τ_j}-cl(A). Therefore A is S_{τ_j}-closed. □

Remark 3.8 Every $S_{\tau_{ij}}$-g^*-closed set is $S_{\tau_{ij}}$-g-closed.
The converse of the above Remark 3.8, is not true as seen from the following example.

Example 3.9 Let X = $\{a, b, c, d\}$,
$S_{\tau_1} = \{\emptyset, X, \{a\}, \{a, d\}, \{b, c, d\}\}$,
$S_{\tau_2} = \{\emptyset, X, \{b\}, \{b, c\}, \{a, c, d\}\}$,
$S_{\tau_{12}}$-gC(X) = $\{\emptyset, X, \{b\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, c, d\}, \{a, b, d\}, \{b, c, d\}\}$,

$S_{\tau_{12}}$-g^*C(X) = $\{\emptyset, X, \{b\}, \{a,d\}, \{a,b,c\}, \{a,c,d\}\}$.
Then the subset $\{a,b\}$ is $S_{\tau_{12}}$-g-closed but it is not $S_{\tau_{12}}$-g^*-closed set.

Remark 3.10 $S_{\tau_{ij}}$-g^*-closed sets and S_{τ_j}-g-closed sets are independent. The following example supports our claim.

Example 3.11 Let X = $\{a,b,c\}$,
$S_{\tau_1} = \{\emptyset, X, \{a,b\}, \{b,c\}\}$,
$S_{\tau_2} = \{\emptyset, X, \{a,c\}, \{b,c\}\}$,
S_{τ_2}-g-closed set = $\{\emptyset, X, \{a\}, \{b\}, \{a,b\}\}$,
$S_{\tau_{12}}$-g^*-closed set = $\{\emptyset, X, \{a\}, \{b\}, \{a,c\}\}$.
Then the subset $\{a,b\}$ is S_{τ_2}-g-closed but not $S_{\tau_{12}}$-g^*-closed set. Also, the subset $\{a,c\}$ is $S_{\tau_{12}}$-g^*-closed set but not S_{τ_2}-g-closed.

Remark 3.12 The union of two $S_{\tau_{ij}}$-g^*-closed sets need not be $S_{\tau_{ij}}$-g^*-closed as seen from the following example.

Example 3.13 Let X = $\{a,b,c\}$,
$S_{\tau_1} = \{\emptyset, X, \{a\}, \{a,b\}, \{b,c\}\}$,
$S_{\tau_2} = \{\emptyset, X, \{c\}, \{a,b\}, \{a,c\}\}$,
$S_{\tau_{12}}$-g^*-C(X) = $\{\emptyset, X, \{b\}, \{c\}, \{a,b\}, \{a,c\}\}$.
Here $\{b\}$ and $\{c\}$ are $S_{\tau_{12}}$-g^*-closed sets but their union $\{b,c\}$ is not $S_{\tau_{12}}$-g^*-closed.

Remark 3.14 The intersection of two $S_{\tau_{ij}}$-g^*-closed sets need not be $S_{\tau_{ij}}$-g^*-closed as seen from the following example.

Example 3.15 In Example 3.9, the subsets $\{a,d\}$ and $\{a,b,c\}$ are $S_{\tau_{12}}$-g^*-closed but their intersection $\{a\}$ is not $S_{\tau_{12}}$-g^*-closed.

Remark 3.16 $S_{\tau_{12}}$-g^*C(X) is generally not equal to $S_{\tau_{21}}$-g^*C(X) as can be seen from the following example.

Example 3.17 Let $X = \{a,b,c\}$,
$S_{\tau_1} = \{\emptyset, X, \{c\}, \{a,b\}, \{b,c\}\}$,
$S_{\tau_2} = \{\emptyset, X, \{b\}, \{a,b\}, \{a,c\}, \{b,c\}\}$,
Then $S_{\tau_{12}}$-g^*C(X) = $\{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a,c\}\}$
and $S_{\tau_{21}}$-g^*C(X) = $\{\emptyset, X, \{a\}, \{c\}, \{a,b\}, \{a,c\}\}$.
Thus $S_{\tau_{12}}$-g^*C(X) $\neq$ $S_{\tau_{21}}$-g^*C(X).

Proposition 3.18 *Let S_{τ_1} and S_{τ_2} be supra topologies on X. If $S_{\tau_1} \subseteq S_{\tau_2}$, then $S_{\tau_{21}}$-g^*C(X) $\subseteq$ $S_{\tau_{12}}$-g^*C(X).*

Proof. Let A be $S_{\tau_{21}}$-g^*C(X). Then S_{τ_1}-cl(A) $\subseteq$ U, Whenever $A \subseteq U$ and U is S_{τ_2}-g-open. Since $S_{\tau_1} \subseteq S_{\tau_2}$, S_{τ_2}-cl(A) $\subseteq$ U, Whenever $A \subseteq U$ and U is S_{τ_1}-g-open. Hence A is $S_{\tau_{12}}$-g^*C(X). □

Proposition 3.19 *Let A be a subset of supra bitopological space $(X, S_{\tau_1}, S_{\tau_2})$. If A is $S_{\tau_{ij}}$-g^*-closed, then S_{τ_j}-cl(A)$-$ A contains no non empty S_{τ_i}-g-closed set, where i,j = 1,2 and $i \neq j$.*

Proof. Let A be $S_{\tau_{ij}}$-g^*-closed set and F be S_{τ_i}-g-closed set such that F $\subseteq$ S_{τ_j}-cl(A)$-$ A. Since A $\in$ $S_{\tau_{ij}} - g^*$C(X), we have S_{τ_j}-cl(A) $\subseteq F^c$. Thus $F \subseteq [S_{\tau_j} - cl(A)] \cap [S_{\tau_j} - cl(A)]^c = \emptyset$. Therefore, F = $\emptyset$. Hence, S_{τ_j}-cl(A)$-$ A contains no non empty S_{τ_i}-g-closed set. □

Remark 3.20 The converse of the above Proposition 3.19, is not true as seen from the following example.

Example 3.21

Let X = $\{a, b, c\}$,

$S_{\tau_1} = \{\emptyset, X, \{a\}, \{a, b\}, \{b, c\}\}$,

$S_{\tau_2} = \{\emptyset, X, \{a, c\}, \{b, c\}\}$.

If A = $\{c\}$, then S_{τ_2}-cl(A) $-$ A = $X - \{c\} = \{a, b\}$ which does not contain any non empty S_{τ_1}-g-closed set. But A is not $S_{\tau_{12}}$-g^*-closed.

Corollary 3.22 *If A is $S_{\tau_{ij}}$-g^*-closed set in $(X, S_{\tau_1}, S_{\tau_2})$, then A is S_{τ_j}-closed if and only if S_{τ_j}-cl(A)$-$ A is S_{τ_i}-g-closed.*

Proof.

Necessity: If A is S_{τ_j}-g-closed, then S_{τ_j}-cl(A)= A. That is S_{τ_j}-cl(A)$-$ A = $\emptyset$ and hence S_{τ_j}-cl(A)$-$ A is S_{τ_i}-g-closed.

Sufficiency:

If S_{τ_j}-cl(A)$-$ A is S_{τ_i}-g-closed, then by Proposition 3.19, S_{τ_j}-cl(A)$-$ A = $\emptyset$, since A is $S_{\tau_{ij}}$-g^*-closed. Therefore A is S_{τ_j}-closed. □

Proposition 3.23 *If A is an $S_{\tau_{ij}}$-g^*-closed set of $(X, S_{\tau_1}, S_{\tau_2})$ such that $A \subseteq B \subseteq S_{\tau_j}$-cl(A), then B is also an $S_{\tau_{ij}}$-g^*-closed set of $(X, S_{\tau_1}, S_{\tau_2})$, where i,j = 1,2 and $i \neq j$.*

Proof. Suppose that A is $S_{\tau_{ij}}$-g^*-closed set and $A \subseteq B \subseteq S_{\tau_j}$-cl(A). Let B $\subseteq$ U and U is S_{τ_i}-g-open. Given A $\subseteq$ B. Then A $\subseteq$ U. Since A is $S_{\tau_{ij}}$-g^*-closed, we have S_{τ_j}-cl(A) $\subseteq$ U. Since B $\subseteq$ S_{τ_j}-cl(A), S_{τ_j}-cl(B) $\subseteq$ S_{τ_j}-cl(A) $\subseteq$ U. Hence, B is $S_{\tau_{ij}}$-g^*-closed set □

Theorem 3.24 *Let $A \subseteq Y \subseteq X$ and suppose that A is $S_{\tau_{ij}}$-g^*-closed in X. Then A is $S_{\tau_{ij}}$-g^*-closed relative to Y.*

Proof. Let A $\subseteq Y \cap U$ and suppose that U is S_{τ_i}-g-open in X. Then $A \subseteq U$ and hence S_{τ_2}-cl(A) $\subseteq U$. It follows that $Y \cap S_{\tau_2}$-cl(A) $\subseteq Y \cap U$. □

Remark 3.25 Intersection of $S_{\tau_{ij}}$-g^*-closed set and S_{τ_i}-g-open set is neither $S_{\tau_{ij}}$-g^*-closed nor S_{τ_i}-g-open as can be seen from the following example.

Example 3.26

Let $X = \{a, b, c\}$,

$S_{\tau_1} = \{\emptyset, X, \{a, b\}, \{b, c\}\}$,

$S_{\tau_2} = \{\emptyset, X, \{a, c\}, \{b, c\}\}$,

Then $S_{\tau_{12}}$-g^*C(X) = $\{\emptyset, X, \{a\}, \{b\}, \{a, c\}\}$ and

S_{τ_1}-g-open = $\{\emptyset, X, \{b\}, \{a, b\}, \{b, c\}\}$.

We have A = $\{b, c\}$ is S_{τ_1}-g-open and B = $\{a, c\}$ is $S_{\tau_{12}}$-g^*-closed set, their intersection $\{c\}$ is neither $S_{\tau_{12}}$-g^*-closed set nor S_{τ_1}-g-open set. □

Proposition 3.27 *For each element $x \in (X, S_{\tau_1}, S_{\tau_2})$, the singleton $\{x\}$ is S_{τ_i}-g-closed or $\{x\}^c$ is $S_{\tau_{ij}}$-g^*-closed set.*

Proof. Suppose that $\{x\}$ is not S_{τ_i}-g-closed. Then $\{x\}^c$ is not S_{τ_i}-g-open and X is the only S_{τ_i}-g-open set which contains $\{x\}^c$ and $\{x\}^c$ is $S_{\tau_{ij}}$-g^*-closed. □

Theorem 3.28 *A subset of supra bitopological space in $(X, S_{\tau_1}, S_{\tau_2})$ is $S_{\tau_{ij}}$-g^*-open set if and only if $B \subseteq S_{\tau_j}$-int(A) whenever B is S_{τ_i}-g-closed and $B \subseteq A$ where i,j = 1,2 and $i \neq j$.*

Proof. Let A be $S_{\tau_{ij}}$-g^*-open set. Let B be a S_{τ_i}-g-closed set such that $B \subseteq A$. Let $A \subseteq B$ and B is S_{τ_i}-g-closed. Then $(A)^c \subseteq B^c$ and $(B)^c$ is S_{τ_i}-g-open, We have A^c is $S_{\tau_{ij}}$-g-closed. Hence, $[S_{\tau_j} - cl(A)^c] \subseteq (B^c)$. Therefore $B \subseteq S_{\tau_j}$-int(A). Conversely, suppose $B \subseteq S_{\tau_j}$-int(A) whenever $B \subseteq A$ and B is S_{τ_i}-g-closed. Let $A^c \subseteq U$ and U is S_{τ_i}-g-open. Then $U^c \subseteq A$ and U^c is S_{τ_i}-g-closed. By hypothesis $U^c \subseteq S_{\tau_j}$-int(A). Hence, $[S_{\tau_j} - int(A)]^c \subseteq U$. That is $[S_{\tau_j} - cl(A)^c] \subseteq U$. Consequently, A^c is $S_{\tau_{ij}}$-g^*-closed set. Hence, A is $S_{\tau_{ij}}$-g^*-open. □

Remark 3.29 Every S_{τ_1}-open set is $S_{\tau_{ij}}$-g^*-open but the converse is not true in general as can be seen from the following example.

Example 3.30
Let X = $\{a, b, c\}$,
$S_{\tau_1} = \{\emptyset, X, \{a, b\}, \{b, c\}\}$,
$S_{\tau_2} = \{\emptyset, X, \{a\} \{a, c\}, \{b, c\}\}$,
$S_{\tau_{12}}$-g^*O(X) = $\{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}\}$.
Here $\{a, c\}$ is $S_{\tau_{12}}$-g^*-open but it is not S_{τ_1}-open.

Remark 3.31 The union of any two $S_{\tau_{ij}}$-g^*-open set is not necessary $S_{\tau_{ij}}$-g^*-open set as seen in the following example.

Example 3.32
In Example 3.13,
$S_{\tau_{12}}$-g^*O(X) = $\{\emptyset, X, \{b\}, \{c\}, \{a, b\}, \{a, c\}\}$.
Here $\{b\}$ and $\{c\}$ are $S_{\tau_{12}}$-g^*-open sets but their union $\{b\} \cup \{c\} = \{b, c\}$ is not $S_{\tau_{12}}$-g^*-open.

Remark 3.33 The intersection of any two $S_{\tau_{ij}}$-g^*-open set is not necessary $S_{\tau_{ij}}$-g^*-open set as in the following example.

Example 3.34
In Example 3.30,
$S_{\tau_{12}}$-g^*O(X) = $\{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}\}$.
Here $\{b, c\}$ and $\{a, c\}$ are $S_{\tau_{12}}$-g^*-open sets but their intersection $\{b, c\} \cap \{a, c\} = \{c\}$ is not $S_{\tau_{12}}$-g^*-open.

Remark 3.35 The intersection of $S_{\tau_{ij}}$-g^*-open set and S_{τ_j}-open set is not $S_{\tau_{ij}}$-g^*-open as seen from the following example.

Example 3.36
Let X = $\{a, b, c\}$,
$S_{\tau_1} = \{\emptyset, X, \{c\}, \{a, b\}, \{b, c\}\}$,
$S_{\tau_2} = \{\emptyset, X, \{b\} \{a, b\}, \{b, c\}, \{a, c\},\}$,
$S_{\tau_{12}}$-g^*O(X) = $\{\emptyset, X, \{b\}, \{a, b\}, \{b, c\}, \{a, c\}\}$.

Here A = $\{b, c\}$ is S_{τ_2}-open and B = $\{a, c\}$ is $S_{\tau_{12}}$-g^*-open but their intersection $\{a, c\} \cap \{b, c\} = \{c\}$ is not $S_{\tau_{12}}$-g^*-open.

Theorem 3.37 *Let A and B be subset of supra bitopological space $(X, S_{\tau_1}, S_{\tau_2})$ such that S_{τ_j}-int(A) $\subseteq B \subseteq A$. If A is $S_{\tau_{ij}}$-g^*-open, then B is $S_{\tau_{ij}}$-g^*-open, where i,j = 1,2 and $i \neq j$.*

Proof. Let A be $S_{\tau_{ij}}$-g^*-open. Let U be a S_{τ_i}-g-closed such that $U \subseteq B$. Since $U \subseteq B$ and $B \subseteq A$, we have $U \subseteq A$. Therefore, $U \subseteq S_{\tau_j}$-int(A). Since S_{τ_j}-int(A) $\subseteq B$, we have $S_{\tau_j} - int(S_{\tau_j} - int(A)) \subseteq S_{\tau_j} - int(B)$. Therefore, S_{τ_j}-int(A) $\subseteq S_{\tau_j}$-int(B). Consequently, U $\subseteq S_{\tau_j}$-int(B). Hence, B is $S_{\tau_{ij}}$-g^*-open □

Theorem 3.38 *A subset A is $S_{\tau_{ij}}$-g^*-closed set then S_{τ_j}-cl(A)$-$ A is $S_{\tau_{ij}}$-g^*-open.*

Proof. Let A be $S_{\tau_{ij}}$-g^*-closed set. Let $F \subseteq S_{\tau_j}$-cl(A)$-$ A where F is S_{τ_i}-g-closed set. Since A is $S_{\tau_{ij}}$-g^*-closed, we have S_{τ_j}-cl(A)$-$ A does not contain non empty S_{τ_i}-g-closed by Proposition 3.19, Consequently, F = $\emptyset$. Therefore, $\emptyset \subseteq S_{\tau_j}$-cl(A)$-$ A, $\emptyset \subseteq S_{\tau_j}$-int($S_{\tau_j}$-cl(A)$-$ A), we obtain $F \subseteq S_{\tau_j}$-int(S_{τ_j}-cl(A)$-$ A). Hence, S_{τ_j}-cl(A)$-$ A is $S_{\tau_{ij}}$-g^*-open. □

Remark 3.39 From the Proposition 3.5, Remark 3.8, and Examples 3.6, 3.9, We get the following implication.

S_{τ_j}-closed $\rightarrow$ $S_{\tau_{ij}}$-g^*-closed $\rightarrow$ $S_{\tau_{ij}}$-g-closed.

The converse of the above implication is not true.

4 $S_{\tau_{ij}}$-$T^*_{1/2}$ and $S_{\tau_{ij}}$-$T^{**}_{1/2}$-spaces in supra bitopological spaces

In this section we introduce, $S_{\tau_{ij}}$-$T^*_{1/2}$ and $S_{\tau_{ij}}$- $T^{**}_{1/2}$-spaces in supra bitopological spaces with the help of $S_{\tau_{ij}}$-g^*-closed set.

Definition 4.1 A supra bitopological space $(X, S_{\tau_1}, S_{\tau_2})$ is said to be $S_{\tau_{ij}}$-$T^*_{1/2}$-space if every $S_{\tau_{ij}}$-g^*-closed set is S_{τ_j}-closed.

Proposition 4.2 *If $(X, S_{\tau_1}, S_{\tau_2})$ is $S_{\tau_{ij}}$-$T_{1/2}$-space, then it is an $S_{\tau_{ij}}$-$T^*_{1/2}$-space.*

Remark 4.3 The converse of the above Proposition 4.2, is not true. The following example supports our claim.

Example 4.4

Let X = $\{a, b, c\}$,

$S_{\tau_1} = \{\emptyset, X, \{b\}\}$,

$S_{\tau_2} = \{\emptyset, X, \{b\}, \{a, c\}\}$.

Then $(X, S_{\tau_1}, S_{\tau_2})$ is $S_{\tau_{ij}}$-$T^*_{1/2}$-space but not a $S_{\tau_{ij}}$-$T_{1/2}$-space.

Theorem 4.5 *A supra bitopological space $(X, S_{\tau_1}, S_{\tau_2})$ is $S_{\tau_{ij}}$-$T^*_{1/2}$-space if and only if $\{x\}$ is S_{τ_j}-open or S_{τ_i}-g-closed for each $x \in X$.*

Proof. Suppose that $\{x\}$ is not S_{τ_i}-g-closed. Then $\{x\}^c$ is $S_{\tau_{ij}}$-g^*-closed by Proposition 3.27. Since $(X, S_{\tau_1}, S_{\tau_2})$ is $S_{\tau_{ij}}$-$T^*_{1/2}$-space, $\{x\}^c$ is S_{τ_j}-closed. Therefore $\{x\}$ is S_{τ_j}-open.

Conversely, let A be an $S_{\tau_{ij}}$-g^*-closed set. By assumption, $\{x\}$ is S_{τ_j}-open or S_{τ_i}-g-closed. for any $x \in S_{\tau_j}$-cl(A).

Case (i): Suppose $\{x\}$ is S_{τ_j}-open. Since $\{x\} \cap A \neq = \emptyset$, We have $x \in A$.

Case (ii): Suppose $\{x\}$ is S_{τ_j}-g-closed. If $x \notin A$, then $\{x\} \subseteq S_{\tau_j}$-cl(A) $-$ A, Which is a contradiction to Proposition 3.27. Therefore $x \in A$.

Thus in both cases, We conclude that A is S_{τ_j}-closed. Hence $(X, S_{\tau_1}, S_{\tau_2})$ is $S_{\tau_{ij}}$-$T^*_{1/2}$-space. □

Remark 4.6 (X, S_{τ_1})-space is not generally S_τ-$T^*_{1/2}$-space even if $(X, S_{\tau_1}, S_{\tau_2})$ is $S_{\tau_{12}}$-$T^*_{1/2}$-space as shown in the following example.

Example 4.7

Let X = $\{a, b, c\}$,

$S_{\tau_1} = \{\emptyset, X, \{c\}, \{a, b\}, \{b, c\}\}$,

$S_{\tau_2} = \{\emptyset, X, \{b\}, \{a, b\}, \{b, c\}, \{a, c\}\}$.

Then (X, S_{τ_1}) is not S_τ-$T^*_{1/2}$-space but $(X, S_{\tau_1}, S_{\tau_2})$ is $S_{\tau_{ij}}$-$T^*_{1/2}$-space.

Definition 4.8 A supra bitopological space $(X, S_{\tau_1}, S_{\tau_2})$ is said to be pairwise S_τ-$T^*_{1/2}$-space if it is both $S_{\tau_{12}}$-$T^*_{1/2}$ and $S_{\tau_{21}}$-$T^*_{1/2}$-space.

Proposition 4.9 *If $(X, S_{\tau_1}, S_{\tau_2})$ is pairwise S_τ-$T_{1/2}$-space then it is pairwise S_τ-$T^*_{1/2}$-space but not conversely.*

Example 4.10

In Example 4.4. Then $(X, S_{\tau_1}, S_{\tau_2})$ is also $S_{\tau_{21}}$-$T^*_{1/2}$and therefore it is pairwise S_τ-$T^*_{1/2}$-space. But $(X, S_{\tau_1}, S_{\tau_2})$ is not a pairwise S_τ-$T_{1/2}$-space, since it is not a $S_{\tau_{12}}$-$T_{1/2}$-space.

Definition 4.11 A supra bitopological space $(X, S_{\tau_1}, S_{\tau_2})$ is said to be $S_{\tau_{ij}}$-$T^{**}_{1/2}$-space if every $S_{\tau_{ij}}$-g-closed set is $S_{\tau_{ij}}$-g^*-closed.

Proposition 4.12 *If $(X, S_{\tau_1}, S_{\tau_2})$ is $S_{\tau_{ij}}$-$T_{1/2}$-space, then it is a $S_{\tau_{ij}}$-$T^{**}_{1/2}$-space.*

Remark 4.13 The converse of the above Proposition 4.12, is not true. The following example supports our claim.

Example 4.14

Let X = $\{a, b, c\}$,

$S_{\tau_1} = \{\emptyset, X, \{a, b\}, \{b, c\}\}$,

$S_{\tau_2} = \{\emptyset, X, \{a\}, \{a, c\}, \{b, c\}\}$.

Then $(X, S_{\tau_1}, S_{\tau_2})$ is $S_{\tau_{12}}$-$T^{**}_{1/2}$-space but not $S_{\tau_{12}}$-$T_{1/2}$-space.

Remark 4.15 If $S_{\tau_{ij}}$-$T^*_{1/2}$ and $S_{\tau_{ij}}$-$T^{**}_{1/2}$-spaces are does not each other as seen from the following examples.

Example 4.16

In example 4.4, $(X, S_{\tau_1}, S_{\tau_2})$ is $S_{\tau_{12}}$-$T^*_{1/2}$-space but not $S_{\tau_{12}}$-$T^{**}_{1/2}$-space.

Example 4.17

Let X = $\{a, b, c\}$,

$S_{\tau_1} = \{\emptyset, X, \{a\}, \{a, b\}, \{b, c\}\}$,

$S_{\tau_2} = \{\emptyset, X, \{a, c\}, \{b, c\}\}$.

Then $(X, S_{\tau_1}, S_{\tau_2})$ is $S_{\tau_{12}}$-$T^{**}_{1/2}$-space but it is not $S_{\tau_{12}}$-$T^*_{1/2}$-space.

Theorem 4.18 *A supra bitopological space $(X, S_{\tau_1}, S_{\tau_2})$ is $S_{\tau_{12}}$-$T_{1/2}$-space if and only if it is both $S_{\tau_{12}}$-$T^*_{1/2}$ and $S_{\tau_{12}}$-$T^{**}_{1/2}$-space.*

Proof. Suppose that $(X, S_{\tau_1}, S_{\tau_2})$ is $S_{\tau_{ij}}$-$T_{1/2}$-space. Then by Proposition 4.9 and Proposition 4.12, $(X, S_{\tau_1}, S_{\tau_2})$ is $S_{\tau_{ij}}$-$T^*_{1/2}$ and $S_{\tau_{ij}}$-$T^{**}_{1/2}$-space.
Converesely, suppose that $(X, S_{\tau_1}, S_{\tau_2})$ is both $S_{\tau_{12}}$-$T^*_{1/2}$and $S_{\tau_{12}}$-$T^{**}_{1/2}$-space. Let A be a $S_{\tau_{ij}}$-g-closed set of $(X, S_{\tau_1}, S_{\tau_2})$. since $(X, S_{\tau_1}, S_{\tau_2})$ is a $S_{\tau_{ij}}$-$T^*_{1/2}$-space, A is a $S_{\tau_{ij}}$-g^*-closed set. Since $(X, S_{\tau_1}, S_{\tau_2})$ is an $S_{\tau_{ij}}$-$T^{**}_{1/2}$-space, A is S_{τ_j}-closed set of $(X, S_{\tau_1}, S_{\tau_2})$. Therefore, $(X, S_{\tau_1}, S_{\tau_2})$ is an $S_{\tau_{ij}}$-$T_{1/2}$-space. □

Remark 4.19 From the above results we illustrate the following relations

$$S_{\tau_{ij}}\text{-}T^*_{1/2}\text{-space} \leftarrow S_{\tau_{ij}}\text{-}T_{1/2}\text{-space} \rightarrow S_{\tau_{ij}}\text{-}T^{**}_{1/2}\text{-space.}$$

The reverse of the implication is not true.

5. Conclusion

In this work, we analysed $S_{\tau_{ij}}$-g^*-closed sets and $S_{\tau_{ij}}$-g^*-open sets. Also, investigate related properties and compared their properties of $S_{\tau_{ij}}$-g-closed sets and$S_{\tau_{ij}}$-g^*-closed sets. Furthermore, we provided some examples wherever necessary. Finally the new separation axioms $S_{\tau_{ij}}$-$T^*_{1/2}$ and $S_{\tau_{ij}}$-$T^{**}_{1/2}$-spaces are introduced and analysed their properties.

References

[1] H. M. Abu-Donia, *New Types of Generalized closed sets in Bi-Topological spaces*, Journal of Egyption Math.soc., **21** (2013), 318-323.

[2] R. Gowri and A. K. R Rajayal, *On supra bitopological spaces*, IOSR-JM **13**(2017), 55-58.

[3] R. Gowri and A. K. R Rajayal, *Supra g-closed sets in Supra Bitopological Spaces*, International Mathematical Forum, **13**(2018), 175-182.

[4] R. Gowri and S. Vembu, *Soft g^*-closed Sets in Soft Biminimal Spaces*, Int.Jr. of Mathematics And its Applications, **5**(2017), 365-370.

[5] M. Kamaraj, G. Ramkumar and O. Ravi, *Supra sg-closed sets and supra gs-closed sets*, International Jr. of Mathematical Archive, **2(11)**(2011), 2413-2419.

[6] A. Kandil, O. A. E. Tantawy, S. A. El-Sheikh and A. M. Abd El-latif, *Supra Generalized Closed Soft Sets with Respect to an soft Ideal in supra soft Topological spaces*, Appl. Math. Inf. Sci., **8(4)**(2014), 1731-1740.

[7] J. C. Kelly, *Bitopological spaces*, Proc. London Math. Soc.No.(3), **13**(1963), 71-89.

[8] S. Lal, *Pairwise concepts in bitopological spaces*, Aust.Jr. Math. soc (ser.A), **26**(2015), 241-250.

[9] H. Maki, K. C. Rao and A. Nagoor Gani, *On generalized semi-open and pre-open sets*, Pure Appl. Math. Sci., **49**(1999), 17-29.

[10] A. S. Mashhour, A.A. Allam, F.S. Mahmoud and F.H. Khedr, *On Supra topological spaces*, Indian Jr.Pure and Appl.Math.No. **4(14)**(1983), 502-510.

[11] C. W. Patty, *Bitopological spaces*, Pure math. Jr. **34**(1967), 387-392.

[12] G. Ramkumar, O. Ravi and M. Joseph Isreal, *On supra g^*-closed sets*, Mathematical Sciences International Research Journal, **5**(2016), 72-76.

[13] O. Ravi, G. Ramkumar, and M. Kamaraj, *On supra g-closed sets*, International Journal of Advances in pure and Applied Mathematics, **1(2)**(2011), 52-66.

[14] M. Sheik John and P. Sundaram, *g^*-cosed sets in bitopological spaces*, Indian Jr. Pure appl. Math **35(1)**(2004), 77-80.