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k^*-Continuous Function In Ideal Closure Spaces

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ABSTRACT. In this paper, we have defined the concept k^*-continuous function in ideal closure spaces. In particular, the properties of open and closed maps, composite of k^*-continuous functions, characterization of k^*-homeomorphic functions in ideal closure spaces are explained in detail.

1. Introduction

In 1983, Mashhour and Ghahim[12] initiated some functions in Čech closure spaces such as Č-continuous function, Č-homeomorphism and composition of Č-continuous function. In addition, Čθ-continuous function and Č almost continuous function in Čech closure spaces were also defined and discussed. Later new types of homeomorphic function for different type of topologies using closure spaces were formed by Chvalina[7]. Recently, Francina Shalini and Arockiarani[9] were derived new functions such as Čech $\pi G\beta$-continuous functions and $\pi G\beta$-Irresolute functions in Čech $\pi G\beta$-Closure spaces. In this paper we have defined and discussed k^*-continuous function between two ideal closure spaces along with some properties and results of k^*-continuous function and composition of k^*-continuous functions. In addition, open and closed maps in ideal closure spaces are defined and their properties are derived. Finally k^*-homeomorphic functions are also introduced and discussed.

2. Preliminaries

In this section, we recall the basic definitions of ideal closure spaces.

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Definition 2.1[1] $(X, \Im)$ be a topological space. An Ideal I on a topological space is a collection of non empty collections of subsets of X which satisfies:

(1) $\emptyset \in I$

(2) $A \in I, B \subseteq A$ implies $B \in I$,

(3) $A \in I, B \in I$ implies $A \cup B \in I$.

If $(X, \Im)$ is a topological space and I is an Ideal on X. Then $(X, \Im, I)$ is called an ideal topological space or an ideal space.

Definition 2.2[11] Let $P(X)$ be the power set of X. Then the operator

$(.)^* : P(X) \to P(X)$ is called a local function of A with respect to $\Im$ and I, is define as follows:

For $A \subseteq X$, $A^*(I, \Im^*) = \{x \in X : U \cap A \notin I for\ every\ open\ set\ U\ containing\ x\}$

Additionally, $cl^*(A) = A \cup A^*$ defines Kuratowski closure operator for a topology $\Im^*$. Here $\Im^*$ is finer than $\Im$.

Definition 2.3[10] let X be a non-empty set. I be an Ideal on X.

Let $A^* : P(X) \to P(X)$ be a function of A with respect to I and $\Im$.

Let $k^*(A) = A \cup A^*$ defines Kuratowski closure operator for a topology.

Then the function $k^* : P(X) \to P(X)$ satisfying,

(1) $k^*(\emptyset) = \emptyset$

(2) $A \subset k^*(A)$

(3) $k^*(A \cup B)$=$k^*(A) \cup k^*(B)$ $\quad \forall A, B \subset X$.

(4) $k^*(A) = k^*(k^*(A))$ $\quad \forall A \subset X$ is called a closure operator on X. The structure (X, I, k^*) is called an ideal closure space.

Example 2.4 $X = \{a, b, c\}$ $\Im = \{X, \emptyset, \{a\}, \{c\}, \{a, c\}\}$. $I = \{\emptyset, \{c\}\}$

(1) $A = \{a, c\}$ $A^* = \{a, b\}$ $\quad k^*(A) = A \cup A^* \implies k^*\{a, c\}$ =X.

(2) $A = \{b, c\}$ $A^* = \{b\}$ $\quad k^*(A) = A \cup A^* \implies k^*\{b, c\}$ =$\{b, c\}$.

(3) $A = \{a, b\}$ $A^* = \{a, b\}$ $\quad k^*(A) = A \cup A^* \implies k^*\{a, b\}$ =$\{a, b\}$.

(4) $A = X$ $\quad A^* = \{a, b\}$ $\quad k^*(A) = A \cup A^* \implies k^*(X)$ = X

(5) $A = \emptyset$ $\quad A^* = \emptyset$ $\quad k^*(A) = A \cup A^* \implies k^*(\emptyset)$ =$\emptyset$.

(6) $A = \{a\}$ $\quad A^* = \{a, b\}$ $\quad k^*(A) = A \cup A^* \implies k^*\{a\}$ =$\{a, b\}$.

(7) $A = \{b\}$ $\quad A^* = \{b\}$ $\quad k^*(A) = A \cup A^* \implies k^*\{b\}$ =$\{b\}$.

(8) $A = \{c\}$ $\quad A^* = \emptyset$ $\quad k^*(A) = A \cup A^* \implies k^*\{c\}$ =$\{c\}$.

Then (X, I, k^*) is an ideal closure space.

Definition 2.5[10] A subset A of an ideal closure space (X, I, k^*) is said to be closed if $k^*(A) = A$.

Definition 2.6[10] A subset A of an ideal closure space (X, I, k^*) is said to be open if $k^*(X - A) = X - A$ (i.e) $Int^*(A) = A$.

Definition 2.7[10] The set Int A with respect to the closure operator k^* is defined as

$Int^*(A) = X - k^*(X - A)(i.e)[k^*(A^C)]^C$, where $A^C = X - A$.

Definition 2.8[10] (X, I, k^*) is an ideal closure space than the associate topology on X is $\Im^* = \{A^C; k^*(A) = A\}$. Here $\Im$ is not equal to $\Im^*$

Definition 2.9[10] A subset A in an ideal closure space (X, I, k^*) is called neighbourhood of x if $x \in Int^*(A)$.

Definition 2.10[10] Let (X, I, k^*) be an ideal closure space. An closure space (Y, I, k^*_Y) is called a subspace of (X, I, k^*) if $Y \subseteq X$ and $k^*_Y(A) = k^*(A) \cap Y, \forall A \subseteq Y$.

3. k^*-Continuous Functions in Ideal Closure Spaces

Definition 3.1 Let (X, I_1, k^*_1) and (Y, I_2, k^*_2) be ideal closure spaces. A function f: $(X, I_1, k^*_1) \to (Y, I_2, k^*_2)$ is said to be k^*-continuous if $f\left(k^*_1(A)\right) \subseteq k^*_2\left(f(A)\right)$, for every $A \subset X$.

Theorem 3.2 *Let (X, I_1, k^*_1) and (Y, I_2, k^*_2) be ideal closure spaces. A function $f : (X, I_1, k^*_1) \to (Y, I_2, k^*_2)$ then the following are equivalent*
(a) f is k^*-continuous.
(b) For every subset A of Y then $k^*_1\left(f^{-1}(A)\right) \subset f^{-1}\left(k^*_2(A)\right)$.
(c) For every closed subset F of Y then $f^{-1}(F)$ is an closed subset of X.
(d) For every open subset U of Y then $f^{-1}(U)$ is an open subset of X.

Proof.
$(a) \implies (b)$
Let $A \subset Y$ then $f^{-1}(A) \subseteq X$. Since f is k^* - continuous, we have $f\left(k^*_1(f^{-1}(A))\right) \subseteq k^*_2\left(f(f^{-1}(A))\right) \subseteq k^*_2(A)$. Therefore $f^{-1}\left(f\left(k^*_1(f^{-1}(A))\right)\right) \subseteq f^{-1}\left(k^*_2(A)\right)$. Hence $k^*_1\left(f^{-1}(A)\right) \subset f^{-1}\left(k^*_2(A)\right)$.
$(b) \implies (c)$
Suppose that F is closed subset of Y. Prove that $f^{-1}(F)$ is closed in X. By the definition of ideal closure spaces

$f^{-1}(F) \subseteq k^*_1(f^{-1}(F))$............(1)

Now we have to prove that $k^*_1(f^{-1}(F)) \subseteq f^{-1}(F)$. Let $x \in k^*_1(f^{-1}(F))$, then $f(x) \in f(k^*_1(f^{-1}(F))$, by the definition of k^*-continuous function, $f(x) \in f(k^*_1(f^{-1}(F)) \subseteq k^*_2(f(f^{-1}(F)) \subseteq k^*_2(F) = F$ then $f(x) = F$ implies $x \in f^{-1}(F)$ then $k^*_1(f^{-1}(F)) \subseteq f^{-1}(F)$........(2).
From (1) and (2), $k^*_1(f^{-1}(F)) = f^{-1}(F)$. Hence $f^{-1}(F)$ is closed in X.
$(c) \implies (d)$
Assume that pre image of closed sets are closed. Suppose that $x \in X$ and U is an open subset of Y. The complement $Y \setminus U$ is closed as U is open. Thus $f^{-1}(Y \setminus U)$ is closed in X but the pre image of the complement is the complement of pre image. (i.e)$f^{-1}(Y \setminus U)$=$f^{-1}(Y) \setminus f^{-1}(U)$. we can replace $f^{-1}(Y)$ with X to obtain $f^{-1}(Y \setminus U)$=$X \setminus f^{-1}(U)$, so $X \setminus f^{-1}(U)$ is closed so that $f^{-1}(U)$ is open.
$(b) \implies (a)$
Let (X, I_1, k^*_1) and (Y, I_2, k^*_2) be ideal closure spaces. A function
$f : (X, I_1, k^*_1) \to (Y, I_2, k^*_2)$ is defined by $f(A) \subseteq B$. Then $A \subseteq f^{-1}(B)$ where $A \subseteq X$ and $B \subseteq Y$. From (b), $B \subseteq Y$, $k^*_1(f^{-1}(B)) \subset f^{-1}(k^*_2(B))$ then $f(k^*_1(f^{-1}(B))) \subset f(f^{-1}(k^*_2(B))$, $f(k^*_1(f^{-1}(B))) \subset k^*_2(B))$, $f(k^*_1((A)) \subset k^*_2(f(A)))$.

Therefore f is k^*-continuous function. □

Example 3.3.

$X = \{a, b, c\};$ $Y = \{x, y, z\}$

$\Im_1 = \{X, \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{c, a\}\},$

$I_1 = \{\emptyset, \{a, b\}\};$ $\Im_2 = \{Y, \{x\}, \{z\}, \{x, z\}\}; I_2 = \{\emptyset\}$

$k_1^*(a) = \{a\};$ $k_2^*(x) = \{x, y\}$

$k_1^*(b) = \{b\};$ $k_2^*(y) = \{y\}$

$k_1^*(c) = \{c\};$ $k_2^*(z) = \{y, z\}$

$k_1^*\{a, b\} = \{a, b\};$ $k_2^*\{x, y\} = \{x, y\}$

$k_1^*\{b, c\} = \{b, c\};$ $k_2^*\{y, z\} = \{y, z\}$

$k_1^*\{c, a\} = \{c, a\};$ $k_2^*\{z, x\} = Y$

$k_1^*(X) = X;$ $k_2^*(Y) = Y$

$k_1^*(\emptyset) = \emptyset;$ $k_2^*(\emptyset) = \emptyset$

f is a mapping from $(X, I_1, k_1^*) \to (Y, I_2, k_2^*)$ defined by $f(a) = y, f(b) = z, f(c) = x$. Here f is k^*-continuous function.

Theorem 3.4 *Let (X, I_1, k_1^*), (Y, I_2, k_2^*) and (Z, I_3, k_3^*) be ideal closure spaces. If the function f: $(X, I_1, k_1^*) \to (Y, I_2, k_2^*)$ and g: $(Y, I_2, k_2^*) \to (Z, I_3, k_3^*)$ are continuous then $g \circ f : (X, I_1, k_1^*) \to (Z, I_3, k_3^*)$ is k^*-continuous.*

Proof. Let $A \subseteq X$, f is k^*-continuous then $f(k_1^*(A)) \subseteq k_2^*(f(A))$.......(1)

Now let $A \subseteq Y$, g is k^*-continuous then $g(k_2^*(A)) \subseteq k_3^*(g(A))$........(2)

To prove that $g \circ f$ is k^*-continuous. Let $A \subseteq X$, $g \circ f(k_1^*(A)) = g(f(k_1^*(A)))$.

Since from (1) and (2) $g(f(k_1^*(A))) \subseteq g(k_2^*(f(A))) \subseteq k_3^*(g(f(A)))$ therefore

$g \circ f(k_1^*(A)) \subseteq k_3^*(g \circ f(A))$. Hence $g \circ f$ is k^*-continuous. □

Theorem 3.5 *If f: $(X, I_1, k_1^*) \to (Y, I_2, k_2^*)$ is k^*-continuous then f: $(X, I_1, \Im_1^*) \to (Y, I_2, \Im_2^*)$ is k^*-continuous.*

Proof. Let F be closed subset of $(Y, I_2, \Im_2^*)$ then we have to show that $f^{-1}(F)$ is closed subset of $(X, I_1, \Im_1^*)$, using the properties of closure operator $f^{-1}(F) \subseteq \Im^* - cl(f^{-1}(F))$........(1)

Now we have to prove that $\Im^* - cl(f^{-1}(F)) \subseteq f^{-1}(F)$. Let $x \in \Im^* - cl(f^{-1}(F))$ then $x \in k_1^*(f^{-1}(F))$ since $k_1^*(f^{-1}(F) \subseteq \Im_1^* - cl(f^{-1}(F)))$ then f is k^*-continuous which implies $f(x) \in f(k_1^*(f^{-1}(F))) \subseteq k_2^*(f(f^{-1}(F))) \subseteq k_2^*(F) \subseteq \Im^* - cl(F) = F$ implies $x \in f^{-1}(F)$ therefore $\Im^* - cl(f^{-1}(F)) \subseteq f^{-1}(F)$.........(2)

From (1) and (2) $\Im^* - cl(f^{-1}(F)) = f^{-1}(F)$. Hence $f^{-1}(F)$ is closed in $(X, I_1, \Im_1^*)$. By using the theorem 3.2 f is k^*-continuous. □

Theorem 3.6 *Let (X, I_1, k_1^*) and (Y, I_2, k_2^*) be ideal closure spaces. Let (A, I, k_A^*) be a closed subspace of (X, I_1, k_1^*). if f: $(X, I_1, k_1^*) \to (Y, I_2, k_2^*)$ is k^*-continuous then $f|_A$: $(A, I, k_A^*) \to (Y, I_2, k_2^*)$ is k^*-continuous.*

Proof. Let $A \subseteq X$, f is k^*-continuous then $f(k_1^*(A)) \subseteq k_2^*(f(A))$. To prove that $f|_A$ is k^*-continuous. Here (A, I, k_A^*) is a closed subspace of (X, I_1, k_1^*) then by the definition of subspace let $B \subseteq A$, $k_A^*(B) = k_1^*(A \cap B)$ therefore $f|_A(k_A^*(B)) = f_{|A}(k_1^*(B \cap A)) = f|_A(k_1^*(B)) = f(k_1^*(B)) \subseteq k_2^*(f(B)) = k_2^*(f|_A(B))$ therefore $f|_A(k_A^*(B)) \subseteq$

$k_2^*(f|_A(B))$. Hence $f|_A$ is k^*-continuous. □

4. Open and Closed Maps in Ideal Closure Spaces

Definition 4.1 Let (X, I_1, k_1^*) and (Y, I_2, k_2^*) be ideal closure spaces. A map $f : (X, I_1, k_1^*) \to (Y, I_2, k_2^*)$ is said to be closed(resp.open) if $f(F)$ is closed(resp.open) subset of Y whenever F is a closed (resp.open) subset of X.

Theorem 4.2 *A map $f : (X, I_1, k_1^*) \to (Y, I_2, k_2^*)$ is closed if and only if for each subset B of Y and each open subset G of X containing $f^{-1}(B)$, there is an open subset U of X such that $B \subseteq U$ and $f^{-1}(U) \subset G$.*

Proof. Suppose f is closed. Let B be a subset of Y and G be an open subset of X such that $f^{-1}(B) \subseteq G$. Then $f(X - G)$ is a closed subset of Y. Let $U = Y - f(X - G)$. Then U is an open subset of Y and $f^{-1}(U) = f^{-1}(Y - f(X - G)) = X - f^{-1}(f(X - G)) \subseteq X - (X - G) = G$. Therefore U is an open subset of Y containing B such that $f^{-1}(U) \subseteq G$.
Conversely, suppose that F is closed subset of X then $f^{-1}(Y - f(F)) \subseteq X - F$ and $X - F$ is an open subset of X. Then by the hypothesis there is an open subset U of Y such that $Y - f(F) \subseteq U$ and $f^{-1}(U) \subseteq X - F$. Therefore $F \subseteq X - f^{-1}(U)$ consequently $Y - U \subseteq f(F) \subseteq f(X - f^{-1}(U)) \subseteq Y - U$ which implies that $f(F) = Y - U$. Thus $f(F)$ is a closed subset of Y. Hence f is closed. □

Theorem 4.3 *Let (X, I_1, k_1^*), (Y, I_2, k_2^*) and (Z, I_3, k_3^*) be ideal closure spaces. Let f: $(X, I_1, k_1^*) \to (Y, I_2, k_2^*)$ and g: $(Y, I_2, k_2^*) \to (Z, I_3, k_3^*)$ be maps. Then*
(1) If f and g are closed then $g \circ f$ is also closed.
(2) If $g \circ f$ is closed and g is k^*-continuous and injection then f is closed.
(3) If $g \circ f$ is closed and f is k^*-continuous and surjection then g is closed.

Proof. .

(1) Since f is closed, F is a closed subset of (X, I_1, k_1^*) then $f(F)$ is a closed subset of (Y, I_2, k_2^*). Since g is closed, G is a closed subset of (X, I_1, k_1^*) then $f(G)$ is a closed subset of (Y, I_2, k_2^*). To prove that $g \circ f$ is closed. Suppose H is a closed subset of (X, I_1, k_1^*). We have to prove that $g \circ f(H)$ is closed.
Let $g \circ f : (X, I_1, k_1^*) \to (Z, I_3, k_3^*)$ then $g \circ f^{-1}(Z - g \circ f(H)) \subseteq X - H$ and $X - H$ is an open subset of (X, I_1, k_1^*) using theorem 4.2 there is an open subset U of (Z, I_3, k_3^*) such that $Z - g \circ f(H) \subseteq U$ and $g \circ f^{-1}(U) \subseteq X - H$. Therefore, $H \subseteq X - g \circ f^{-1}(U)$, consequently $Z - U \subseteq g \circ f(H) \subseteq g \circ f(X - g \circ f^{-1}(U)) \subseteq Z - U$, which implies that $g \circ f(H) = Z - U$. Thus $g \circ f(H)$ is closed subset of (Y, I_2, k_2^*). Hence $g \circ f$ is closed.

(2) If F be a closed subset of (X, I_1, k_1^*). since $g \circ f$ is closed, $g(f(F))$ is closed in (Z, I_3, k_3^*). As g is k^*-continuous then $g^{-1}(g(f(F)))$ is closed in (Y, I_2, k_2^*) but g is injective so $g^{-1}(g(f(F))) = f(F)$ is closed in (Y, I_2, k_2^*). Hence f is closed.

(3) If F be a closed subset of (Y, I_2, k_2^*) since $g \circ f$ is closed, $g(f(F))$ is closed in (Z, I_3, k_3^*). As f is k^*-

continuous then $f^{-1}(g(f(F)))$ is closed in (X, I_1, k_1^*) but f is surjective, $f^{-1}(g(f(F))) = g(f^{-1}(f(F))) = g(F)$ is closed in (Z, I_3, k_3^*). Hence g is closed. □

5. k^*-Homeomorphic Functions in Ideal Closure Spaces

Definition 5.1 Let $f : (X, I_1, k_1^*) \to (Y, I_2, k_2^*)$ be a bijection between ideal closure spaces. The function f^{-1}:(Y, I_2, k_2^*) $\to (X, I_1, k_1^*)$ given by the rule $f^{-1}(y) = x$ if and only if $f(x) = y$ is called the inverse function or inverse of f.

Definition 5.2 Let $f : (X, I_1, k_1^*) \to (Y, I_2, k_2^*)$ be a bijective between ideal closure spaces. If f and f^{-1} are k^*-continuous then f is called a k^*-homeomorphism. Then (X, I_1, k_1^*) and (Y, I_2, k_2^*) are said to be k^*-homeomorphic if there exists a k^*-homeomorphism between (X, I_1, k_1^*) and (Y, I_2, k_2^*).

Proposition 5.3 *Let (X, I_1, k_1^*) and (Y, I_2, k_2^*) be ideal closure spaces.*
If f: $(X, I_1, k_1^*) \to (Y, I_2, k_2^*)$ be a bijection then the following statements are equivalent.
(1) The inverse map f^{-1} is k^*-continuous.
(2) f is an open map.
(3) f is a closed map.

Proof. .
(1) $\implies$ (2)
Let f^{-1}: $(Y, I_2, k_2^*) \to (X, I_1, k_1^*)$ be k^*-continuous and U be an open set in X. Then $(f^{-1})^{-1}(U)$ is open, which implies $f(U)$ is open. Then f is an open map.
(2) $\implies$ (3)
Let F be closed in X. Then $X - F$ be open in X. Since f is open, $f(X - F)$ is open in Y. Then $Y - f(F)$ is open in X. Hence $f(F)$ is closed in X. Then f is a closed map.
(3) $\implies$ (1)
Let F be closed in X. As f is closed, $f(A)$ is closed in Y but $f(A) = (f^{-1})^{-1}(A)$. Thus f^{-1} is k^*continuous. □

Theorem 5.4 *Let (X, I_1, k_1^*) and (Y, I_2, k_2^*) be ideal closure spaces and f be a bijection mapping from (X, I_1, k_1^*) onto (Y, I_2, k_2^*). Then the following conditions are equivalent.*
(1) f is open and k^*-continuous.
(2) f is closed and k^*-continuous.
(3) f is k^*-homeomorphism.

Proof. .
(1) $\implies$ (3) Let f is bijective open and k^*-continuous. Let U be an open set in X. Then f is open then $f(U)$ is open in Y, (i.e) $(f^{-1})^{-1}(U) = f(U)$ is open in Y. Thus f^{-1} is k^*-continuous. Hence f is k^*-homeomorphic.
(3) $\implies$ (2)
Assume that f is k^*-homeomorphism. Then by definition, f is k^*-continuous. Let F be a closed set in X. Then $X - F$ is open and $f^{-1} = g$ is k^*-continuous, $g^{-1}(X - F)$ is open (i.e) $g^{-1}(X - F) = Y - g^{-1}(F)$ is open. Thus

$g^{-1}(F)$ is closed (i.e) $f(F)$ is closed. Hence f is closed map.

(3) $\implies$ (1)

Assume that f is k^*-homeomorphism. Then by definition, f is k^*-continuous. Let U be an open set in X. Then $X-U$ is closed and $f^{-1} = g$ is k^*-continuous, $g^{-1}(X-U)$ is closed (i.e) $g^{-1}(X-U) = Y - g^{-1}(F)$ is closed. Thus $g^{-1}(U)$ is open (i.e) $f(U)$ is open. Hence f is open map.

(2) $\implies$ (3)

If f is bijective closed and k^*-continuous then we have to prove that f^{-1} also k^*-continuous. Let U be an open set. Then $X-U$ is closed. Since f is closed then $f(X-G)$ is closed therefore $g^{-1}(X-G) = Y - g^{-1}(U)$ is closed implies $g^{-1}(U)$ is open. Thus inverse image under g of every open is open. Here $g = f^{-1}$ is continuous. Therefore f is k^*-homeomorphism.

6. Conclusion

In this paper, the basic properties of k^*-continuous functions in ideal closure space is introduced. Also the open and closed maps and k^*-homeomorphic function between two ideal closure spaces are analyzed.

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