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Painleve Analysis and Symmetry Analysis of the Two Dimensional Variable Coefficient Burgers Equation

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ABSTRACT. We discuss the painleve analysis of the variable coefficient of the Burgers equation. Next discuss the symmetries of the variable coefficient of the Burgers equation. We classify one-dimensional and two-dimensional subalgebras of the Burgers equation. Further reduction of these equations to second order equations.

1 Introduction

The Korteweg-de Vries Burgers equation $(u_t + uu_x + \mu u_{xxx} - vu_{xx})_x + \sigma u_{yy} = 0$ is a prototype example of an evolution equation in $(2+1)$-dimensions which is not completely integrable. Here μ, ν are real constants and $\sigma = \pm 1$. Although it has an infinite-dimensional symmetry algebra it does not have a Virasoro structure. The presence of a Virasoro algebra generally signals integrability for $(2+1)$-dimensional evolution equations. The history of the KdV equation started with experiments by John Scott Russell in 1834, followed by theoretical investigations by Lord Rayleigh and Joseph Boussinesq around 1870 and, finally, Korteweg and De Vries in 1895. The KdV equation is a nonlinear, dispersive partial differential equation for a function of two real variables, space x and time t: $u_t + 6uu_x + u_{xxx} = 0$. The Generalized KdV equation has been studied for its solutions by many authors name list. Senthilkumaran, Pandiaraja and Mayil Vaganan [1] reported invariant solutions of another GKdVE in the form

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$u_t + u^n u_x + \alpha(t)u + \beta(t)u_{xxx} = 0$ using Lie's group of infinitesimal transformations [2], [3]. In 1969, Zabolotskaya and Khokhlove derived the ZK equation $(u_t + uu_x - \beta u_{xx})_x + \gamma u_{yy} = 0$. Yet another model equation is derived by Kadomtsev and Petviashvili [5] $(u_t + uu_x + \epsilon 2u_{xx})_x + \lambda u_{yy} = 0, \lambda = \pm 1$ is the generalization of two spatial dimensions, x and y, of the KdV equation. But David, Levi and Winternitz [4], [5], [6] generalized KP equation to describe water waves in straits or rivers.

The classical Lie symmetries of the partial differential equations (PDEs) which can be obtained through the Lie group method of infinitesimal transformations were originally developed by Lie [7]. Symmetry group method plays an important role in the analysis of differential equations. The primary objective of the group classification methods advocated by Sophus Lie is to find one or several parameter local continuous transformations leaving the equations invariant and then exploit those to obtain the so called invariant or similarity solutions Ovsiannikov [8] and Olver [2]. Recently, there have been several generalizations of the classical Lie group method for symmetry reductions.

Similarity solutions of the super KdV equation were obtained by Huidan and Jiefang [9]. Zhong Zhou Dong and Yong Chen [10], obtained the exact solutions of the dispersive long wave equation. Pan Xiu-de [11] found the similarity solutions of the combined KdV equation.

Painleve property of KdV equation with nonunifromities Brugarino T[12]. In 1985, Painleve property of a Space-Dependent Burgers Equation by W. H. STEEB and W. STRAMPP [13]. Painleve analysis and reducibility to the canonical form for the generalized KP equation by Tommaso Brugarino and Antaonio M. Greaco [14]. Discuss Painleve analysis and Backlund transformation in the KS equation in Robert Conte and Micheline Musette [15].

In this paper, we discuss Painleve property and symmetries of the variable coefficient of the Burgers equation

$$(u_t + uu_x - p(t)u_{xx})_x + r(t)u_{yy} = 0 \tag{1.1}$$

This paper is organised as follows: In section 2, General view of Painleve analysis. In section 3, Painleve analysis of the variable coefficient of the Burgers equation. In section 4, we perform a symmetry classification of the equation using Lie classical method. In section 5, using one-dimensional subalgebras we reduce the given PDE. In section 6, we summarize the results of the present work.

2 Painleve analysis: generalities

An essential question in the study of NLPDE is the nature of the singularities of the solutions (poles, branch points or essential singularities) and their position (fixed or movable).

For this purpose, the Painleve analysis, which has been renewed by Ablowitz et al [16] for ordinary differential equations (ODE). It consists in looking for the general solution of the PDE in the form (written here in the case of one dependent and two independent variables):

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t)[\varnothing(x,t)]^{n+k} \tag{2.1}$$

where k is negative, $\varnothing(x,t) = 0$ is the equation of a non-characteristic ($\varnothing_x \varnothing_t \neq 0$) singular manifold, and the functions u_n have to be determined by substitution of expansion (2.1) in the PDE, which becomes:

$$\sum_{n=0}^{\infty} E_n(u_0, ...u_n, \varnothing)[\varnothing]^{n+q} = 0 \tag{2.2}$$

where q is some negative constant. E_n depends on $\varnothing$ only by the derivatives of $\varnothing$.
The successive practical steps of Painleve analysis are the following.

1. Determine the possible leading orders k by balancing two or more terms of the PDE and expressing that they dominate the other terms.
2. Solve equation $E_0 = 0$ for non-zero values of u_0; this may lead to several solutions, called branches.
3. Find the resonances, i.e. the values of n for which u_n cannot be determined from equation $E_n = 0$. This last equation has usually the form:

$$E_n \equiv (n+1)P(n)\varnothing_x^j \varnothing_t^{m-j} u_n + Q(u_0, ...u_{n-1}, \varnothing) = 0 \quad \forall n > 0 \tag{2.3}$$

   where m is the order of the PDE, $0 \leq j \leq m$ and P a polynomial of degree $m-1$. The values of the resonances are the zeros of P.
4. Determine if the resonances are compatible or not. At a resonance, after substitution in (2.3) of the previously computed $u_l, l \leq n-1$, the function Q is either zero, in which case u_n can be arbitrarily chosen and the resonance is said to be compatible, or non-zero and the expansion (2.1) does not exist for arbitrary $\varnothing$.

The Painleve property is characterised by the fact that k is a negative integer and all resonances occur at positive integer values of n and are compatible.

3 Painleve analysis of (1.1)

We are looking for a solution of (1.1) in the Laurent series expansion

$$u(x,y,t) = \sum_{n=0}^{\infty} u_n(x,y,t)[\varnothing(x,t,p(t))]^{n+k} \tag{3.1}$$

where $u_n(x,y,t)$ are analytic functions in a neighbourhood of the singular manifold $\varnothing(x,t,p(t)) = 0$ and k is an integer to be determined. Inserting the ansatz

$$u(x,y,t) \cong u_0 \varnothing^k$$

In (1.1) and comparing the exponents, we find that the leading-order analysis gives the value $k = -1$. Insertng the ansatz (3.1) together with $k = -1$ in (1.1), then we find the recursion relations for $u_n(x,y,t)$

$$u_{n-3,xt} + u_{n-2,x}(n-3)\varnothing_t + u_{n-2,t}(n-3)\varnothing_x + u_{n-1,x}(n-2)(n-3)\varnothing_x\varnothing_t + u_{n-2}(n-3)\varnothing_{xt}$$
$$+ u_{n-2,x}(n-3)\varnothing_p p_t + u_{n-1}(n-2)(n-3)\varnothing_x\varnothing_p p_t + u_{n-2}(n-3)\varnothing_{xp} p_t + \sum_{m=0}^{n} u_{m-2,x}u_{n-m,x}$$
$$+ \sum_{m=0}^{n} u_m u_{n-m}(n-m-1)(m-1)\varnothing_x^2 + 2\sum_{m=0}^{n} u_{m-1,x}u_{n-m}(n-m-1)\varnothing_x$$
$$+ \sum_{m=0}^{n} u_{m-2,x}u_{n-m,xx} + 2\sum_{m=0}^{n} u_{m-1}u_{n-m,x}(n-m-1)\varnothing_x + \sum_{m=0}^{n} u_m u_{n-m}(n-m-1)(n-m-2)\varnothing_x^2 \quad (3.2)$$
$$+ \sum_{m=0}^{n} u_{m-1}u_{n-m}(n-m-1)\varnothing_{xx} - p(t)u_{n-3,xxx} - 3p(t)u_{n-2,xx}(n-3)\varnothing_x - 3p(t)u_{n-1,x}(n-2)(n-3)\varnothing_x^2$$
$$- 3p(t)u_{n-2,x}(n-3)\varnothing_{xx} - p(t)u_n(n-3)\varnothing_{xx} - p(t)u_n(n-1)(n-2)(n-3)\varnothing_x^3$$
$$- 3p(t)u_{n-2}(n-3)\varnothing_x\varnothing_{xx} - p(t)u_{n-2}(n-3)\varnothing_{xxx}r(t)u_{n-3,yy} = 0; \quad n = 0,1,2,3,...$$

In collecting terms involving u_n, it is found that

$$(n+1)(n-2)(n-3)\varnothing_x^3 u_n = F(x,y,t,u_{n-1},,u_0,\varnothing_t,\varnothing_y,\varnothing_x,..) \quad (3.3)$$

Equation (3.3) determines the coefficients u_n of the series expansion (3.1), provided that $n \neq -1,2,3$. These values of n are called the resonance of the recursion relations and allow the introduction of arbitrary functions u_2 and u_3. For $n = -1$, the series (3.1) is undefined and therefore the resonance at $n = -1$ corresponds to arbitrary function $\varnothing$ defining the singular manifold. Put $n = 0$ in (3.2), we get

$$u_0 = -2p(t)\varnothing_x \quad (3.4)$$

Put $n = 1,2,3$ in (3.2) and using (3.4), we get the following system of equations

$$\varnothing_p pt + \varnothing_t + u_1\varnothing_x - p(t)\varnothing_{xx} = 0$$

$$2u_{1x}\varnothing_x^2 - 2p(t)\varnothing_x\varnothing_{xxx} + 3u_1\varnothing_x\varnothing_{xx} + 2\varnothing_x\varnothing_{xp}p_t + \varnothing_{xx}\varnothing_p p_t - p(t)\varnothing_{xx}^2 + \varnothing_{xx}\varnothing_t + 2\varnothing_x\varnothing_{xt} + \frac{p_t}{p(t)}\varnothing_x^2 = 0$$
$$p_t\varnothing_{xx} + p(t)\varnothing_{xxt} + p(t)\varnothing_{xxp}p_t + 2p(t)\varnothing_{xx}u_{1x} + p(t)\varnothing_x u_{1xx} + p(t)u_1\varnothing_{xxx} - [(p(t))]^2\varnothing_{xxxx} = 0$$

Furthermore, if we set the arbitrary functions u_2 and u_3 equal to zero. Then $u_n = 0$ for $n \geq 2$ and u_1 is a solution of the equation (1.1) $(u_{1t} + u_1u_{1x} - p(t)u_{1xx})_x + r(t)u_{1yy} = 0$.

In this case, we find the following Backlund transformation for the $(2+1)$-dimensional variable coefficient Burgers equation $u = -\frac{2p(t)\varnothing_x}{\varnothing} + u_1$ where (u, u_1) satisfy the (2+1)-dimensional variable coefficient Burgers equation. When $u_1 = 0$, the Cole - Hopf tranform is obtained. Thus $u = -\frac{2p(t)\varnothing_x}{\varnothing} + u_1$ is the Cole-Hopf transformation of the (2+1)-dimensional variable coefficient Burgers equation.

Next, we discuss the singular manifold is of the form $\varnothing(x,y,t,p(t))$. We continue the above process, we get

$k = -1$ and the recursion relations for $u_n(x, y, t)$

$$
\begin{aligned}
& u_{n-3,xt} + u_{n-2,x}(n-3)\varnothing_t + u_{n-2,t}(n-3)\varnothing_x + u_{n-1,x}(n-2)(n-3)\varnothing_x\varnothing_t + u_{n-2}(n-3)\varnothing_{xt} \\
& + u_{n-2,x}(n-3)\varnothing_p p_t + u_{n-1}(n-2)(n-3)\varnothing_x\varnothing_p p_t + u_{n-2}(n-3)\varnothing_{xp} p_t + \sum_{m=0}^{n} u_{m-2,x}u_{n-m,x} \\
& + \sum_{m=0}^{n} u_m u_{n-m}(n-m-1)\varnothing_x^2 + 2\sum_{m=0}^{n} u_{m-1,x}u_{n-m}(n-m-1)(m-1)\varnothing_x \\
& + \sum_{m=0}^{n} u_{m-2,x}u_{n-m,xx} + 2\sum_{m=0}^{n} u_{m-1}u_{n-m,x}(n-m-1)\varnothing_x + \sum_{m=0}^{n} u_m u_{n-m}(n-m-1)(n-m-2)\varnothing_x^2 \\
& + \sum_{m=0}^{n} u_{m-1}u_{n-m}(n-m-1)\varnothing_{xx} - p(t)u_{n-3,xxx} - 3p(t)u_{n-2,xx}(n-3)\varnothing_x - 3p(t)u_{n-1,x}(n-2)(n-3)\varnothing_x^2 \\
& - 3p(t)u_{n-2,x}(n-3)\varnothing_{xx} - p(t)u_n(n-3)\varnothing_{xx} \\
& - p(t)u_n(n-1)(n-2)(n-3)\varnothing_x^3 \\
& - 3p(t)u_{n-1}(n-2)(n-3)\varnothing_x\varnothing_{xx} - p(t)u_{n-2}(n-3)\varnothing_{xxx} \\
& + p(t)u_{n-2}(n-3)\varnothing_{xxx} + r(t)u_{n-3,yy} + 2r(t)u_{n-2,y}(n-3)\varnothing_y + r(t)u_{n-2}(n-3)\varnothing_y^2 \\
& + 2r(t)u_{n-2}(n-3)\varnothing_{yy} = 0; \quad n = 0, 1, 2, 3, ...
\end{aligned} \tag{3.5}
$$

In collecting terms involving u_n, it is found that

$$
(n+1)(n-2)(n-3)\varnothing_x^3 u_n = F(x, y, t, u_{n-1}..., u_0, \varnothing_t, \varnothing_y, \varnothing_x, ...) \tag{3.6}
$$

Equation (3.6) determines the coefficients u_n of the series expansion (3.1), provided that $n \neq -1, 2, 3$. These values of n are the resonance of the recursion relations and allow the introduction of arbitrary functions u_2 and u_3. For $n = -1$, the series (3.1) is undefined and therefore the resonance at $n = -1$ corresponds to arbitrary function $\varnothing$ defining the singular manifold.

Put $n = 0$ in (3.5), we get

$$
u_0 = -2p(t)\varnothing_x \tag{3.7}
$$

Put $n = 1, 2, 3$ in (3.5) and using (3.7), we get the following system of equations

$$
\varnothing_x\varnothing_t + \varnothing_x\varnothing_p p_t - p(t)\varnothing_x\varnothing_{xx} + u_1\varnothing_x^2 - r(t)\varnothing_y^2 = 0 \tag{3.8}
$$

$$
p(t)\varnothing_{xt} + p(t)\varnothing_{xp}p_t + p_t\varnothing_x + p_t u_{1x}\varnothing_x + p_t u_1\varnothing_{xx} - [p(t)]^2\varnothing_{xxx} + p(t)r(t)\varnothing_{yy} = 0 \tag{3.9}
$$

$$
p_t\varnothing_{xx} + p(t)\varnothing_{xxt} + p(t)\varnothing_{xxp}p_t + 2p(t)\varnothing_{xx}u_{1x} + p(t)\varnothing_x u_{1xx} + p(t)u_1\varnothing_{xxx} - [p(t)]^2\varnothing_{xxxx} + p(t)r(t)\varnothing_{xyy} = 0 \tag{3.10}
$$

By (3.9) the compatibility condition (3.10) is satisfied identically. Furthermore, if we set the arbitrary functions u_2 and u_3 equal to zero. Then $u_n = 0$ for $n \geq 2$ and u_1 is a solution of the equation (1.1)

$$
(u_{1t} + u_1 u_{1x} - p(t)u_{1xx})_x + r(t)u_{1yy} = 0.
$$

In this case, we find the following Backlund transformation for the (2+1)-dimensional variable coefficient Burgers equation $u = -\frac{2p(t)\varnothing_x}{\varnothing} + u_1$ where (u, u_1) satisfy the (2+1)-dimensional variable coefficient Burgers equation. When $u_1 = 0$, the Cole-Hopf tranform is obtained. Thus $u = -\frac{2p(t)\varnothing_x}{\varnothing} + u_1$ is the Cole-Hopf transformation of the (2+1)-dimensional variable coefficient Burgers equation.

4 The symmetry group and its Lie algebra

We consider the oneparameter Lie group of infinitesimal transformations (Olver [3], Blumen and Kumei [18]) in x, y, t and u given by $x_i^* = x_i + \epsilon\xi_i(x, y, t; u) + 0(\epsilon^2), i = 1, 2, 3$ where $x_1 = x, x_2 = y, x_3 = t, x_4 = u, \xi_1 = X(x, y, t; u), \xi_2 = Y(x, y, t; u), \xi_3 = T(x, y, t; u), \xi_4 = U(x, y, t; u)$, and the corresponding vector field $V = X(x, y, t; u)\partial_x + Y(x, y, t; u)\partial_y + T(x, y, t; u)\partial_t + U(x, y, t; u)\partial_u$ then the third prolongation $pr^{(3)}V$ of the corresponding above vector field
$pr^{(3)}V = V + U^x \frac{\partial}{\partial u_x} + U^y \frac{\partial}{\partial u_y} + U^t \frac{\partial}{\partial u_t} + U^{xx} \frac{\partial}{\partial u_{xx}} + U^{xy} \frac{\partial}{\partial u_{xy}} + U^{xt} \frac{\partial}{\partial u_{xt}} + U^{yy} \frac{\partial}{\partial u_{yy}} + U^{yt} \frac{\partial}{\partial u_{yt}} + U^{tt} \frac{\partial}{\partial u_{tt}} + U^{xxx} \frac{\partial}{\partial u_{xxx}} + U^{xxy} \frac{\partial}{\partial u_{xxy}} + U^{xxt} \frac{\partial}{\partial u_{xxt}} + U^{xyy} \frac{\partial}{\partial u_{xyy}} + U^{xyt} \frac{\partial}{\partial u_{xyt}} + U^{yyt} \frac{\partial}{\partial u_{yyt}} + U^{yyy} \frac{\partial}{\partial u_{yyy}} + U^{ttx} \frac{\partial}{\partial u_{ttx}} + U^{tty} \frac{\partial}{\partial u_{tty}} + U^{ttt} \frac{\partial}{\partial u_{ttt}}$ Equation (1.1) can be written as

$$\Omega \equiv (u_t + uu_x - p(t)u_{xx})_x + r(t)u_{yy} = 0$$

the vector field satisfies $pr^{(3)}V\Omega(x, y, t; u)|_{\Omega(x,y,t;u)=0} = 0$ using the above equation, we get the infinitesimals X, Y, T and U are

$$X = f_2(t) - \frac{yf_1'(t)}{2r(t)}, Y = f_1(t), T = 0, U = \frac{yr'f_1'(t)}{2r^2(t)} + f_2'(t) - \frac{yf_1''(t)}{2r(t)}$$

where $f_1(t)$ and $f_2(t)$ are arbitrary smooth functions and primes denote time derivatives. For any function $r(t)$ the symmetry algebra of (1.1) is an infinite-dimensional Lie algebra which we denote by L_p. A general element of L_p for an arbitrary $r(t) \neq$ constant is represented by

$$V = X(f_1) + Y(f_2) \tag{4.1}$$

$$X(f_1) = -\frac{yf_1'(t)}{2r(t)}\partial_x + f_1(t)\partial_y + (\frac{yr'f_1'(t)}{2r^2(t)} - \frac{yf_1''(t)}{2r(t)})\partial_u = -\frac{yf_1'(t)}{2r(t)}\partial_x + f_1(t)\partial_y - (-\frac{yf_1'(t)}{2r(t)})'\partial_u$$

$$Y(f_2) = f_2(t)\partial_x + f_2'(t)\partial_u$$

The commutation relations are

$$[X(f_1), X(g_1)] = Y(\frac{1}{2r}(f_1'g_1 - f_1g_1')) \quad [[X(f_1), Y(f_2)] = 0] \quad [[Y(f_2), Y(g_2)] = 0] \tag{4.2}$$

Where [,] stands for the Lie bracket. It is readily seen that the coefficients of the vector fields $X(f_1)$ and $Y(f_2)$ multiplying ∂_t are necessarily zero. This implies that the symmetry algebra does not have the structure of a Virasoro algebra. This stems from the fact that the equation under study is non-integrable. All known integrable equations in 2+1 dimensions have symmetry algebras of Virasoro type.

The vector fields $X(f_1)$ and $Y(f_2)$ can be integrated to obtain the Lie group of transformations. Thus if $u(x, y, t)$ is any solution to equation (1.1) then so are
$\tilde{u} = u(x - \alpha f_1(t), y, t) + \alpha f_1'(t) \quad \alpha \in \mathbb{R}$ and $\tilde{u} = u(x + \frac{\alpha f_2'(t)}{2r(t)}[y - \frac{\alpha f_2(t)}{2}], y - \alpha f_2(t), t) - \frac{\alpha}{2}(\frac{f_2'(t)}{r(t)})'[y - \frac{\alpha f_2(t)}{2}]$ respectively. If $r(t) = \sigma$ =constant and restricting $f_1(t)$ and $f_2(t)$ to be linear polynomials we obtain obvious physical symmetries spanned by

$$X \equiv X(1) = \partial_y \qquad Y \equiv Y(1) = \partial_x$$

$$B \equiv X(t) = -\frac{y}{2\sigma}\partial_x + t\partial_y \qquad R \equiv Y(t) = t\partial_x + \partial_u$$

Commutator Table

[,]	X	Y	B	R
X	0	0	$-\frac{1}{2\sigma}Y$	0
Y	0	0	0	0
B	$-\frac{1}{2\sigma}Y$	0	0	0
R	0	0	0	0

4.1 Low-dimensonal subalgebras of the symmetry algebra

In order to be able to perform symmetry reductions in a systematic way, we need to classify subalgebras of the infinite-dimensional algebras. We use the approach followed in [10] as an adaptation of the methods developed for the classification of subalgebras of the finite-dimensional algebras to infinite-dimensional ones. The difference is that we obtain differential conditions on the arbitrary functions labelling the group elements, rather than algebraic conditions on the parameters labelling the group elements of the finite-dimensional group. We present a classification of the one-dimensional subalgebras of the symmetry algebra into conjugacy classes under the adjoint action of the symmetry group.
If $r(t)$ = arbitrary$\neq$ constant,
Conjugating the general element $V = X(f_1) + Y(f_2)$ by $X(G)$ and using the commutation relations (4.2) we obtain $\text{Ad}\{exp(\lambda X(G))\}V = Y(f_2 - \frac{\lambda}{2r}(G' f_1 - G f_1')) + X(f_1)$.
If we choose a function G(t) to be defined by

$$G(t) = 2af_1 \int_0^1 (rf_2 f_1^{-2})(u)du + cf_1$$

where a and c are arbitrary constants, as the function labelling the element $X(G)$ of the symmetry algebra and $\lambda = a^{-1}$ as the value of the parameter λ of the one-parameter subgroup associated with $X(G)$, we see that if $f_1 \neq 0$, V is conjugate to $X(f_1)$, otherwise to $Y(f_2)$.

5 Symmetry reductions of (1.1) by one-dimensional subalgebras

As there are two generators, we consider the reductions of (1.1) under each generator separately. **Case 1:** Subalgebra $L_{s,1} = \{X(f_1)\}$ The characteristic equation associated the generator $X(f_1)$ is

$$\frac{dx}{-\frac{yf_1'(t)}{2r(t)}} = \frac{dy}{f_1(t)} = \frac{dt}{0} = \frac{du}{-\frac{y}{2}(\frac{f_1'(t)}{r(t)})'} \tag{5.1}$$

Integration of (5.1) yields the similarity transformation

$$u = w(\xi, \eta) - \frac{y^2}{4f_1(t)}(-\frac{f_1'(t)}{r(t)})'; \ \ \xi = x + \frac{y^2 f_1'(t)}{4f_1(t)r(t)}; \eta = t \tag{5.2}$$

Using (5.2) in (1.1), obtain the reduced PDE

$$w_{\xi\eta} + ww_{\xi\xi} + w_\xi^2 - p(t)w_{\xi\xi\xi} + \frac{f_1'(t)}{2f_1(t)}w_\xi - \frac{yr(t)}{2f_1(t)}(\frac{f_1'(t)}{r(t)})' = 0$$

Integrate once w.r.t. ξ, we get

$$w_\xi + ww_\xi - p(t)w_{\xi\xi} + \frac{f_1'(t)}{2f_1(t)}w - \frac{yr(t)}{2f_1(t)}(\frac{f_1'(t)}{r(t)})'\xi = f(\eta)$$

where $f(\eta)$ is an arbitrary function of integration. For $f(\eta) = 0, p(t) = 1$ and $f_1(t) =$ constant this equation is the one-dimensional Burgers equation

$$w_\xi + ww_\xi - w_{\xi\xi} = 0$$

Case 2: Subalgebra $L_{s,2} = \{Y(f_2)\}$

The characteristic equation associated the generator $Y(f_2)$ is

$$\frac{dx}{f_2(t)} = \frac{dy}{0} = \frac{dt}{0} = \frac{du}{f_2'(t)} \tag{5.3}$$

Integration of (5.3) yields the similarity transformation

$$u = w(\xi, \eta) + x\frac{f_2'(t)}{f_2(t)}; \xi = y; \eta = t \tag{5.4}$$

Using (5.4) in (1.1), obtain the reduced PDE

$$\frac{f_2''(t)}{f_2(t)} + r(t)w_{\xi\xi} = 0 \tag{5.5}$$

Integrate twice w.r.t. ξ, we get

$$w = -\frac{f_2''(t)}{r(t)f_2(t)}\frac{\xi^2}{2}H(\eta)\xi + L(\eta) \tag{5.6}$$

where $H(\eta)$ and $L(\eta)$ are arbitrary functions of integration. Using (5.4) n (5.6), we get

$$u = -\frac{f_2''(t)}{r(t)f_2(t)}\frac{y^2}{2} + H(t)y + L(t) + x\frac{f_2'(t)}{f_2(t)}$$

6 Conclusion

The results can be summarized as follows:

- We summarize the generalzation of painleve property .
- We analyze the painleve property of generalized (2+1)-dimensional variable coefficient Burger equation.
- We determined the symmetry generators of the variable coefficient Burger equation using Lie classical method.
- We found a classification of the one-dimensional subalgebras of the symmetry algebra under the adjoint (conjugate) action of the symmetry group.

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