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Optimal Expected Value of Assets Under Black-Scholes Equation with Transaction Costs

Joy Ijeoma Adindu-Dick

Department of Mathematics, Imo State University,

Owerri, Nigeria.

Email: ji16adindudick@yahoo.com

ABSTRACT. This paper deals with optimal expected value of assets under Black-Scholes equation with transaction costs. The partial differential equation for option pricing with transaction costs on the domain $(P, T) \in (0, \infty) \times (0, T)$ with terminal condition $C(P, T) = max(P - E, 0), P \in (0, \infty)$ for European call options with strike price E, and a suitable terminal condition for European puts was obtained and then solved using Eulers substitution method. Keywords: Black-Scholes equation with transaction costs, Optimal value, Eulers substitution method, Option pricing.

1 Introduction

In this work, we consider a market model in which trading the asset requires paying transaction fees which are proportional to the quantity and the value of the asset traded. This is a problem with a long history in mathematical finance. In a complete frictionless (i.e. without transaction costs) financial market, the Black-Scholes model (1973) [1] provides a hedging strategy for any European type contingent claim. However, the Black-Scholes hedging portfolio requires trading at all time instants, and the total turnover of stock in the time interval [0,T] is infinite. Accordingly, when transactions cost that is directly proportional to trading is incorporated in the Black-Scholes model the resulting hedging portfolio is prohibitively expensive. It is therefore acceptable that in

* Corresponding Author.

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a continuous time model with transaction costs, there is no portfolio that can replicate the European call option with finite transaction costs. To proceed, the condition under which hedging can take place has to be relaxed such that the portfolio only dominates rather than replicates the value of the European call option at maturity. With this relaxation, there is always the trivial dominating hedging strategy of buying and holding one share of the stock on which the call is written. From arbitrage pricing theory, the price of an option should not be greater than the smallest initial capital that can support a dominating portfolio. Interesting results have evolved from this line of approach to pricing option without transaction costs, however, in the presence of constraints (see Cvitanic and Karatzas [2]), and Broadie et al [3]), in the presence of transaction costs, Soner et al [4], proved that the minimal hedging portfolio that dominates a European call option is the trivial one. Leland [5] uses a relaxation with the effect that his model allows transactions only at discrete times. Lelands derivation makes an assumption of the convexity of the resulting option price. Avellanda and Paras [6] have made an extension to this approach to general prices. In another approach by Boyle and Vorst [7], pricing option is looked at in discrete time with a binomial tree model for the stock value. By applying the theorem of central limit, they showed that as the time step and the transaction cost tend to zero, the price of the discrete option converges to a Black-Scholes price with adjusted volatility. In a related paper [8], Bensaid et al investigated the discrete time, dominating policies. In [9], Hodges and Neuberger maximized the utility of the difference between final wealth when there is no option liability and when there is such a liability. They then postulate that the price of the option should be equal to the unique cash increment which offsets this differences. It is therefore surprising that in the absence of market frictions, the price obtained from maximizing utility is the same as the Black-Scholes option price. Thus, the Black-Scholes option pricing theory can be extended by use of utility maximization approach. Davis [10] has further work on this in the presence of transaction costs. This is to say that the price depends on the initial wealth of the investor, the chosen portfolio, the type of utility function, and the mean return rate of the stock.

2 Basic Tools and Preliminaries

The random work is governed in discrete time by

$$\partial P = \sigma P \varphi \sqrt{\partial t} + \mu P \delta t \tag{1}$$

where φ is drawn from a standardized normal distribution. The expected return on the hedged portfolio is equivalent to the return on a bank deposit. The portfolio is redefined or revised after a finite fixed time-step δt. Rehedging the portfolio could be done weekly. Transaction costs in buying and selling the asset are proportional to the monetary value of the transaction. At a price of P and a constant k depending on an individuals aversion to risk, the transaction costs are $P|N|k$, where N is the number of shares bought $(N > 0)$ or sold $(N < 0)$. On an infinitesimal level, rebalancing the portfolio between stocks and bonds does not change the value of the hedging portfolio.

3 Euler's Method

The second order equation $x^2\frac{d^2y}{dx^2} + a_1x\frac{dy}{dx} + a_2y = \varphi(x)$ where $x^2 \neq 0$ and a_1 and a_2 are constants is called the Euler-Cauchy type differential equation of second order. To solve the equation, we first reduce it to an equation with constant coefficients using the substitution $x = e^t$, or=ln $x; x > 0$. When we use $x = e^t$ or $t = \ln(-x)$. Then we have the following $\frac{dt}{dx} = \frac{1}{x}$ and $\frac{dy}{dx} = \frac{dy}{dt}.\frac{dt}{dx} = \frac{1}{x}\frac{dy}{dt}$.
By product rule we have $\frac{d^2y}{dx^2} = \frac{d}{dx}(\frac{1}{x}\frac{dy}{dt}) = \frac{1}{x}\frac{d}{dx}(\frac{dy}{dt}) + \frac{dy}{dt}.\frac{d}{dx}(\frac{1}{x}) = \frac{1}{x^2}(\frac{d^2y}{dx^2} - \frac{dy}{dt})$. Substituting these values into the given equation, we have $x^2[\frac{1}{x^2}(\frac{d^2y}{dx^2} - \frac{dy}{dt})] + a_1x.\frac{1}{x}\frac{dy}{dx} + a_2y(t) = \varphi(e^t)$ This implies that $\frac{d^2y}{dx^2} + (a_1 - 1)\frac{dy}{dy} + a_2y = \varphi(e^t)$. The resulting equation is a linear equation with constant coefficients and can be solved in the usual way.

4 The Model

We first create a hedging portfolio that has α stocks and β bonds in X. Hence, the initial value of the hedging portfolio that is at time t_0 is

$$\alpha_{t_0}P_{t_0} + \beta_{t_0}B_{t_0} = \alpha_{t_0}P_{t_0} + \beta_{t_0}B_{t_0} \tag{2}$$

At the final time T, we intend to find

$$\alpha_(T)P(T) + \beta(T)B(T) = \rho(P(T)), \tag{3}$$

where ρ is called the contract function. Equation (3) is the value of the portfolio. If we denote the value of the option at time t by $F(t, P(t))$, we have

$$F(t, P(t)) = \alpha(t)P(t) + \beta(t)B(t) \tag{4}$$

Since both sides should have the same dynamics, we have

$$d(F(t, P(t))) = d(\alpha(t)P(t) + \beta(t)B(t)), \tag{5}$$

where d is an infinitesimal change. If the portfolio is self-financing, we have

$$d(\alpha(t)P(t) + \beta(t)B(t)) = \alpha(t)dP(t) + \beta(t)dB(t). \tag{6}$$

But

$$dB = rBdt \tag{6i}$$

Equation (6) becomes

$$\alpha(\mu P\delta t + \sigma\varphi P\sqrt{\delta t}) + \beta rB\delta t = (\alpha\mu P + \beta rB)\delta t + \alpha\sigma\varphi P\sqrt{\delta t} \tag{7}$$

Since $\delta P = \mu P\delta t + \sigma\varphi P\sqrt{\delta t}$. Thus

$$\delta(F(t, Pt)) = (\alpha\mu P + \beta rB)\delta t + \alpha\sigma\varphi P\sqrt{\delta t} \tag{8}$$

By Itos lemma and replacing $F(t, P(t))$ by F, we have

$$\delta F = \frac{\partial F}{\partial t}\delta t + \frac{\partial F}{\partial P}\delta P + \frac{1}{2}\frac{\partial^2 F}{\partial P^2}(\partial P)^2, \tag{9}$$

where $\partial P = \sigma P\varphi\sqrt{\delta t} + \mu P\delta t$. Hence $(\delta P)^2 = (\sigma P\varphi\sqrt{\delta t} + \mu P\delta t)^2 = \sigma^2\varphi^2P^2\delta t$, since $(\delta t)^2 \approx 0$ and $\delta t\sqrt{\delta t} = 0$. Equation (9) becomes

$$\delta F = \frac{\partial F}{\partial t}\delta t + \frac{\partial F}{\partial P}(\mu P\delta t + \sigma P\varphi\sqrt{\delta t}) + \frac{1}{2}\frac{\partial^2 F}{\partial P^2}\sigma^2\varphi^2P^2\delta t \tag{10}$$

Rearranging, we have

$$\delta F = (\frac{\partial F}{\partial t} + \frac{\partial F}{\partial P}\mu P + \frac{1}{2}\frac{\partial^2 F}{\partial P^2}\sigma^2\varphi^2P^2)\delta t + \sigma P\varphi\sqrt{\delta t}\frac{\partial F}{\partial P} \tag{11}$$

By the assumption about the existence of transaction costs, we have

$$\delta F = (\frac{\partial F}{\partial t} + \frac{\partial F}{\partial P}\mu P + \frac{1}{2}\frac{\partial^2 F}{\partial P^2}\sigma^2\varphi^2P^2)\delta t + \sigma P\varphi\sqrt{\delta t}\frac{\partial F}{\partial P} - k|N|P. \tag{12}$$

From equations (8) and (12), we have

$$\begin{aligned}\delta F &= (\frac{\partial F}{\partial t} + \frac{\partial F}{\partial P}\mu P + \frac{1}{2}\frac{\partial^2 F}{\partial P^2}\sigma^2\varphi^2P^2)\delta t + \sigma P\varphi\sqrt{\delta t}\frac{\partial F}{\partial P} - k|N|P \\ &= (\alpha\mu P + \beta r B)\delta t + \alpha\varphi\sigma P\sqrt{\delta t}\end{aligned} \tag{13}$$

Let $\alpha = \frac{\partial F}{\partial P}$ where α is the number of assets. Given that α is evaluated at the asset value P and time t, we have $\alpha = \frac{\partial F}{\partial P}(P,t)$. Re-hedging after a finite time δt gives $\frac{\partial F}{\partial P}(P + \delta P, t + \delta t)$. To find the number N of assets bought or sold at the new time and asset price, we have

$$N = \frac{\partial F}{\partial P}(P + \delta P, t, \delta t) - \frac{\partial F}{\partial P}(P,t) \tag{14}$$

For small δP and δt and by Taylors series expansion of the first term, we have $\delta P = \sigma P\varphi\sqrt{\delta t} + \mu P\delta t = \sigma P\varphi\sqrt{\delta t} + O(\delta t)$. Therefore, the first term expands as $\frac{\partial F}{\partial P}(P + \delta P, t + \delta t) = \frac{\partial F}{\partial P}(P,t) + \frac{\partial^2 F}{\partial P^2}(\delta P) + \frac{\partial^3 F}{\partial P^3}(\delta P)^2 + ...$ The third P partial derivative of the option price is removed since it does not play any role. Hence,

$$\frac{\partial F}{\partial P}(P + \delta P, t + \delta t) = \frac{\partial F}{\partial P}(P,t) + \frac{\partial^2 F}{\partial P^2}(\delta P) \tag{15}$$

Substituting for δP gives

$$N = \frac{\partial^2 F}{\partial P^2}(\delta P) \approx \frac{\partial^2 F}{\partial P^2}\sigma\varphi P\sqrt{\delta t} \tag{16}$$

Hence, the expected transaction cost in a finite time-step, $E[k|N|P]$ is given by

$$E[k|N|P] = kpE[|N|] = kPE[|\frac{\partial^2 F}{\partial P^2}\sigma\varphi P\sqrt{\delta t}|] = k\sigma P^2|\frac{\partial^2 F}{\partial P^2}|E[|\varphi|]\sqrt{\delta t}. \tag{17}$$

Now,

$$E[|\varphi|] = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\varphi e^{-0.5\varphi^2}d\varphi = 2\sqrt{\frac{1}{2\pi}}\int_{-\infty}^{\infty}\varphi e^{-0.5\varphi^2}d\varphi = \sqrt{\frac{2}{\pi}}\int_{-\infty}^{\infty}\varphi e^{-0.5\varphi^2}d\varphi \tag{18}$$

Solving equation (17) by substitution: let $\varphi^2 = V$, then $\varphi d\varphi = \frac{dV}{2}$. Equation (17) becomes $\frac{1}{2}\int_0^{\infty} e^{\frac{V}{2}}dV = -[e^{\infty} - e^0] = 1$. Hence

$$E|\varphi| = \sqrt{\frac{2}{\pi}} \tag{19}$$

From equations (13) and (16) we have $(\frac{\partial F}{\partial t}) + \mu P\frac{\partial F}{\partial P} + \frac{1}{2}\sigma^2\varphi^2P^2\frac{\partial^2 F}{\partial p^2} - \sqrt{\frac{2}{\pi}}kP^2\sigma\sqrt{\delta t}|\frac{\partial^2 F}{\partial P^2}|)\delta t + \sigma\varphi P\frac{\partial F}{\partial P}\sqrt{\delta t}$. Substituting into equation (12) gives

$$\begin{aligned}\delta F &= (\frac{\partial F}{\partial t} + \mu P\frac{\partial F}{\partial P} + \frac{1}{2}\sigma^2\varphi^2P^2\frac{\partial^2 F}{\partial p^2})\delta t + \sigma\varphi P\frac{\partial F}{\partial P}\sqrt{\delta t} - \sqrt{\frac{2}{\pi}}kP^2\sigma\sqrt{\delta t}|\frac{\partial^2 F}{\partial P^2}|\delta t \\ &= (\frac{\partial F}{\partial t} + \mu P\frac{\partial F}{\partial P} + \frac{1}{2}\sigma^2\varphi^2P^2\frac{\partial^2 F}{\partial p^2} - \sqrt{\frac{2}{\pi}}kP^2\sigma\sqrt{\delta t}|\frac{\partial^2 F}{\partial P^2}|)\delta t + \sigma\varphi P\frac{\partial F}{\partial P}\sqrt{\delta t}\end{aligned}$$

It follows that

$$E|\delta F| = \frac{\partial F}{\partial t} + \mu P\frac{\partial F}{\partial P} + \frac{1}{2}\sigma^2 P^2\frac{\partial^2 F}{\partial p^2} - \sqrt{\frac{2}{\pi}}kP^2\sigma\sqrt{\delta t}|\frac{\partial^2 F}{\partial P^2}| \tag{20}$$

since $E[\emptyset^2] = 1$. From equation(8), we have $\alpha = \frac{\partial F}{\partial P}$ and

$$\alpha\mu P + \beta rB = \frac{\partial F}{\partial t} + \mu P\frac{\partial F}{\partial P} + \frac{1}{2}\sigma^2 P^2\frac{\partial^2 F}{\partial p^2} - \sqrt{\frac{2}{\pi}}kP^2\sigma\sqrt{\delta t}|\frac{\partial^2 F}{\partial P^2}| \tag{21}$$

From equation (4), we have $\beta B = F - \alpha P = F - P\frac{\partial F}{\partial P}$. Since $\alpha = \frac{\partial F}{\partial P}$, equation (20) becomes

$$\beta rB = (\frac{\partial F}{\partial t} + \frac{1}{2}\sigma^2 P^2\frac{\partial^2 F}{\partial p^2} - \sqrt{\frac{2}{\pi}}kP^2\sigma\sqrt{\delta t}|\frac{\partial^2 F}{\partial P^2}|)(\frac{1}{r})$$

$$F - P\frac{\partial F}{\partial P} = (\frac{\partial F}{\partial t} + \frac{1}{2}\sigma^2 P^2\frac{\partial^2 F}{\partial p^2} - \sqrt{\frac{2}{\pi}}kP^2\sigma\sqrt{\delta t}|\frac{\partial^2 F}{\partial P^2}|)(\frac{1}{r}) \tag{22}$$

Equation (21) becomes

$$rF - rP\frac{\partial F}{\partial t} = (\frac{\partial F}{\partial t} + \frac{1}{2}\sigma^2 P^2\frac{\partial^2 F}{\partial p^2} - \sqrt{\frac{2}{\pi}}kP^2\sigma\sqrt{\delta t}|\frac{\partial^2 F}{\partial P^2}|) \tag{23}$$

Hence

$$(\frac{\partial F}{\partial t} + rP\frac{\partial F}{\partial t} + \frac{1}{2}\sigma^2 P^2\frac{\partial^2 F}{\partial p^2} - \sqrt{\frac{2}{\pi}}kP^2\sigma\sqrt{\delta t}|\frac{\partial^2 F}{\partial P^2}|) = rF \tag{24}$$

Equation (22) is one form of Black-Scholes equation that incorporates transaction costs. Take

$$\sqrt{\frac{2}{\pi}}k\sigma\sqrt{\delta t} = \varphi; z = \frac{\alpha}{P}; F(P) = z^\beta W(z) \tag{25}$$

Thus

$$\begin{aligned}\frac{dz}{dp} &= -\frac{\alpha}{P^2} = -\frac{1}{\alpha}z^2\\ \frac{dF}{dP} &= \frac{dF}{dz}.\frac{dz}{dP}\\ &= -\frac{1}{\alpha}z^2(\beta z^{\beta-1}W + z^\beta\frac{dW}{dz})\\ &= -\frac{1}{\alpha}(\beta z^{\beta+1}W + z^{\beta+2}\frac{dW}{dz})\end{aligned}$$

Hence

$$\begin{aligned}\frac{d^2F}{dP^2} &= \frac{d}{dP}(\frac{dF}{dz}).\frac{dz}{dP}\\ &= -\frac{1}{\alpha}z^2(\beta(\beta+1)z^\beta W + \beta z^{\beta+1}\frac{dw}{dz} + (\beta+2)z^{\beta+1}\frac{dW}{dz} + z^{\beta+2}\frac{d^2W}{dz^2})\end{aligned}$$

In this case F is not dependent on r. Substituting into the given differential equation we have

$$\begin{aligned}rz^\beta W = &-\frac{\sigma^2}{2}\alpha(\beta(\beta+1)z^\beta W + \beta z^{\beta+1}\frac{dw}{dz} + (\beta+2)z^{\beta+1}\frac{dW}{dz} + z^{\beta+2}\frac{d^2W}{dz^2})\\ &+ (\frac{r\alpha}{2})(\frac{-1}{\alpha})(\beta z^{\beta+1}W + z^{\beta+2}\frac{dW}{dz})\\ &- \varphi(\frac{\alpha}{2})^2(-\frac{z^2}{\alpha}(\beta(\beta+1)z^\beta W + \beta z^{\beta+1}\frac{dw}{dz} + (\beta+2)z^{\beta+1}\frac{dW}{dz} + z^{\beta+2}\frac{d^2W}{dz^2}))\end{aligned} \tag{26}$$

Cancelling by z^β and collecting like terms we have

$$\begin{aligned}rW = &-\frac{\sigma^2}{2}\alpha(\beta(\beta+1)W + \beta z\frac{dw}{dz} + (\beta+2)z^{\beta+1}\frac{dW}{dz} + z^2\frac{d^2W}{dz^2})\\ &- (\frac{r}{z})(\beta zW + z^2\frac{dW}{dz})\\ &+ \varphi\alpha(\beta(\beta+1)W + \beta z\frac{dw}{dz} + (\beta+2)z\frac{dW}{dz} + z^2\frac{d^2W}{dz^2}))\end{aligned}$$

or

$$rW = -\frac{\sigma^2}{2}\alpha z^2\frac{d^2W}{dz^2} + (\frac{dW}{dz}(-\frac{\sigma^2}{2}\alpha(\beta+1)2z - rz) + w(-\frac{\sigma^2}{2}\alpha(\beta+1) - r\beta) + \varphi\alpha z^2\frac{d^2W}{dz^2} + \frac{dW}{dz}(\varphi\alpha(\beta+1)2z) + w(\beta+1)\beta)$$

Hence

$$\begin{aligned}0 = (\varphi\alpha - \frac{\sigma^2}{2}\alpha)z^2\frac{d^2W}{dz^2} - \frac{dW}{dz}z(\sigma^2\alpha(\beta+1) - r + 2\varphi\alpha(\beta+1))\\ + w(-\frac{\sigma^2}{2}\alpha(\beta+1)\beta - r\beta + (\beta+1)\beta - r)\end{aligned} \tag{27}$$

Let $\beta = 0$, equation (25) becomes

$$0 = (\varphi\alpha - \frac{\sigma^2}{2})z^2\frac{d^2W}{dz^2} - \frac{dW}{ds}z(\sigma^2\alpha - r + 2\varphi\alpha) - wr \tag{28}$$

Divide both sides by α to have

$$0 = (\varphi - \frac{\sigma^2}{2})z^2\frac{d^2W}{dz^2} - \frac{dW}{dz}z(\sigma^2 - r\alpha^{-1} + 2\varphi) - wr\alpha^{-1} \tag{29}$$

Let $\varphi = \sigma^2$, equation (27) becomes $0 = \frac{\sigma^2}{2}z^2\frac{d^2W}{dz^2} - \frac{dW}{dz}z(3\sigma^2 - r\alpha^{-1}) - wr\alpha^{-1}$. Multiply by $\frac{2}{\sigma^2}$ to have

$$0 = z^2\frac{d^2W}{dz^2} - \frac{dW}{dz}z(6 - \frac{2r}{\sigma^2\alpha}) - \frac{2rw}{\sigma^2\alpha} \tag{30}$$

We then solve equation (28) by change of variable using Eulers substitution method. Let $z = e^t$, then $\ln z = t$, and $\frac{dt}{dz} = \frac{1}{z}$. Hence, $\frac{dW}{dz} = \frac{dW}{dt}\frac{dt}{dz} = \frac{1}{z}\frac{dW}{dt}$.
$\frac{d^2W}{dz^2} = \frac{d}{dz}(\frac{1}{z}\frac{dW}{dt}) = \frac{1}{z}\frac{d}{dz}(\frac{dW}{dt}) + \frac{dW}{dt}\frac{1}{z}(\frac{d}{dz})$, and $\frac{d^2W}{dz^2} = \frac{1}{z^2}\frac{d^2W}{dt^2} - \frac{dW}{dt})$.
Substituting the above equations in equation (28) gives
$0 = \frac{d^2W}{dt^2} - \frac{dW}{dt} - \frac{dW}{dt}(6 - \frac{2r}{\alpha\sigma^2}) - \frac{2rw}{\alpha\sigma^2}$
That is

$$0 = \frac{d^2W}{dt^2} + \frac{dW}{dt}(\frac{2r}{\alpha\sigma^2} - 7) - \frac{2r}{\alpha\sigma^2}w. \tag{31}$$

Equation (29) has constant coefficients, solving gives the auxiliary equation as

$$m^2 + (\frac{2r}{\alpha\sigma^2} - 7)m - \frac{2r}{\alpha\sigma^2} = 0$$

Solving equation (30): $a = 1; b = (\frac{2r}{\alpha\sigma^2} - 7)$; and $c = -\frac{2r}{\alpha\sigma^2}$.
Hence,

$$\begin{aligned}m &= \frac{(7 - \frac{2r}{\alpha\sigma^2}) \pm \sqrt{(\frac{2r-7\alpha\sigma^2}{\alpha\sigma^2})^2 + 4(1)\frac{2r}{\alpha\sigma^2}}}{2}\\ &= \frac{(7 - \frac{2r}{\alpha\sigma^2}) \pm \frac{1}{\alpha\sigma}\sqrt{(\frac{2r-7\alpha\sigma^2}{\sigma})^2 + 8r\alpha}}{2}\end{aligned}$$

Let the roots be λ_1 and λ_2, hence $m_1 = \lambda_1$ and $m_2 = \lambda_2$. That is

$$\lambda_1 = \frac{(7 - \frac{2r}{\alpha\sigma^2}) + \frac{1}{\alpha\sigma}\sqrt{(\frac{2r-7\alpha\sigma^2}{\sigma})^2 + 8r\alpha}}{2} \tag{31a}$$

and

$$\lambda_2 = \frac{(7 - \frac{2r}{\alpha\sigma^2}) - \frac{1}{\alpha\sigma}\sqrt{(\frac{2r-7\alpha\sigma^2}{\sigma})^2 + 8r\alpha}}{2} \tag{31b}$$

Therefore, from equation (29),
$w = Ae^{\lambda_1 t} + Be^{\lambda_2 t}$ where A and B are arbitrary constants. But $t = \ln z$, this implies that $w = Ae^{\lambda_1 \ln z} + Be^{\lambda_2 \ln z} = Ae^{\ln z^{\lambda_1}} + Be^{\ln z^{\lambda_2}}$. Therefore,

$$w = Az^{\lambda_1} + Bz^{\lambda_2} \tag{32}$$

But $z = \frac{\alpha}{p}$, equation (32) becomes

$$w = A(\frac{\alpha}{p})^{\lambda_1} + B(\frac{\alpha}{p})^{\lambda_2} \tag{33}$$

Also,

$$F(p) = z^{\beta} w(z). \tag{34}$$

Hence, substituting equation (33) into equation (34) gives

$$F(p) = (\frac{\alpha}{p})^{\beta}[A(\frac{\alpha}{p})^{\lambda_1} + B(\frac{\alpha}{p})^{\lambda_2}]. \tag{35}$$

where $\lambda_1 = \frac{(7 - \frac{2r}{\alpha\sigma^2}) + \frac{1}{\alpha\sigma}\sqrt{(\frac{2r - 7\alpha\sigma^2}{\sigma})^2 + 8r\alpha}}{2}$ $\lambda_2 = \frac{(7 - \frac{2r}{\alpha\sigma^2}) - \frac{1}{\alpha\sigma}\sqrt{(\frac{2r - 7\alpha\sigma^2}{\sigma})^2 + 8r\alpha}}{2}$

5 Conclusion

The random component does not feature in the Black-Scholes equation and this is precisely because of the manner in which we chose the number of stocks α. The second derivative of the option price in the transaction cost term apparently suggests that it is closely linked to the extent of re-hedging in the following time interval. The equation is nonlinear, hence, the value of a portfolio of options is different from the sum of the individual components. If we have a position in two call options which have the same exercise price and the same expiry date and on the same underlying asset. If one is held short and the other long, our net position is zero. There is a cancellation effect of the two opposite positions that goes unnoticed if our basket of options is large. Due to this effect we decide to hedge each of them separately. Re-hedging at each time step on both options implies that we drain the option price. At maturity the costs are still present and we have a negative net balance, since the two payoffs offset each other.

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