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Fixed Point Theorems for Generalized Rational Contractions in Complex Valued *b*-Metric Spaces

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ABSTRACT. In this paper, some fixed point theorems for generalized rational contractions in complex valued b-metric space are proved. These results extend and improve several well known results obtained previously.

1 Introduction

In the last fifty years, Fixed point theory has become one of the most famous and interesting area of research. For example, this theory is being used in control theory, differential equation, game theory, mathematical economics etc. In 1922, Banach[8] established the first important and useful results in this area, generally known as the Banach contraction principle. This principle includes applications in different direction as well as different spaces adopted by mathematician: for example, D-metric space, partial metric space, G-metric space, cone metric spaces had already been obtained. The existing literature of fixed point theory contains a great number of generalizations of Banach contraction principle. Czerwik[9] introduced the concept of a b-metric space. Since then, several research paper have appeared in this direction(see e.g,[1,5,7, 11,12,16,18]).

Azam et al.[6] introduced the notion of complex valued metric spaces which is more general than well-known metric spaces and also proved common fixed point theorems for mapping satisfying rational expressions. Subsequently, many authors studied the existence and uniqueness of the fixed point and common fixed points of self

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mappings in different contractive conditions. Some of these results are referred in ([2,3,4,14,17,19]). Further, Rao et al. [15] introduced the concept of complex valued b-metric space. Very recently, A.A. Mukheimer[13] proved the existence and uniqueness of the common fixed point in complete complex valued b-metric spaces. In sequel, Dubey et al. [10] established some fixed point theorems in complex valued b-metric spaces.

The aim of this paper is to provide some fixed point theorems satisfying generalized rational contractive type condition in a complex valued b-metric space. The results presented in this paper are generalization of work done by Sarwar et al. [18] and Dubey et al. [10].

2 Preliminaries

We recall some definitions and properties for complex valued b-metric spaces given by Rao et al. [15].
Let $\mathbb{C}$ be the set of complex numbers and $z_1, z_2 \in \mathbb{C}$. Define a partial order $\precsim$ on $\mathbb{C}$ as follows:
$z_1 \precsim z_2$ if and only if $Re(z_1) \leq Re(z_2), lm(z_1) \leq lm(z_2)$. Thus $z_1 \precsim z_2$ if one of the following holds:

1. $Re(z_1) = Re(z_2) \;\; and \;\; lm(z_1) = lm(z_2),$
2. $Re(z_1) < Re(z_2) \;\; and \;\; lm(z_1) = lm(z_2),$
3. $Re(z_1) = Re(z_2) \;\; and \;\; lm(z_1) < lm(z_2),$
4. $Re(z_1) < Re(z_2) \;\; and \;\; lm(z_1) < lm(z_2).$

We will write $z_1 \precnsim z_2$ if $z_1 \neq z_2$ and one of (2), (3) and (4) is satisfied, also we will write $z_1 \prec z_2$ if only (4) is satisfied. It follows that

1. $0 \preceq z_1 \precnsim z_2$ implies $|z_1| < |z_2|$,
2. $z_1 \preceq z_2$ and $z_2 \prec z_3$ imply $z_1 \prec z_3$,
3. $0 \preceq z_1 \preceq z_2$ implies $|z_1| < |z_2|$,
4. $a, b \in \mathbb{R}$ and $a \leq b$ imply $az \preceq bz$ for all $z \in \mathbb{C}$

Definition 2.1.[15] Let X be a nonempty set and let $s \geq 1$ be a given real number. A function $d : X \times X \to \mathbb{C}$ is called a complex valued b-metric on X if for all $x, y, z \in X$ the following conditions are satisfied:

1. $0 \preceq d(x, y)$ and $d(x, y) = 0$ if and only if $x = y$;
2. $d(x, y) = d(y, x)$;
3. $d(x, y) \preceq s[d(x, z) + d(z, y)]$.

The pair (X, d) is called a complex valued b-metric space.
Example 2.1[15]: Let $X = [0, 1]$. Define the mapping $d : X \times X \to \mathbb{C}$ by $d(x, y) = |x-y|^2 + i|x-y|^2$ for all $x, y \in X$. Then (X, d) is a complex valued b-metric space with $s = 2$.
Definition 2.2[15]: Let (X, d) be a complex valued b-metric space.

1. A point $x \in X$ is called interior point of a set $A \subseteq X$ whenever there exists $0 \prec r \in \mathbb{C}$ such that $B(x, r) = \{y \in X : d(x, y) \prec r\} \subseteq A$.

2. A point $x \in X$ is called limit point of a set A whenever, for every $0 \prec r \in \mathbb{C}, B(x,r) \cap (A - \{x\}) \neq \emptyset$.

3. A subset $A \subseteq X$ is called open set whenever each element of A is an interior point of A.

4. A subset $A \subseteq X$ is called closed set whenever each limit point of A belongs to A.

5. A sub-basis for a Hausdorff topology τ on X is a family $F = \{B(x,r) : x \in X \ \ and \ \ 0 \prec r\}$.

Definition 2.3[15]: Let (X,d) be a complex valued b-metric space; let $\{x_n\}$ be a sequence in X and $x \in X$.

1. If for every $c \in \mathbb{C}$ with $0 \prec c$ there is $N \in \mathbb{N}$ such that for all $n > N, d(x_n, x) \prec c$, then $\{x_n\}$ is said to be convergent, $\{x_n\}$ converges to x and x is the limit point of $\{x_n\}$. One denotes this by $lim_{n\to\infty}x_n = x$ or $\{x_n\} \to x$ as $n \to \infty$.

2. If for every $c \in \mathbb{C}$ with $0 \prec c$ there is $N \in \mathbb{N}$ such that for all $N, d(x_n, x_{n+m}) \prec c$, where $m \in \mathbb{N}$ then $\{x_n\}$ is said to be Cauchy sequence.

3. If every Cauchy sequence in X is convergent, then (X,d) is said to be a complete complex valued b-metric space.

Lemma 2.1[15]: Let (X,d) be a complex valued b-metric space and let $\{x_n\}$ be a sequence in X. Then $\{x_n\}$ converges to x if and only if $|d(x_n,x)| \to 0$ as $n \to \infty$.

Lemma 2.2[15]: Let (X,d) be a complex valued b-metric space and let $\{x_n\}$ be a sequence in X Then $\{x_n\}$ is a Cauchy sequence if and only if $|d(x_n, x_{n+m})| \to 0$ as $n \to \infty$ where $m \in \mathbb{N}$.

3 Main Results

Theorem 3.1. Let (X,d) be a complete complex valued b-metric space with the coefficient $s \geq 1$ and let $T : X \to X$ be a mapping satisfying the condition

$$d(Tx,Ty) \preceq \lambda d(x,y) + \mu \frac{d(x,Tx)d(x,Ty) + d(y,Ty)d(y,Tx)}{d(x,Ty) + d(y,Tx)} \tag{3.1}$$

for all $x, y \in X$ such that $x \neq y, \ \ d(x,Ty) + d(y,Tx) \neq 0$, where λ, μ are non-negative reals with $s(\lambda + \mu) < 1$. Then T has a unique fixed point.

Proof. For any arbitrary point $x_0 \in X$, we define a sequence $\{x_n\}$ by the rule,

$x_0, x_1 = Tx_0, x_2 = Tx_1, ...x_{n+1} = Tx_n$, for all $n \in \mathbb{N}$.

Now, we show that the sequence $\{x_n\}$ is Cauchy, consider $d(x_n, x_{n+1}) = d(Tx_{n-1}, Tx_n)$.

From equation (3.1) we have

$$\begin{aligned} d(x_n, x_{n+1}) &\preceq \lambda d(x_{n-1}, x_n) + \mu \frac{d(x_{n-1}, Tx_{n-1})d(x_{n-1}, Tx_n) + d(x_n, Tx_n)d(x_n, Tx_{n-1})}{d(x_{n-1}, Tx_n) + d(x_n, Tx_{n-1})} \\ &\preceq \lambda d(x_{n-1}, x_n) + \mu \frac{d(x_{n-1}, x_n)d(x_{n-1}, x_{n+1}) + d(x_n, x_{n+1})d(x_n, x_n)}{d(x_{n-1}, x_{n+1}) + d(x_n, x_n)} \\ &\preceq \lambda d(x_{n-1}, x_n) + \mu d(x_{n-1}, x_n) \\ &= (\lambda + \mu) d(x_{n-1}, x_n). \end{aligned} \tag{3.2}$$

Since $s(\lambda+\mu)<1$ and $s\geq 1$, we get $\lambda+\mu<1$. Therefore, with $\delta=\lambda+\mu<1$ and for all $n\geq 0$ and consequently, we have

$$\begin{aligned}|d(x_n,x_{n+1})| &\leq \delta|d(x_{n-1},x_n)|\\ &\leq \delta^2|d(x_{n-2},x_{n-1})|\\ &\vdots\\ &\leq \delta^n|d(x_0,x_1)|.\end{aligned} \tag{3.3}$$

Thus, for any $m>n; m,n\in\mathbb{N}$, we have

$$\begin{aligned}|d(x_n,x_m)| &\leq s|d(x_n,x_{n+1})|+s|d(x_{n+1},x_m)|\\ &\leq s|d(x_n,x_{n+1})|+s^2|d(x_{n+1},x_{n+2})|+s^2|d(x_{n+2},x_m)|\\ &\vdots\\ &\leq s|d(x_n,x_{n+1})|+s^2|d(x_{n+1},x_{n+2})|+...+s^{m-n-1}|d(x_{m-2},x_{m-1})|+s^{m-n}|d(x_{m-1},x_m)|.\end{aligned}$$

By using (3.3), we get

$$\begin{aligned}|d(x_n,x_m)| &\leq s\delta^n|d(x_0,x_1)|+...+s^{m-n-1}\delta^{m-2}|d(x_0,x_1)|+s^{m-n}\delta^{m-1}|d(x_0,x_1)|\\ &=\sum_{i=1}^{m-n}s^i\delta^{i+n-1}|d(x_0,x_1)|.\end{aligned} \tag{3.4}$$

Therefore,

$$\begin{aligned}|d(x_n,x_m)| &\leq \sum_{i=1}^{m-n}s^{i+n-1}\delta^{i+n-1}|d(x_0,x_1)|\\ &=\sum_{t=n}^{m-1}s^t\delta^t|d(x_0,x_1)|\\ &\leq\sum_{t=n}^{m-1}(s\delta)^t|d(x_0,x_1)|\\ &=\frac{(s\delta)^t}{1-s\delta}|d(x_0,x_1)|\end{aligned}$$

and hence

$$|d(x_n,x_m)|\leq\frac{(s\delta)^t}{1-s\delta}|d(x_0,x_1)|\to 0 \;\; as \;\; m,n\to\infty. \tag{3.5}$$

Thus, $\{x_n\}$ is a Cauchy sequence in X. Since X is complete, there exists some $u\in X$ such that $x_n\to u$ as $n\to\infty$. Suppose this is not possible, then there exists $z\in X$ such that

$$|d(u,Tu)|=|z|>0. \tag{3.6}$$

Now,

$$\begin{aligned}z=d(u,Tu) &\preceq sd(u,x_{n+1})+sd(x_{n+1},Tu)\\ &=sd(u,x_{n+1})+sd(Tx_n,Tu)\\ &\preceq sd(u,x_{n+1})+s\lambda d(x_n,u)+s\mu\frac{d(x_n,Tx_n)d(x_n,Tu)+d(u,Tu)d(u,Tx_n)}{d(x_n,Tu)+d(u,Tx_n)}\\ &=sd(u,x_{n+1})+s\lambda d(x_n,u)+s\mu\frac{d(x_n,x_{n+1})d(x_n,Tu)+d(u,Tu)d(u,x_{n+1})}{d(x_n,Tu)+d(u,x_{n+1})}.\end{aligned}$$

which implies that

$$|z| = |d(u, Tu)| \leq s|d(u, x_{n+1})| + s\lambda|d(x_n, u)| + s\mu\frac{|d(x_n, x_{n+1})||d(x_n, Tu)| + |d(u, Tu)||d(u, x_{n+1})|}{|d(x_n, Tu)| + |d(u, x_{n+1})|} \tag{3.7}$$

Taking the limit of (3.7) as $n \to \infty$, we obtain that $|z| = |d(u, Tu)| \leq 0$, a contradiction with (3.6). So, $|z| = 0$. Hence $Tu = u$.

Now, we show that T has a unique fixed point in X. To show this, assume that u^* is another fixed point of T. Then,

$$\begin{aligned} d(u, u^*) &= d(Tu, Tu^*) \\ &\preceq \lambda d(u, u^*) + \mu\frac{d(u, Tu)d(u, Tu^*) + d(u^*, Tu^*)d(u^*, Tu)}{d(u, Tu^*)d(u^*, Tu)}. \end{aligned}$$

So that

$$|d(u, u^*)| \preceq \lambda|d(u, u^*)| + \mu\frac{|d(u, Tu)||d(u, Tu^*)| + |d(u^*, Tu^*)||d(u^*, Tu)|}{|d(u, Tu^*)||d(u^*, Tu)|}$$

$$< \lambda|d(u, u^*)|,$$

a contradiction. So $u = u^*$.

Therefore, the fixed point of T is unique. This completes the proof of the Theorem.

Corollary 3.1. Let (X, d) be a complete complex valued b-metric space with the coefficient $s \geq 1$ and let $T : X \to X$ be a mapping satisfying the condition (for some fixed n)

$$d(T^n x, T^n y) \preceq \lambda d(x, y) + \mu\frac{d(x, T^n x)d(x, T^n y) + d(y, T^n y)d(y, T^n x)}{d(x, T^n y) + d(y, T^n x)} \tag{3.8}$$

for all $x, y \in X$ such that $x \neq y$, $d(x, Ty) + d(y, Tx) \neq 0$, where λ, μ are non-negative reals with $s(\lambda + \mu) < 1$. Then T has a unique fixed point.

Proof: From Theorem 3.1, we obtain $u \in X$ such that

$$T^n u = u \tag{3.9}$$

The uniqueness follows from

$$\begin{aligned} d(Tu, u) &= d(TT^n u, T^n u) = d(T^n Tu, T^n u) \\ &\preceq \lambda d(Tu, u) + \mu\frac{d(Tu, T^n Tu)d(Tu, T^n u) + d(u, T^n u)d(u, T^n Tu)}{d(Tu, T^n u) + d(u, T^n Tu)} \\ &= \lambda d(Tu, u). \end{aligned} \tag{3.10}$$

By taking modulus (3.10), we get $|d(Tu, u)| < \lambda|d(Tu, u)|$, a contradiction. So, $Tu = u$.

Hence $Tu = T^n u = u$. Therefore, the fixed point of T is unique. This completes the proof. **Theorem 3.2.** Let (X, d) be a complete complex valued b-metric space with the coefficient $s \geq 1$ and let $T : X \to X$ be a mapping satisfying the condition

$$d(Tx, Ty) \preceq \alpha d(x, y) + \beta\frac{d(y, Ty)[1 + d(x, Tx)]}{1 + d(x, y)} + \gamma\frac{d(y, Ty) + d(y, Tx)}{1 + d(y, Ty)d(y, Tx)} \tag{3.11}$$

for all $x, y \in X$, where α, β, γ are non-negative reals with $s(\alpha + \beta + \gamma) < 1$. Then T has a unique fixed point.

Proof. Let x_0 be arbitrary in X, we define a sequence $\{x_n\}$ by the rule,

$x_0, x_1 = Tx_0, x_2 = Tx_1, ...x_{n+1} = Tx_n$, for all $n \in \mathbb{N}$.

Now, we show that the sequence $\{x_n\}$ is Cauchy, consider $d(x_n, x_{n+1}) = d(Tx_{n-1}, Tx_n)$.

From equation (3. 11), we have

$$\begin{aligned} d(x_n, x_{n+1}) &\preceq \alpha d(x_{n-1}, x_n) + \beta \frac{d(x_n, Tx_n)[1 + d(x_{n-1}, Tx_{n-1})]}{1 + d(x_{n-1}, x_n)} + \gamma \frac{d(x_n, Tx_n) + d(x_n, Tx_{n-1})}{1 + d(x_n, Tx_n)d(x_n, Tx_{n-1})} \\ &\preceq \alpha d(x_{n-1}, x_n) + \beta \frac{d(x_n, x_{n+1})[1 + d(x_{n-1}, x_n)]}{1 + d(x_{n-1}, x_n)} + \gamma d(x_n, x_{n+1}). \end{aligned}$$

Therefore,

$$d(x_n, x_{n+1}) \preceq \frac{\alpha}{1 - (\beta + \gamma)} d(x_{n-1}, x_n). \tag{3.12}$$

Since $s(\alpha + \beta + \gamma) < 1$ and $s \geq 1$, we get $\alpha + \beta + \gamma < 1$.

Therefore, with $h = \frac{\alpha}{1-(\beta+\gamma)} < 1$ and for all $n \geq 0$ and consequently, we have

$$\begin{aligned} |d(x_n, x_{n+1})| &\leq h|d(x_{n-1}, x_n)| \\ &\leq h^2|d(x_{n-2}, x_{n-1})| \\ &\vdots \\ &\leq h^n|d(x_0, x_1)|. \end{aligned} \tag{3.13}$$

Thus, for any $m > n;\ \ m, n \in \mathbb{N}$ we have

$$\begin{aligned} |d(x_n, x_m)| &\leq s|d(x_n, x_{n+1})| + s|d(x_{n+1}, x_m)| \\ &\leq s|d(x_n, x_{n+1})| + s^2|d(x_{n+1}, x_{n+2})| \leq s^2|d(x_{n+2}, x_m)| \\ &\vdots \\ &\leq s|d(x_n, x_{n+1})| + s^2|d(x_{n+1}, x_{n+2})| + ... + s^{m-n-1}|d(x_{m-2}, x_{m-1})| + s^{m-n}|d(x_{m-1}, x_m)|. \end{aligned}$$

By using (3.13), we get

$$\begin{aligned} |d(x_n, x_m)| &\leq sh^n|d(x_0, x_1)| + ... + s^{m-n-1}h^{m-2}|d(x_0, x_1)| + s^{m-n}h^{m-1}|d(x_0, x_1)| \\ &= \sum_{i=1}^{m-n} s^i h^{i+n-1}|d(x_0, x_1)|. \end{aligned} \tag{3.14}$$

Therefore,

$$\begin{aligned} |d(x_n, x_m)| &\leq \sum_{t=n}^{m-1} (sh)^t|d(x_0, x_1)| \\ &= \frac{(sh)^t}{1 - sh}|d(x_0, x_1)| \end{aligned} \tag{3.15}$$

and hence

$$|d(x_n, x_m)| \leq \frac{(sh)^t}{1 - sh}|d(x_0, x_1)| \to 0 \ \ as \ \ m, n \to \infty. \tag{3.16}$$

Thus, $\{x_n\}$ is a Cauchy sequence in X.Since X is complete, there exists some $u \in X$ such that $x_n \to u$ as $n \to \infty$.

Suppose this is not possible, then there exists $z \in X$ such that

$$|d(u, Tu)| = |z| > 0. \tag{3.17}$$

Now,

$$\begin{aligned} z = d(u,Tu) &\preceq sd(u,x_{n+1}) + sd(x_{n+1},Tu) \\ &= sd(u,x_{n+1}) + sd(Tx_n,Tu) \\ &\preceq sd(u,x_{n+1}) + s\alpha d(x_n,u) + s\beta \frac{d(u,Tu)[1+d(x_n,Tx_n)]}{1+d(x_n,u)} + s\gamma \frac{d(u,Tu)+d(u,Tx_n)}{1+d(u,Tu)d(u,Tx_n)} \\ &= sd(u,x_{n+1}) + s\alpha d(x_n,u) + s\beta \frac{d(u,Tu)[1+d(x_n,x_{n+1})]}{1+d(x_n,u)} + s\gamma \frac{d(u,Tu)+d(u,x_{n+1})}{1+d(u,Tu)d(u,x_{n+1})} \end{aligned}$$

which implies that

$$\begin{aligned} |z| &= |d(u,Tu)| \\ &\leq s|d(u,x_{n+1})| + s\alpha|d(x_n,u)| + s\beta \frac{|d(u,Tu)||[1+d(x_n,x_{n+1})]|}{|1+d(x_n,u)|} + s\gamma \frac{|d(u,Tu)|+|d(u,x_{n+1})|}{|1+d(u,Tu)d(u,x_{n+1})|} \end{aligned} \tag{3.18}$$

Taking the limit of (3.17) as $n \to \infty$, we obtain that $|z| = |d(u,Tu)| \leq 0$, a contradiction with (3.16). So $|z| = 0$. Hence $Tu = u$.

Now, we show that T has a unique fixed point in X .To show this, assume that u^* is another fixed point of T. Then,

$$\begin{aligned} d(u,u^*) &= d(Tu,Tu^*) \\ &\preceq \alpha d(u,u^*) + \beta \frac{d(u^*,Tu^*)[1+d(u,Tu)]}{1+d(u,u^*)} + \gamma \frac{d(u^*,Tu^*)+d(u^*,Tu)}{1+d(u^*,Tu^*)d(u^*,Tu)} \end{aligned}$$

So that

$$|d(u,u^*)| \leq \alpha|d(u,u^*)| + \beta \frac{|d(u^*,Tu^*)||[1+d(u,Tu)]|}{|1+d(u,u^*)|} + \gamma \frac{|d(u^*,Tu^*)|+|d(u^*,Tu)|}{|1+d(u^*,Tu^*)d(u^*,Tu)|}$$

$$< (\alpha+\gamma)|d(u,u^*)|,$$

a contradiction. So $u = u^*$. Therefore, the fixed point of T is unique.This completes the proof of the Theorem.

Corollary 3.2. Let (X,d) be a complete complex valued b-metric space with the coefficient $s \geq 1$ and let $T : X \to X$ be a mapping satisfying the condition(for some fixed n)

$$d(T^n x, T^n y) \preceq \alpha d(x,y) + \beta \frac{d(y,T^n y)[1+d(x,T^n x)]}{1+d(x,y)} + \gamma \frac{d(y,T^n y)+d(y,T^n x)}{1+d(y,T^n y)d(y,T^n x)} \tag{3.19}$$

for all $x,y \in X$, where α,β,γ are non-negative reals with $s(\alpha+\beta+\gamma) < 1$. Then T has a unique fixed point.

Proof: The proof of this Corollary is similar as the Corollary 3.1.

Corollary 3.3. Let (X,d) be a complete complex valued b-metric space with the coefficient $s \geq 1$ and let $T : X \to X$ be a mapping satisfying the condition

$$d(Tx,Ty) \preceq \alpha d(x,y) + \beta \frac{d(y,Ty)[1+d(x,Tx)]}{1+d(x,y)} \tag{3.20}$$

for all $x,y \in X$, where α,β are non-negative reals with $s(\alpha+\beta) < 1$. Then T has a unique fixed point.

Proof: We can prove this result by applying Theorem 3.2 by setting $\gamma = 0$.

Corollary 3.4. Let (X,d) be a complete complex valued b-metric space with the coefficient $s \geq 1$ and let $T : X \to X$ be a mapping satisfying the condition

$$d(Tx,Ty) \preceq \alpha d(x,y) + \beta \frac{d(y,Ty)[1+d(x,Tx)]}{1+d(x,y)} + \gamma d(y,Ty) \tag{3.21}$$

for all $x, y \in X$, where α, β, γ are non-negative reals with $s(\alpha + \beta + \gamma) < 1$. Then T has a unique fixed point.

Proof: We can prove this result by applying Theorem 3.2 by setting $d(y, Tx) = 0$.

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