International Journal of Advances in Mathematics
Volume 2018, Number 1, Pages 73-82, 2018
eISSN 2456-6098
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Common Fixed Point Theorems for (ϕ, ψ)-Weak Contractions in Intuitionistic Fuzzy Cone Metric Spaces

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ABSTRACT. In this manuscript, we extend the concept of (ϕ, ψ)-weak contractions in intuitionistic fuzzy cone metric space by using an altering distance function and prove some common fixed point theorems for (ϕ, ψ)-weak contractions in intuitionistic fuzzy cone metric spaces.

1 Introduction

In 1965, L.A. Zadeh [1] introduced the concept of fuzzy set as a function with domain X and values in $[0, 1]$. After that many authors extensively developed the theory of fuzzy sets and its applications. In 1975, Kramosil and Michaleck [2] introduced fuzzy metric space and generalized the statistical (probabilistic) metric space. In 1986, K.T. Atanassorv [3] gave the concept of intuitionistic fuzzy set which is an extension of fuzzy metric space. Park [4] with the notion of intuitionistic fuzzy metric space, generalized the notion of fuzzy metric space due to George and Veeramani [5] in 1994. After that Saadati et al. [6] modified the idea of intuitionistic fuzzy metric space and obtained various fixed point results in modified intuitionistic fuzzy metric spaces.

* Corresponding Author.
Received September 27, 2017; revised November 04, 2017; accepted November 08, 2017.
2010 Mathematics Subject Classification: 47H09, 54H25.
Key words and phrases: Fuzzy cone metric space, generalized weak contraction, common fixed point.
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In 2007, Huang and Zhang [7] introduced the concept of cone metric space and proved some fixed point theorems for contractive mappings. Recently, Tarkan Oner et al. [8] itroduced the concept of fuzzy cone metric space that generalized the corresponding notions of fuzzy metric space by George and Veeramani [5] and proved the fuzzy cone Banach contraction theorem. In 2010, C. Vetro et al.[13] extended the notion of (ϕ, ψ)-weak contraction to fuzzy metric spaces and proved some common fixed point theorems for four mappings in fuzzy metric spaces by using the idea of an altering distance function. In recent past, several authors proved various fixed point theorems employing more generalized conditions. For more results on fuzzy metric space and cone metric space one can see research papers in [14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29].

The aim of this paper is to extend and prove some common fixed point theorems for (ϕ, ψ)-weak contractions in intuitionistic fuzzy cone metric spaces.

2 Preliminaries

Now in this section, we start with some basic concepts on intuitionistic fuzzy cone metric spaces.

Definition 2.1. *[1] Let X be any set. A fuzzy set A in X is a function with domain X and values in* $[0, 1]$.

Definition 2.2. *[7] Let E be a real Banach space, θ the zero of E and P a subset of E. Then P is called a cone if and only if*

1. *P is closed, nonempty and* $P \neq \{\theta\}$,
2. *if* $a, b \in R$, $a, b \geq 0$ *and* $x, y \in P$, *then* $ax + by \in P$,
3. *if both* $x \in P$ *and* $-x \in P$, *then* $x = \theta$.

For a given cone P, a partial ordering $\preceq$ on E with respect to P is defined by $x \preceq y$ if and only if $y - x \in P$. The notation $x \prec y$ will stand for $x \preceq y$ and $x \neq y$, while $x \ll y$ will stand for $y - x \in int(P)$. Throughout this paper, we assume that all cones have nonempty interior.

A cone P is called normal if there exists a constant $K > 0$ such that for all $t, s \in E$, $\theta \preceq t \preceq s$ implies $\|t\| \leq K\|s\|$ and the least positive number K satisfying this property is called normal constant of P [7]. Rezapour and Hamlbarani [9] showed that there is no cone with normal constant $K < 1$ and there exist cones of normal constant 1 and cones of normal constant $M > K$ for each $K > 1$.

Definition 2.3. *[10] A binary operation* $* : [0, 1] \times [0, 1] \to [0, 1]$ *is a continuous t-norm if* $*$ *satisfies the following conditions;*

1. $*$ *is associative and commutative,*
2. $*$ *is continuous,*
3. $a * 1 = a$ *for all* $a \in [0, 1]$,
4. $a * b \leq c * d$ *whenever* $a \leq c$ *and* $b \leq d$, $\forall a, b, c, d \in [0, 1]$.

Example 1. $a * b = \min\{a, b\}$.

Definition 2.4. *[8] A 3-tuple $(X, M, *)$ is said to be a fuzzy cone metric space if P is a cone of E, X is an arbitrary set, $*$ is a continuous t-norm and M is a fuzzy set on $X^2 \times int(P)$ satisfying the following conditions;*
For all $x, y, z \in X$ and $t, s \in int(P)$ (that is $t \gg \theta$, $s \gg \theta$)

FCM1. $M(x, y, t) > 0$,

FCM2. $M(x, y, t) = 1$ *if and only if* $x = y$,

FCM3. $M(x, y, t) = M(y, x, t)$,

FCM4. $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$,

FCM5. $M(x, y, .) : int(P) \to [0, 1]$ *is continuous.*

If $E = \mathbb{R}$, $P = [0, \infty)$ and $a * b = ab$, then each fuzzy metric space becomes a fuzzy cone metric space.

Example 2. *[8] Let $E = \mathbb{R}^2$. Then $P = \{(k_1, k_2) : k_1, k_2 \geq 0\} \subset E$ is a normal cone with normal constant $K = 1$. Let $X = \mathbb{R}$, $a * b = ab$ and $M : X^2 \times int(P) \to [0, 1]$ defined by $M(x, y, t) = \frac{1}{e^{\frac{|x-y|}{\|t\|}}}$ for all $x, y \in X$ and $t \gg \theta$.*

Definition 2.5. *[8] Let $(X, M, *)$ be a fuzzy cone metric space, $x \in X$ and $\{x_n\}$ be a sequence in X. Then*

1. *$\{x_n\}$ is said to converge to x if for any $t \gg \theta$ and any $r \in (0, 1)$ there exists a natural number n_0 such that $M(x_n, x, t) > 1 - r$ for all $n \geq n_0$. We denote this by $\lim_{n\to\infty} x_n = x$ or $x_n \to x$ as $n \to \infty$.*
2. *$\{x_n\}$ is said to be a Cauchy sequence if for any $0 < \varepsilon < 1$ and any $t \gg \theta$ there exists a natural number n_0 such that $M(x_n, x_m, t) > 1 - \varepsilon$ for all $n, m \geq n_0$.*
3. *$(X, M, *)$ is called complete if every Cauchy sequence is convergent.*

Definition 2.6. *[10] A binary operation $\diamond : [0, 1] \times [0, 1] \to [0, 1]$ is a continuous t-conorm if $\diamond$ satisfies the following conditions;*

1. *$\diamond$ is associative and commutative,*
2. *$\diamond$ is continuous,*
3. *$a \diamond 0 = a$ for every $a \in [0, 1]$,*
4. *$a \diamond b \leq c \diamond d$ if $a \leq c$ and $b \leq d$, $\forall a, b, c, d \in [0, 1]$.*

Example 3. $a \diamond b = \max\{a, b\}$.

Definition 2.7. *A 5-tuple $(X, M, N, *, \diamond)$ is an intuitionistic fuzzy cone metric space if X is an arbitrary set, $*$ is a continuous t-norm, $\diamond$ is a continuous t-conorm and M, N are fuzzy sets in $X^2 \times int(P)$ satisfying the following conditions:*

1. $M(x, y, t) > 0$,
2. $M(x, y, t) = 1$ *if and only if* $x = y$,
3. $M(x, y, t) = M(y, x, t)$,
4. $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$,

5. $M(x, y, .) : int(P) \to [0, 1]$ *is continuous,*
6. $N(x, y, t) < 0$,
7. $N(x, y, t) = 0$ *if and only if* $x = y$,
8. $N(x, y, t) = N(y, x, t)$,
9. $N(x, y, t) \diamond N(y, z, s) \geq N(x, z, t + s)$,
10. $N(x, y, .) : int(P) \to [0, 1]$ *is continuous,*
11. $M(x, y, t) + N(x, y, t) \leq 1, \forall x, y, z \in X$ *and* $t \gg \theta$, $s \gg \theta$.

Then (M, N) is called an intuitionistic fuzzy cone metric space on X. The functions $M(x, y, t)$ and $N(x, y, t)$ denote the degree of nearness and the degree of non-nearness between x and y with respect to t respectively.

Remark 1. *In an intuitionistic fuzzy cone metric space* $(X, M, N, *, \diamond)$, $M(x, y, .)$ *is non-decreasing and* $N(x, y, .)$ *non-increasing for all* $x, y \in X$.

Example 4. *Let* (X, d) *be a metric space and let* $P = \mathbb{R}^+$. *Denote* $a * b = ab$ *and* $a \diamond b = \min\{1, a + b\}$ *for all* $a, b \in [0, 1]$ *and let M and N be fuzzy sets on* $X^2 \times intP$ *defined as follows:*

$$M(x, y, t) = \frac{kt^n}{kt^n + ld(x, y)} \text{ and } N(x, y, t) = \frac{d(x, y)}{mt^n + ld(x, y)},$$

for all $k, l, m, n \in intP$ *and for each* $t \gg \theta$. *Then,* $(X, M, N, *, \diamond)$ *is an intuitionistic fuzzy cone metric space.*

Definition 2.8. *Let* $(X, M, N, *, \diamond)$ *be an intuitionistic fuzzy cone metric space,* $x \in X$ *and* $\{x_n\}$ *be a sequence in X. Then*

1. $\{x_n\}$ *is said to converge to x if for each* $t \gg \theta$, *we have* $\lim_{n\to\infty} M(x_n, x, t) = 1$ *and* $\lim_{n\to\infty} N(x_n, x, t) = 0$.
2. $\{x_n\}$ *is said to a Cauchy sequence if and only if for each* $r \in (0, 1)$ *and* $t \gg \theta$, *there exists* $n_0 \in \mathbb{N}$ *such that* $M(x_n, x_m, t) > 1 - r$ *and* $N(x_n, x_m, t) < r$ *for all* $n, m \geq n_0$.
3. $(X, M, N, *, \diamond)$ *is called complete if and only if every Cauchy sequence is convergent.*
4. $(X, M, N, *, \diamond)$ *is compact if every sequence contains a convergent subsequence.*

Definition 2.9. *[11] A function* $\phi : [0, \infty) \to [0, \infty)$ *is an altering distance function if* $\phi(t)$ *is monotone non-decreasing and continuous and* $\phi(t) = 0$ *if and only if* $t = 0$.

Definition 2.10. *Let* $(X, M, N, *, \diamond)$ *be an intuitionistic fuzzy cone metric space and* $f, g : X \to X$ *be two mappings. The mapping g is called intuitionistic* (ϕ, ψ)*-weak contraction with respect to f if there exists a function* $\psi : [0, \infty) \to [0, \infty)$ *with* $\psi(r) > 0$ *for* $r > 0$ *and* $\psi(0) = 0$ *and altering distance* ϕ *such that*

$$\phi\left(\frac{1}{M(g(x), g(y), t)} - 1\right) \leq \phi\left(\frac{1}{M(f(x), f(y), t)} - 1\right) - \psi\left(\frac{1}{M(f(x), f(y), t)} - 1\right) \tag{1}$$

and

$$\phi(N(g(x), g(y), t)) \leq \phi(N(f(x), f(y), t)) - \psi(N(f(x), f(y), t)) \tag{2}$$

hold for all $x, y \in X$ *and each* $t \gg \theta$. *If f is the identity mapping, then g is called* (ϕ, ψ)*-weak contraction.*

Definition 2.11. *Let $(X, M, N, *, \diamond)$ be an intuitionistic fuzzy cone metric space and $f, g : X \to X$ be two mappings. A point z in X is called coincidence point (common fixed point) of f and g if $z = f(z) = g(z)$.*

Definition 2.12. *[12] Let $\{f_i\}$ and $\{g_j\}$ be two finite families of self mappings on X are said to be pairwise commuting if*

1. *$f_i f_j = f_j f_i$, where $i, j \in \{1, 2, \ldots, m\}$,*
2. *$g_i g_j = g_j g_i$, where $i, j \in \{1, 2, \ldots, n\}$,*
3. *$f_i g_j = g_j f_i$, where $i \in \{1, 2, \ldots, m\}$ and $j \in \{1, 2, \ldots, n\}$.*

3 Main results

In this section, we prove the following fixed point theorems for intuitionistic (ϕ, ψ)-weak contraction mappings.

Theorem 3.1. *Let $(X, M, N, *, \diamond)$ be an intuitionistic fuzzy cone metric space and $g : X \to X$ be a generalized intuitionistic (ϕ, ψ)-weak contraction with respect to $f : X \to X$. If $g(X) \subseteq f(X)$ and $f(X)$ or $g(X)$ is a complete subset of X, then f and g have a unique common fixed point in X provided that ψ is a continuous function.*

Proof. Let $x_0 \in X$ be an arbitrary point. Choose a point $x_1 \in X$ such that $g(x_0) = f(x_1)$. This can be done since $g(X) \subseteq f(X)$. Continuing this process, we obtain a sequence $\{x_n\}$ in X such that $y_n = g(x_n) = f(x_{n+1})$. We assume that $y_{n+1} \neq y_n$ for all $n \in \mathbb{N}$, otherwise f and g have a coincidence point.
Now we get

$$\begin{aligned}\phi\Big(\frac{1}{M(y_n, y_{n+1}, t)} - 1\Big) &= \phi\Big(\frac{1}{M(g(x_n), g(x_{n+1}), t)} - 1\Big) \\ &\leq \phi\Big(\frac{1}{M(f(x_n), f(x_{n+1}), t)} - 1\Big) - \psi\Big(\frac{1}{M(f(x_n), f(x_{n+1}), t)} - 1\Big) \\ &\leq \phi\Big(\frac{1}{M(y_{n-1}, y_n, t)} - 1\Big) - \psi\Big(\frac{1}{M(y_{n-1}, y_n, t)} - 1\Big) \\ &< \phi\Big(\frac{1}{M(y_{n-1}, y_n, t)} - 1\Big),\end{aligned}$$

which, considering that the ϕ function is non-decreasing, implies that $M(y_n, y_{n+1}, t) > M(y_{n-1}, y_n, t)$ for all $n \in \mathbb{N}$ and hence $\{M(y_{n-1}, y_n, t)\}$ is an increasing sequence of positive real numbers in $(0, 1]$.

Let $U(t) = \lim_{n\to\infty} M(y_{n-1}, y_n, t)$, we show that $U(t) = 1$ for all $t \gg \theta$. If not, there exists $t \gg \theta$ such that $U(t) < 1$, then from the above inequality on taking $n \to \infty$, we obtain

$$\phi\Big(\frac{1}{U(t)} - 1\Big) \leq \phi\Big(\frac{1}{U(t)} - 1\Big) - \psi\Big(\frac{1}{U(t)} - 1\Big),$$

which is a contradiction. Therefore $M(y_n, y_{n+1}, t) \to 1$ as $n \to \infty$.
Now, for each positive integer p, we have

$$M(y_n, y_{n+p}, t) \geq M(y_n, y_{n+1}, \frac{t}{p}) * M(y_{n+1}, y_{n+2}, \frac{t}{p}) * \cdots * M(y_{n+p-1}, y_{n+p}, \frac{t}{p}).$$

It follows that

$$\lim_{n\to\infty} M(y_n, y_{n+p}, t) \geq 1 * 1 * \cdots * 1 = 1.$$

At the same time, we have

$$\begin{aligned}\phi(N(y_n, y_{n+1}, t)) &= \phi(N(g(x_n), g(x_{n+1}), t)) \\ &\leq \phi(N(f(x_n), f(x_{n+1}), t)) - \psi(N(f(x_n), f(x_{n+1}), t)) \\ &\leq \phi(N(y_{n-1}, y_n, t)) - \psi(N(y_{n-1}, y_n, t)) \\ &< \phi(N(y_{n-1}, y_n, t)),\end{aligned}$$

which, considering that the ϕ function is non-decreasing, implies that $N(y_n, y_{n+1}, t) < N(y_{n-1}, y_n, t)$ for all $n \in \mathbb{N}$ and hence $\{N(y_{n-1}, y_n, t)\}$ is a decreasing sequence of positive real numbers in $[0, 1)$.

Let $V(t) = \lim_{n\to\infty} N(y_{n-1}, y_n, t)$, we show that $V(t) = 0$ for all $t \gg \theta$. If not, there exists $t \gg \theta$ such that $V(t) > 0$, then from the above inequality on taking $n \to \infty$, we obtain

$$\phi(V(t)) \leq \phi(V(t)) - \psi(V(t)),$$

which is a contradiction. Therefore $N(y_n, y_{n+1}, t) \to 0$ as $n \to \infty$.
Now, for each positive integer p, by definition 2.7, we have

$$M(y_n, y_{n+p}, t) + N(y_n, y_{n+p}, t) \leq 1$$

and

$$\lim_{n\to\infty}(M(y_n, y_{n+p}, t) + N(y_n, y_{n+p}, t)) \leq 1.$$

It follows that

$$\lim_{n\to\infty} N(y_n, y_{n+p}, t) = 0.$$

Hence $\{y_n\}$ is a Cauchy sequence. If $f(X)$ is complete, then there exists $q \in f(X)$ such that $y_n \to q$ as $n \to \infty$. The same holds if $g(X)$ is complete with $q \in g(X)$. Let $p \in X$ be such that $f(p) = q$. Now, we shall show that p is a coincidence point of f and g.

In fact, we have

$$\begin{aligned}\phi\left(\frac{1}{M(g(p), f(x_{n+1}), t)} - 1\right) &= \phi\left(\frac{1}{M(g(p), g(x_n), t)} - 1\right) \\ &\leq \phi\left(\frac{1}{M(f(p), f(x_n), t)} - 1\right) - \psi\left(\frac{1}{M(f(p), f(x_n), t)} - 1\right)\end{aligned}$$

for every $t \gg \theta$, which on taking $n \to \infty$ gives that

$$\begin{aligned}\lim_{n\to\infty} M(g(p), f(x_{n+1}), t) &= \lim_{n\to\infty} M(g(p), g(x_n), t) \\ &= M(g(p), f(p), t) \\ &= 1.\end{aligned}$$

Therefore $f(p) = g(p) = q$. Now, we shall show that $f(q) = q$. If it is not so, then we consider

$$\begin{aligned}\frac{1}{M(f(q),q,t)} - 1 &= \frac{1}{M(g(q),g(p),t)} - 1 \\ &\leq \Big(\frac{1}{M(f(q),f(p),t)} - 1\Big) - \psi\Big(\frac{1}{M(f(q),f(p),t)} - 1\Big) \\ &= \Big(\frac{1}{M(f(q),q,t)} - 1\Big) - \psi\Big(\frac{1}{M(f(q),q,t)} - 1\Big)\end{aligned}$$

which is a contradiction that leads our result.

The uniqueness of fixed point easily follows from the inequality (1) or (2) and so the proof is completed. □

Example 5. *Let $(X, M, N, *, \diamond)$ be a complete intuitionistic fuzzy cone metric space and let $X = \{\frac{1}{n} : n \in \mathbb{N}\} \cup \{0\}$, $*$ be a minimum norm and $\diamond$ be a maximum norm. Let (M, N) be defined by*

$$M(x,y,t) = \begin{cases} \frac{t}{t+|x+y|}, & \text{if } t > 0; \\ 0, & \text{if } t = 0, \end{cases} \quad \text{and } N(x,y,t) = \begin{cases} \frac{|x+y|}{t+|x+y|}, & \text{if } t > 0; \\ 1, & \text{if } t = 0. \end{cases}$$

Also, define $\phi, \psi : [0,\infty) \to [0,\infty)$ by $\phi(t) = \frac{t}{2}$, $\psi(t) = \frac{t}{8}$, for all $t \gg \theta$, $f(x) = \frac{x}{2}$ and $g(x) = \frac{x}{4}$. Obviousely, $g(X) \subseteq f(X)$ and ψ is a continuous function. Then we have

$$\begin{aligned}\phi\Big(\frac{1}{M(f(x),f(y),t)} - 1\Big) - \psi\Big(\frac{1}{M(f(x),f(y),t)} - 1\Big) &= \frac{3}{16}\frac{|x+y|}{t} \\ &\geq \frac{1}{8}\frac{|x+y|}{t} \\ &= \phi\Big(\frac{1}{M(g(x),g(y),t)} - 1\Big).\end{aligned}$$

From the above inequality and the fact that $N = 1 - M$, we conclude that the conditions (1) and (2) are satisfied. Thus g is an intuitionistic (ϕ,ψ)-weak contraction with respect to f.
All the hypothesis of Theorem 3.1 are satisfied, then f and g have a unique common fixed point.

Corollary 3.1. *Let $(X, M, N, *, \diamond)$ be an intuitionistic fuzzy cone metric space and $g : X \to X$ be a (ϕ,ψ)-weak contraction. If ψ is continuous then g has a unique fixed point.*

Corollary 3.2. *Let $(X, M, N, *, \diamond)$ be an intuitionistic fuzzy cone metric space and $g : X \to X$ be a mapping satisfying*

$$\frac{1}{M(g(x),g(y),t)} - 1 \leq k\Big(\frac{1}{M(x,y,t)} - 1\Big)$$

and

$$M(g(x),g(y),t) \leq kN(x,y,t)$$

for each $x, y \in X$, $t \gg \theta$ and $k \in (0,1)$. Then g has a unique fixed point.

Theorem 3.2. *Let $(X, M, N, *, \diamond)$ be an intuitionistic fuzzy cone metric space and $\{f_i\}$, $\{g_j\}$, where $i \in \{1,2,\ldots,m\}$ and $j \in \{1,2,\ldots,n\}$, be two finite families of self mappings on X with $f = f_1.f_2 \ldots f_n$ and $g = g_1.g_2 \ldots g_m$. Let g be a generalized intuitionistic (ϕ,ψ)-weak contraction with respect to f. If $g(X) \subseteq f(X)$ and $f(X)$ or $g(X)$ is a complete subset of X then g_j and f_i have a unique common fixed point in X, provided that ψ is a continuous function and the families $\{f_i\}$ and $\{g_j\}$ commute pairwise.*

Proof. Using Theorem 3.1, we conclude that f and g have a unique common fixed point, say q.

Now, we want to show that q remains the fixed point of all the component mappings. For this consider

$$\begin{aligned} gg_i(q) &= (g_1.g_2 \cdots g_m)g_i(q) \\ &= (g_1.g_2 \cdots g_{m-1})g_m g_i(q) \\ &= (g_1.g_2 \cdots g_{m-1})g_i g_m(q) \\ &= \dots \\ &= g_1.g_i(g_2.g_3 \cdots g_m)(q) \\ &= g_i.g_1(g_2.g_3 \cdots g_m)(q) \\ &= g_i(g(q)) \\ &= g_i(q). \end{aligned}$$

Similarly, we can show that $gf_j(q) = f_j g(q) = f_j(q)$, $ff_j(q) = f_j f(q) = f_j(q)$ and $fg_i(q) = g_i f(q) = g_i(q)$, which implies that, for all i and j, $g_i(q)$ and $I_j(q)$ are other fixed points of the pair $\{g, f\}$. Now the uniqueness of common fixed point of mappings f and g, we get $q = g_i(q) = I_j(q)$, which shows that q is a common fixed point of f_i and g_j for all i and j. □

Example 6. *Set M, N, $*$, $\diamond$, ϕ as in Example 5 and let $X = [0, \infty)$ and $P = \mathbb{R}^+$. Define $\psi : [0, \infty) \to [0, \infty)$ by $\psi(t) = \frac{t}{2}$, for all $t \gg \theta$ and two families of self mappings $\{f_i\}$ and $\{g_j\}$, where $i, j \in \{1, 2, \dots, n\}$, by*

$$f_i(x) = \begin{cases} 0, & \text{if } x = 0; \\ \frac{1}{x^{\sqrt[n]{6}}}, & \text{if } x \in intP, \end{cases} \quad \text{and } g_j(x) = \begin{cases} 0, & \text{if } x = 0; \\ \frac{1}{x^{\sqrt[n]{2}}}, & \text{if } x \in intP. \end{cases}$$

Therefore, we have

$$\begin{aligned} \left(\frac{1}{M(f(x), f(y), t)} - 1\right) - \psi\left(\frac{1}{M(f(x), f(y), t)} - 1\right) &= \frac{|x^6 + y^6|}{2tx^6y^6} \\ &\geq \frac{|x^2 + y^2|}{tx^2y^2} \\ &= \left(\frac{1}{M(g(x), g(y), t)} - 1\right). \end{aligned}$$

From the above inequality and the fact that $N = 1 - M$, we conclude that the conditions (1) and (2) are satisfied. All the hypothesis of Theorem 3.2 are satisfied, then f_i and g_j have a unique common fixed point.

Conclusion

We extend the concept of (ϕ, ψ)-weak contractions in intuitionistic fuzzy cone metric space by using an altering distance function. Our results generalize the results of fixed point theorems by employing intuitionistic fuzzy cone metric space.

Acknowledgements: Authors would like to thank the editor and the referees for their useful suggestions which improve the contents of this paper.

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