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Some results on Slant submanifolds of (k,μ)-contact manifolds

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ABSTRACT. The objective of this paper is to study slant sub manifolds of (k,μ)-contact manifold when structure tensor field ϕ is killing and have obtained some results under certain conditions.

1 Introduction

It is very interesting to study the geometry of unknown manifold. It is also very convenient to study the geometry of unknown manifold along with the study of ambient manifold which resulted in the new theory of submanifold. Later on it has become an independent research topic.

Slant Submanifolds play a very important role in the field of submanifolds because of the presence of invariant and anti-invariant conditions. Chen [3] defined the slant submanifold for an almost Hermitian manifold, as a generalisation of both holomorphic and totally real submanifolds. Slant submanifolds of C^2 and C^4 were given by Chen [2], also the examples were discussed by Chen and Tazawa [4, 5]. The notion of slant immersions into almost contact metric manifolds was introduced by Lotta [7, 8] where he has obtained the results of fundamental importance. Later on, the geometry of slant submanifold is studied by Cabrerizo et al [1, 12] in more specialized settings of K-contact and sasakian manifolds, while slant submanifolds of a Kaehler manifold were given

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by Maeda et al [9]. Also Gupta [6] et al. defined and studied about Slant submanifold of a Kenmotsu manifold. Siddesha and Bagewadi [10] studied about (k,μ)-contact manifolds. Killing structures on contact manifolds was studied by Blair [11].

In this paper we extend the study to find the condition for killing structure of slant submanifolds of (k,μ)-contact manifold.

2 Preliminaries

Let $(\overline{M}, g)$ be an almost contact metric manifold of dimension $(2n+1)$ equipped with structure (ϕ, ξ, η, g) consisting of a (1,1) tensor field ϕ, a vector field ξ, a 1-form η and a Riemannian metric g satisfying

$$\phi^2 X = -X + \eta(X)\xi, \eta(\xi) = 1, \phi\xi = 0, \eta(\phi X) = 0, \tag{2.1}$$

$$g(\phi X, \phi Y) = g(X,Y) - \eta(X)\eta(Y), \eta(X) = g(X, \xi). \tag{2.2}$$

An almost contact metric manifold is called (k,μ) contact metric manifold if

$$(\overline{\nabla}_X \phi)Y = g(X + hX, Y)\xi - \eta(Y)(X + hX), \tag{2.3}$$

and

$$(\overline{\nabla}_X \xi) = -\phi X - \phi h X, \tag{2.4}$$

where $\overline{\nabla}$ denotes the Levi-civita connection on $\overline{M}$.
Let M be a n-dimensional Riemannian manifold with induced metric g isometrically immersed in $\overline{M}$. We denote by TM, the Lie algebra of vector fields on M and by $T^{\perp}M$ the set of all vector fields normal to M.
For $X \in TM$ and $N \in T^{\perp}M$, we write

$$\phi X = TX + NX, \tag{2.5}$$

$$\phi V = tV + nV, \tag{2.6}$$

where TX and NX denotes the tangential and normal component of ϕX. tV and nV denotes the tangential and normal component of ϕV.
Let ∇ be the Riemannian connection on M, then the Gauss and Weingarten formulae are given by,

$$\overline{\nabla}_X Y = \nabla_X Y + h(X,Y), \tag{2.7}$$

$$\overline{\nabla}_X N = \nabla^{\perp}_X N - A_N X, \tag{2.8}$$

for any $X, Y \in TM$ and $N \in T^{\perp}M$ of $\overline{M}$.
$\nabla^{\perp}$ is the connection in the normal bundle $T^{\perp}M$ of M, h is the second fundamental form of M and A_N is the Weingarten endomorphism associated with N. The second fundamental form h and the shape operator A related by

$$g(h(X,Y), N) = g(A_N X, Y). \tag{2.9}$$

If T is the endomorphism defined by (2.5) then,

$$g(TX, Y) = -g(X, TY). \tag{2.10}$$

Thus T^2 which is denoted by Q is self adjoint.

$$T^2 = Q. \tag{2.11}$$

On the otherhand, Gauss and Weingarten formulae together with (2.3) and (2.5) implies that

$$(\nabla_X T)Y = A_{NY}X + t\sigma(X, Y) + g(Y, X + hX)\xi - \eta(Y)(X + hX), \tag{2.12}$$

$$(\nabla_X N)Y = n\sigma(X, Y) - \sigma(X, TY), \tag{2.13}$$

for any $X, Y \in TM$, A tensor field ϕ is said to be killing if,

$$(\overline{\nabla}_X \phi)Y + (\overline{\nabla}_Y \phi)X = 0. \tag{2.14}$$

3 slant submanifold of (k,μ)-contact manifold with killing structure

In this section we consider slant submanifold of (k,μ)-contact manifold with killing structure tensor field ϕ of type (1,1).

From [6] we know that If M is a submanifold of an almost contact metric manifold $\overline{M}$ such that $\xi \in TM$. Then, M is slant if and only if there exists a constant $\lambda \in [0, 1]$ *suchthat*

$$T^2(X) = -\lambda(X - \eta(X)\xi). \tag{3.1}$$

Further more, if θ is the slant angle of M, then $\lambda = cos^2\theta$ Also for any $X, Y \in TM$,
we have,

$$g(TX, TY) = cos^2\theta(g(X, Y) - \eta(X)\eta(Y)), \tag{3.2}$$

$$g(NX, NY) = sin^2\theta(g(X, Y) - \eta(X)\eta(Y)). \tag{3.3}$$

Theorem 3.1. *Let M be a n dimensional submanifold of (k,μ)-contact manifold $\overline{M}$ with killing structure tensor field ϕ, then M is slant if*

$$g(2X + hX, TY)\xi - g(ThY, X)\xi - \eta(X)[TY - ThY] = 0, \tag{3.4}$$

and

$$\eta(X)NhY - \eta(X)NY = 0. \tag{3.5}$$

Proof: Consider (2.3)

$$(\overline{\nabla}_X \phi)Y = g(X + hX, Y)\xi - \eta(Y)(X + hX),$$

interchange X to Y we have

$$(\overline{\nabla}_Y \phi)X = g(Y + hY, X)\xi - \eta(X)(Y + hY). \tag{3.6}$$

Add (2.3) and (3.6) we get,

$$(\overline{\nabla}_X\phi)Y + (\overline{\nabla}_Y\phi)X = g(X + hX, Y)\xi - \eta(Y)(X + hX) + g(Y + hY, X)\xi - \eta(X)(Y + hY). \tag{3.7}$$

If ϕ has killing structure field then,

$$(\overline{\nabla}_X\phi)Y + (\overline{\nabla}_Y\phi)X = 0. \tag{3.8}$$

Therefore we have,

$$g(X + hX, Y)\xi - \eta(Y)(X + hX) + g(Y + hY, X)\xi - \eta(X)(Y + hY) = 0. \tag{3.9}$$

Replace Y by ϕY

$$g(X + hX, \phi Y)\xi + g(\phi Y + h\phi Y, X)\xi - \eta(X)(\phi Y + h\phi Y) = 0, \tag{3.10}$$

and by the application of (2.5), comparing tangential and normal part we get

$$g(2X + hX, TY)\xi - g(ThY, X)\xi - \eta(X)[TY - ThY] = 0,$$

and

$$\eta(X)NhY - \eta(X)NY = 0.$$

Hence the proof.

Theorem 3.2. *Let M be a n dimensional submanifold of (k,μ)-contact manifold $\overline{M}$ with killing structure tensor field φ, then M is slant if*

$$(\nabla_X Q)Y + (\nabla_Y Q)X = 0, \tag{3.11}$$

and

$$2\eta(X)TY - 2g(X, TY)\xi + \eta(Y)TX + \eta(Y)ThX + 2NYNhX + 2\eta(X)NhY = 0. \tag{3.12}$$

Proof: We know that from (3.1) and (2.11)

$$QX = -\lambda(X - \eta(X)\xi). \tag{3.13}$$

Applying covariant differentiation on Q , we get

$$(\nabla_X Q)Y = -\lambda[\nabla_X Y - (\nabla_X\eta)Y\xi + \eta(X)\nabla_X\xi]. \tag{3.14}$$

Consider

$$(\nabla_X Q)Y = \lambda[g(Y, \phi X + \phi hX)\xi - \eta(Y)(\phi X + \phi hX), \tag{3.15}$$

also we have

$$\phi X = TX + NX, \tag{3.16}$$

$$\phi hX = ThX + NhX, \tag{3.17}$$

we get,

$$(\nabla_X Q)Y = \lambda[g(Y, \phi X)\xi + g(Y, \phi hX)\xi - \eta(Y)\phi X - \eta(Y)\phi hX). \tag{3.18}$$

Similarly we get,

$$(\nabla_X Q)Y = \lambda[g(Y, TX + NX)\xi + g(Y, ThX + NhX)\xi - \eta(Y)(TX + NX) - \eta(Y)(ThX + NhX)], \quad (3.19)$$

$$(\nabla_X Q)Y = \lambda[g(Y, TX)\xi + g(Y, ThX)\xi - \eta(Y)(TX + NX) - \eta(Y)(ThX + NhX). \quad (3.20)$$

Interchange X to Y and Y to X we get

$$(\nabla_Y Q)X = \lambda[g(X, TY)\xi + g(X, ThY)\xi - \eta(X)(TY + NY) - \eta(X)(ThY + NhY). \quad (3.21)$$

Add equations (3.20) and (3.21) we get,

$$\begin{aligned}(\nabla_X Q)Y + (\nabla_Y Q)X = \lambda[g(Y, ThX)\xi - \eta(Y)(TX + NX) - \eta(Y)(ThX + NhX)] \\ +\lambda[g(X, ThY)\xi - \eta(X)(TY + NY) - \eta(X)(ThY + NhY).\end{aligned} \quad (3.22)$$

If

$$(\nabla_X Q)Y + (\nabla_Y Q)X = 0. \quad (3.23)$$

Using the result (3.5) obtained from Theorem (3.1), we get

$$2\eta(X)TY - 2g(X, TY)\xi + \eta(Y)TX + \eta(Y)ThX + 2NYNhX + 2\eta(X)NhY = 0. \quad (3.24)$$

Hence the proof.

Theorem 3.3. *Let M be a n dimensional submanifold of (k,μ)-contact manifold, then*

$$(\nabla_X N)Y + (\nabla_Y N)X = 0, \quad (3.25)$$

if and only if

$$2n\sigma(X, Y) = \sigma(X, TY) + \sigma(Y, TX). \quad (3.26)$$

Proof: We know that from (2.13)

$$(\nabla_X N)Y = n\sigma(X, Y) - \sigma(X, TY),$$

Interchange X to Y and Y to X , we get

$$(\nabla_Y N)X = n\sigma(Y, X) - \sigma(Y, TX). \quad (3.27)$$

Add equation (2.13) and (3.27) we have,

$$(\nabla_X N)Y + (\nabla_Y N)X = 2n\sigma(X, Y) - \sigma(X, TY) - \sigma(Y, TX). \quad (3.28)$$

If N has killing structure field then

$$(\nabla_X N)Y + (\nabla_Y N)X = 0, \quad (3.29)$$

Therefore

$$2n\sigma(X, Y) = \sigma(X, TY) + \sigma(Y, TX). \quad (3.30)$$

Hence the proof.

Theorem 3.4. *Let M be a n dimensional submanifold of (k,μ)-contact manifold $\overline{M}$ with killing structure tensor field ϕ, then M is slant if and only if T satisfies*

$$(\nabla_X T)\phi Y + (\nabla_{\phi Y} T)X = 0, \tag{3.31}$$

and

$$\begin{aligned} &g(h(X,Y), N\phi Y) + 2g(t\sigma(X,\phi Y), Y) + \eta(Y)g(2X + hX, TY) \\ &+g(h(\phi Y, Y), NX) + \eta(Y)g(X, ThY) + \eta(X)(ThY, Y) = 0. \end{aligned} \tag{3.32}$$

Proof: We know that from equation (2.12)

$$(\nabla_X T)Y = A_{NY}X + t\sigma(X,Y) + g(Y, X + hX)\xi - \eta(Y)(X + hX),$$

Replace X by Y and Y by X we get,

$$(\nabla_Y T)X = A_{NX}Y + t\sigma(Y,X) + g(X, Y + hY)\xi - \eta(X)(Y + hY). \tag{3.33}$$

Add equations (2.12) and (3.33) we get,

$$\begin{aligned} (\nabla_X T)Y + (\nabla_Y T)X = A_{NY}X + 2t\sigma(X,Y) + g(Y, X + hX)\xi - \eta(Y)(X + hX) \\ +A_{NX}Y + g(X, Y + hY)\xi - \eta(X)(Y + hY). \end{aligned} \tag{3.34}$$

If

$$(\nabla_X T)Y + (\nabla_Y T)X = 0. \tag{3.35}$$

Then we have,

$$\begin{aligned} A_{NY}X + 2t\sigma(X,Y) + g(Y, X + hX)\xi - \eta(Y)(X + hX) + A_{NX}Y \\ +g(X, Y + hY)\xi - \eta(X)(Y + hY) = 0. \end{aligned} \tag{3.36}$$

Now replace Y by ϕY in (3.36) we get

$$A_{N\phi Y}X + 2t\sigma(X,\phi Y) + g(\phi Y, X + hX)\xi + A_{NX}\phi Y + g(X, \phi Y + h\phi Y)\xi - \eta(X)(\phi Y + h\phi Y) = 0. \tag{3.37}$$

Also by virtue of (2.5) and Theorem 3.1 we have,

$$A_{N\phi Y}X + 2t\sigma(X,\phi Y) + g(2X + hX, TY)\xi + A_{NX}\phi Y + g(X, ThY)\xi - \eta(X)(TY - ThY) = 0. \tag{3.38}$$

Contract (3.38) with respect to Y Also using (2.9) we get,

$$\begin{aligned} &g(h(X,Y), N\phi Y) + 2g(t\sigma(X,\phi Y), Y) + \eta(Y)g(2X + hX, TY) \\ &+g(h(\phi Y, Y), NX) + \eta(Y)g(X, ThY) + \eta(X)(ThY, Y) = 0. \end{aligned} \tag{3.39}$$

Hence the proof.

4 Conclusion

In this paper we have obtained some results on slant submanifolds of (k,μ)-contact manifold obeying certain conditions with ϕ as a killing structure tensor field.

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