International Journal of Advances in Mathematics
Volume 2018, Number 1, Pages 26-30, 2018
eISSN 2456-6098

On Feebly-Locally Open Sets in Bitopological Spaces

S. Rajakumar
Department of Science and Humanities,
Krishnasamy College of Engineering and Technology,
Cuddalore, Tamil Nadu, India.
E-mail:srkumar277@gmail.com

ABSTRACT. In this paper a new class of sets namely feebly-locally open sets(in short fLO-sets), fLO^{*} sets, fLO^{**} sets in bitopological spaces are introduced. Some examples are provided to illustrate the behaviour of these new class of sets.

1 Introduction

In 1963, Kelly [3] defined a bitopological space (X, τ_1, τ_2) to be a set X equipped with two topologies τ_1, τ_2 on $X$and he initiated a systematic study of bitopological spaces. The study of locally closed set in a bitopological space was introduced by Kuratowski and Sierpinski [4]. According to Bourbaki [1], a subset A of a topological space X is called locally closed in X if it is the intersection of an open set and a closed set in X. Recently Maheswari S.N. and Jain P.C., introduced and studied f-open sets in topological spaces. In this paper a new class of sets namely feebly-locally open sets(in short fLO sets), fLO^{*} sets, fLO^{**} sets in bitopological spaces are introduced and obtained several characterizations.

* Corresponding Author.
Received August 08, 2017; revised October 22, 2017; accepted October 25, 2017.
2010 Mathematics Subject Classification: 54A05, 54A10, 54C05, 54C08.
Key words and phrases: Bitopological spaces, f-open sets, f-closed sets, f-locally open sets, fLO^{*} sets, fLO^{**} sets.

2 Preliminaries

Throughout this paper X represent a non-empty bitopological spaces (X, τ_1, τ_2) on which no seperation axioms are assumed unless explicitly mentioned. For a subset A of X, τ_i-$cl(A)$ (respectively. $\tau_i int(A)$) denotes the closure (respectively. interior) of A with respect to τ_i for i = 1,2.

Let us recall the following definitions which are used in the sequel.

Definition 2.1 A subset A of a topological space (X, τ) is called semi open[5] if $A \subseteq cl(int(A))$. The complement of semi open set is called semi closed set. The intersection of all semi closed sets containing A is called the semi closure[2] of A and is denoted by $scl(A)$.

Definition 2.2 A subset A of a topological space (X, τ) is called locally closed set[7] (briefly, lc) [8] if $A = U \cap F$ where U is open and F is closed in X.

Definition 2.3 A subset A of a topological space (X, τ) is said to be feebly open[6](f-open) (respectively. feebly closed) if $A \subset scl(int(A))$ (respectively. $sint(cl(A)) \supset A$).

3 Feebly-Locally Open Sets in Bitopological Spaces

In this section the notion of locally open sets (in short LO sets), feebly-locally open sets(in short fLOsets), fLO^* sets, fLO^{**} sets in bitopological spaces are introduced.

Definition 3.1 A subset A of a bitopological space (X, τ_1, τ_2) is called (τ_1, τ_2)-locally open (briefly, (τ_1, τ_2)-LO) set if $A = G \cup F$ where G is τ_1-closed and F is τ_2-open in (X, τ_1, τ_2).

Definition 3.2 A subset A of a bitopological space (X, τ_1, τ_2) is called a (τ_1, τ_2)-feebly-locally open (briefly, (τ_1, τ_2)-fLO) set if $A = G \cup F$ where G is τ_1-f-closed and F is τ_2-f-open in (X, τ_1, τ_2).

Definition 3.3 A subset A of a bitopological space (X, τ_1, τ_2) is called a (τ_1, τ_2)-fLO^* (briefly, (τ_1, τ_2)-fLO^*) if there exist a τ_1-f-closed set G and F is τ_2-open set F of (X, τ_1, τ_2) such that $A = G \cup F$.

Definition 3.4 A subset A of a bitopological space (X, τ_1, τ_2) is called a (τ_1, τ_2)-fLO^{**} (briefly, (τ_1, τ_2)-fLO^{**}) if there exist a τ_1-closed set G and τ_2-$f$open set F of (X, τ_1, τ_2) such that $A = G \cup F$.

The collection of all (τ_1, τ_2)-LO(respectively (τ_1, τ_2)-fLO, (τ_1, τ_2)-fLO^*, (τ_1, τ_2)-fLO^{**} sets of (τ_1, τ_2)) will be denoted by (τ_1, τ_2)-$LO(X)$ (respectively (τ_1, τ_2)-$fLO(X)$, (τ_1, τ_2)-$fLO^*(X)$, (τ_1, τ_2)-$fLO^{**}(X)$.)

Theorem 3.1 Let A be a subset of a space (X, τ_1, τ_2). Then if $A \in (\tau_1, \tau_2)$-$LO(X)$

a) $A \in (\tau_1, \tau_2)$-$fLO^*(X)$.

b) $A \in (\tau_1, \tau_2)$-$fLO^{**}(X)$.

Proof. (a) Since $A \in (\tau_1, \tau_2)$-$LO(X)$, there exist a τ_1-closed set G and a τ_2-open set F such that $A = G \cup F$. Since G is τ_1-closed, we have $G \subset scl(int(G))$. Therefore G is τ_1-f-closed. Thus $A = G \cup F$, where G is τ_1-f-closed and F is τ_2-open. Hence $A \in (\tau_1, \tau_2)$-$fLO^*(X)$.

(b) Let $A \in (\tau_1, \tau_2)$-$LO(X)$. Then we have $A = G \cup F$, where G is τ_1-closed and F is τ_2-open. Since F is τ_2-open, we have $sint(cl(F)) \supset F$. Therefore F is τ_2-f-open. Now we have $A = G \cup F$, where G is τ_1-closed and F is τ_2-f-open. Hence $A \in (\tau_1, \tau_2)$-$fLO^{**}(X)$. This completes the proof. □

Remark 3.1 The converse of the Theorem 3.1 is not necessarily true. It is clear from the following example.

Example 3.1 Let $X = \{a, b, c\}$, $\tau_1 = \{X, \phi, \{a\}, \{a, b, \}\}$, $\tau_2 = \{X, \phi, \{a\}, \{a, c\}\}$. Then the subset $\{b\} \in (\tau_1, \tau_2)$-$fLO^*(X)$, but $\{b\} \notin (\tau_1, \tau_2)$-$LO(X)$, and $\{a, b\} \in (\tau_1, \tau_2)$-$fLO^{**}(X)$ but $\{a, b\} \notin (\tau_1, \tau_2)$-$fLO(X)$.

Theorem 3.2. Let A be a subset of the bitopological space (X, τ_1, τ_2). If $A \in (\tau_1, \tau_2)$-$fLO^*(X)$, then $A \in (\tau_1, \tau_2)$-$fLO(X)$.

Proof. Let $A \in (\tau_1, \tau_2)$-f-$LO^*(X)$. Then there exists a τ_1-f-closed set R and a τ_2-open set S such that $A = R \cup S$. Since S is τ_2-open we have $S \subset sint(cl(A))$. Thus S is τ_2-f-open set. Therefore there exist a τ_1-f-closed set R and a τ_2-f-open set S such that $A = R \cup S$. Hence $A \in (\tau_1, \tau_2)$-$fLO(X)$. □

Remark 3.2 The converse of the Theorem 3.2 is not necessarily true. It is clear from the following example.

Example 3.2 Let $X = \{a, b, c\}$, $\tau_1 = \{X, \phi, \{a\}, \{c\}, \{a, c\}, \{b, c\}\}$, $\tau_2 = \{X, \phi, \{a\}\}$. Then the subset $\{a, c\} \in (\tau_1, \tau_2)$-$fLO(X)$ but $\{a, c\} \notin (\tau_1, \tau_2)$-$LO^*(X)$.

Theorem 3.3 Let A be a subset of the bitopological space (X, τ_1, τ_2). If $A \in (\tau_1, \tau_2)$-f-$LO^{**}(X)$ then $A \in (\tau_1, \tau_2)$-$fLO(X)$.

Proof. The Proof is easy. so omitted. □

Remark 3.3 The converse of the Theorem 3.3 is not necessarily true. It follows from the following example.

Example 3.3 Let $X = \{a, b, c\}$, $\tau_1 = \{X, \phi, \{a\}, \{a, b\}\}$, $\tau_2 = \{X, \phi, \{a\}, \{a, c\}\}$. Then the subset $\{b\} \in (\tau_1, \tau_2)$-$fLO(X)$ but $\{b\} \notin (\tau_1, \tau_2)$-$fLO^{**}(X)$.

4 Characterisation

Theorem 4.1 Let R and S be any two subsets of a space (X, τ_1, τ_2). If $R \in (\tau_1, \tau_2)$-$fLO(X)$ and S is τ_1-f-closed and τ_2-f-open then $R \cap S \in (\tau_1, \tau_2)$-$fLO(X)$.

Proof. Since $R \in (\tau_1, \tau_2)$-$fLO(X)$ then there exists a τ_1-f-closed set A and τ_2-f-open set S such that $R = A \cup B$. We have $R \cap S = (A \cup B) \cap S = (A \cap S) \cup (B \cap S)$. Since S is τ_1-f-closed, then $A \cap S$ is τ_1-f-closed. Since S is τ_2-f-open, $B \cap S$ is τ_2-f-open. Then there exist a τ_1-f-closed set $A \cap S$ and a τ_2-f-open set $B \cap S$ such that $R \cap S = (A \cap S) \cup (B \cap S)$. Hence $R \cap S \in (\tau_1, \tau_2)$-$fLO(X)$. □

Theorem 4.2 Let $A \in (\tau_1, \tau_2)$-$fLO^*(X)$ and B be a τ_1-closed and τ_2-open subsets of (X, τ_1, τ_2), then $A \cap B \in (\tau_1, \tau_2)$-$f$-$LO^*(X)$.

Proof. Since $A \in (\tau_1, \tau_2)$-$fLO^*(X)$. Then there exist τ_1-f-closed set R and τ_2-open S such that $A = R \cup S$. We have $A \cap B = (R \cup S) \cap B = (R \cap B) \cup (S \cap B)$. Since B is τ_1-closed, $R \cap B$ is τ_1-f-closed.

Further B is τ_2-open, therefore $S \cap B$ is τ_2-open. Thus there exists a τ_1-f-closed set $R \cap B$ and a τ_2-open set $S \cap B$ such that $A \cap B = (R \cap B) \cup (S \cap B)$. Hence $A \cap B \in (\tau_1, \tau_2)$-$fLO^*(X)$. □

Theorem 4.3 Let $A \in (\tau_1, \tau_2)$-$fLO^{**}(X)$ and B is τ_1-closed and τ_2-open subsets of (X, τ_1, τ_2), then $A \cap B \in (\tau_1, \tau_2)$-$fLO^{**}(X)$.

Proof. Since $A \in (\tau_1, \tau_2)$-$fLO^{**}(X)$. Then there exist τ_1-closed set R and τ_2-f-open S such that $A = R \cup S$. We have $A \cap B = (R \cup S) \cap B = (R \cap B) \cup (S \cap B)$. Since B is τ_1-closed, $R \cap B$ is τ_1-closed.
Again B is τ_2-open, therefore $S \cap B$ is τ_2-f-open. Then there exists a τ_1-closed set $R \cap B$ and a τ_2-f-open set $S \cap B$ such that $A \cap B = (R \cap B) \cup (S \cap B)$. Hence $A \cap B \in (\tau_1, \tau_2)$-$fLO^{**}(X)$. □

Theorem 4.4 Let A be a subset of a space (X, τ_1, τ_2). Then $A \in (\tau_1, \tau_2)$-$fLO^*(X)$ if and only if $A = G \cup \tau_2$-$int(A)$ for some τ_1-f-closed set G.

Proof. Let $A \in (\tau_1, \tau_2)$-$fLO^*(X)$. Then $A = G \cup F$,where G is τ_1-f-closed and F is τ_2-open set in (X, τ_1, τ_2). Since $G \subset A$ and $\tau_2 - int(A) \subset A$.

$$G \cup \tau_2 - int(A) \subset A \tag{4.1}$$

Further $\tau_2 - int(A) \supset F$. Therefore

$$G \cup \tau_2 - int(A) \supset G \cup F = A \tag{4.2}$$

From (4.1) and (4.2) we have$A = G \cup \tau_2 - int(A)$. Conversely given that G is τ_1-f-closed. We have $\tau_2 - int(A)$ is τ_2-open. Thus there exist a τ_1-f-closed set G and a τ_2-open set in (X, τ_1, τ_2) such that $A = G \cup \tau_2$-$int(A)$. Hence $A \in (\tau_1, \tau_2)$-$fLO^*(X)$. □

Theorem 4.5 Let A and B be any two subsets of the bitopological space (X, τ_1, τ_2). If $A \in (\tau_1, \tau_2)$-$fLO(X)$ and B is either τ_1-f-closed or τ_2-f-open, then $A \cup B \in (\tau_1, \tau_2)$-$fLO(X)$.

Proof. Since $A \in (\tau_1, \tau_2)$-$fLO(X)$, then there exists a τ_1-f-closed R and a τ_2-f-open set S such that $A = R \cup S$. We have $A \cup B = (R \cup S) \cup B = (R \cup B) \cup S)$. If B is τ_1-f-closed, then $R \cup B$ is also τ_1-f-closed. Hence $A \cup B \in (\tau_1, \tau_2)$-$fLO(X)$.

Let B be τ_2-f-open, then $A \cup B = (R \cup S) \cup B = R \cup (S \cup B)$, where $S \cup B$ is τ_2-f-open. Thus $A \cup B \in (\tau_1, \tau_2)$-$fLO(X)$. □

Theorem 4.6 If $A \in (\tau_1, \tau_2)$-$fLO^*(X)$ and B is either τ_1-closed or τ_2-open subset of (X, τ_1, τ_2) then $A \cup B \in (\tau_1, \tau_2)$-$fLO^*(X)$.

Proof. Since $A \in (\tau_1, \tau_2)$-$fLO^*(X)$, then $A = R \cup S$ where R is τ_1-f-closed set and S is τ_2-open set of (X, τ_1, τ_2). Now $A \cup B = (R \cup S) \cup B = (R \cup B) \cup S$. If B is τ_1-closed, then $R \cup B$ is also τ_1-f-closed, where R is τ_1-f-closed set. Hence $A \cup B \in (\tau_1, \tau_2)$-$fLO^*(X)$.

If B is τ_2-open, then $S \cup B$ is τ_2-open. Now $A \cup B = (R \cup S) \cup B = R \cup (S \cup B)$. Thus $A \cup B \in (\tau_1, \tau_2)$-$fLO^*(X)$. □

Theorem 4.7 If $A \in (\tau_1, \tau_2)$-$fLO^{**}(X)$ and B is either τ_1-closed or τ_2-open subset of (X, τ_1, τ_2) then $A \cup B \in$

(τ_1, τ_2)-$fLO^{**}(X)$.

Proof. The proof is easy , so omitted. □

Theorem 4.8 If $A, B \in (\tau_1, \tau_2)$-$fLO(X)$ then $A \cup B \in (\tau_1, \tau_2)$-$fLO(X)$.

Proof. Let $A, B \in (\tau_1, \tau_2)$-$fLO(X)$. Then there exists τ_1-f-closed sets R,T and τ_2-f-open sets S,U such that $A = R \cup S$ and $B = T \cup U$. We have $A \cup B = (R \cup S) \cup (T \cup U) = (R \cup T) \cup (S \cup U)$, Where $(R \cup T)$ is τ_1-f-closed and $(S \cup U)$ is τ_2-f-open. Hence $A \cup B \in (\tau_1, \tau_2)$-$fLO(X)$. □

Theorem 4.9 If $A, B \in (\tau_1, \tau_2)$-$fLO^*(X)$ then $A \cup B \in (\tau_1, \tau_2)$-$fLO^*(X)$.

Proof. Since $A, B \in (\tau_1, \tau_2)$-$fLO^*(X)$, then by Theorem 4.4 there exists τ_1-f-closed sets R and S such that $A = R \cup \tau_2 - int(A)$ and $B = S \cup \tau_2 - int(B)$. We have
$A \cup B = [R \cup \tau_2 - int(A)] \cup [(S \cup \tau_2 - int(B)]$
=$(R \cup S) \cup (\tau_2 - int(A) \cup \tau_2 - int(B))$, Where $(R \cup T)$ is τ_1-f-closed and τ_2-$int(A) \cup \tau_2 - int(B)$ is τ_2-open set. Hence $A \cup B \in (\tau_1, \tau_2)$-$fLO^*(X)$. □

Acknowledgements: The author wish to thank the referees for useful comments and suggestions, which will help to improve the paper.

References

[1] N. Bourbaki, General Topology, Part I reading MA: Addison Wesley, 1996.

[2] S. G. Crossely and S. K. Hildebrand, Semi-topological properties, Fund. Math., 74(1972), 235-254.

[3] J. C. Kelly, Bitopological spaces, Proc. London Math. Soc., (3), 13(1962), 71-89.

[4] C. Kuratowski and W. Sierpinski, Surles differences deux ensembles fermes, Tohoku Math. J., 20(1921), 22-25.

[5] N. Levine, Semi-open sets and Semi-continuity in topological spaces, Amer. Math. Monthly, 70(1963), 36-41.

[6] Maheswari S.N. and Jain P.C., Some new mappings, Mathematica 24(47) (1-2): 53 55, 1982.

[7] M. Ganster, and I. L. Reilly, Locally Closed Sets and LC-Continuous Functions, Int. J. Math. Sci., 12,(1989), 417-424.