

## Eccentricity Version of Atom-Bond Connectivity Index of $NA_m^n$ Nanotube

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ABSTRACT. The eccentricity of a vertex  $v$  is the distance between  $v$  and a vertex farthest from  $v$  in a graph  $G$ . In this paper, we compute the eccentricity version of the atom-bond connectivity index of a  $NA_m^n$  nanotube.

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## 1 Introduction

Graph theory has provided variety of useful tools, such as topological indices to chemists. Molecules and molecular compounds are often modeled by molecular graph. A molecular graph is a representation of the structural formula of a chemical compound in terms of graph, whose vertices correspond to the atoms of the compound and edges correspond to chemical bonds [13]. Cheminformatics is new subject which is a combination of chemistry, mathematics and information science. It studies Quantitative structure-activity relationships (QSAR) and Quantitative structure-property relationships (QSPR) that are used to predict the biological activities and properties of chemical compounds.

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Received October 15, 2017; revised December 04, 2017; accepted December 09, 2017.

2010 Mathematics Subject Classification: 05C12, 05C90.

Key words and phrases: Molecular graph, eccentricity, atom-bond connectivity index, nanotube.

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A graph can be recognized by a numeric number, a polynomial, a sequence of numbers or a matrix. A topological index is a numeric quantity associated with a graph which characterizes the topology of a graph and is invariant under graph automorphism. There are some major classes of topological indices such as distance based topological indices, eccentricity based topological indices, degree based topological indices and counting related polynomials and indices of graphs. In more precise way, a topological index  $Top(G)$  of a graph  $G$ , is a number with the property that for every graph  $H$  isomorphic to  $G$ ,  $Top(H) = Top(G)$  [3].

Let  $G$  be a molecular graph with vertex set  $V(G)$  and edge set  $E(G)$ . The vertices of  $G$  denote atoms and an edge between two vertices denotes the chemical bond between these vertices. The edge between two vertices  $u$  and  $v$  is denoted by  $uv$ . If no vertices in  $u$ - $v$  walk are repeated then it is called  $u$ - $v$  path in graph  $G$ . The length of a path is the number of edges in it. The distance between two vertices  $u$  and  $v$ , denoted by  $d(u, v)$ , is the length of a shortest  $u - v$  path in a graph  $G$ . In a connected graph  $G$ , the eccentricity of a vertex  $v$ , denoted by  $\varepsilon_v$ , is the distance between  $v$  and a vertex farthest from  $v$  in  $G$ .

The concept of topological indices came from Harold Wiener [16] while he was working on boiling point of paraffin, named this index as *path number*. Later on, the path number was renamed as *Wiener index*, defined as half of the sum of distances between all ordered pairs of vertices in a graph [16].

That is the Wiener index  $W(G)$  of a connected graph  $G$  is defined as

$$W(G) = \frac{1}{2} \sum_{(u,v)} d(u, v), \tag{1.1}$$

where  $(u, v)$  is any ordered pair of vertices in  $G$  and  $d(u, v)$  is the distance between the vertices  $u$  and  $v$ .

Among the topological indices, the connectivity indices are very important and they have a prominent role in chemistry. For a connected graph  $G$  the *atom-bond connectivity index* is defined by Estrada et al. [4] as

$$ABC(G) = \sum_{uv \in E(G)} \sqrt{\frac{d_u + d_v - 2}{d_u d_v}}, \tag{1.2}$$

where  $d_u$  and  $d_v$  denotes the degrees of the vertices  $u$  and  $v$  respectively. Other results on atom-bond connectivity index can be seen in [14, 15].

A new version of atom-bond connectivity index called *second atom-bond connectivity index* is introduced by Graovac and Ghorbani [11]. The second atom-bond connectivity index is defined as

$$ABC_2(G) = \sum_{uv \in E(G)} \sqrt{\frac{n_u + n_v - 2}{n_u n_v}}, \tag{1.3}$$

where  $n_u$  is the number of vertices closer to vertex  $u$  than vertex  $v$  and  $n_v$  is the number of vertices closer to vertex  $v$  than vertex  $u$ .

The *eccentricity version of atom-bond connectivity index* of a graph  $G$ , denoted by  $ABC_5(G)$ , is defined as [5]

$$ABC_5(G) = \sum_{uv \in E(G)} \sqrt{\frac{\varepsilon_u + \varepsilon_v - 2}{\varepsilon_u \varepsilon_v}}, \tag{1.4}$$

where  $\varepsilon_u$  and  $\varepsilon_v$  denotes the eccentricities of the vertices  $u$  and  $v$  respectively.

The topological indices based on eccentricity are considered in [12, 17]. The eccentricity based topological indices of nanostar dendrimers have been reported in [1, 8]. Second atom-bond connectivity index of special

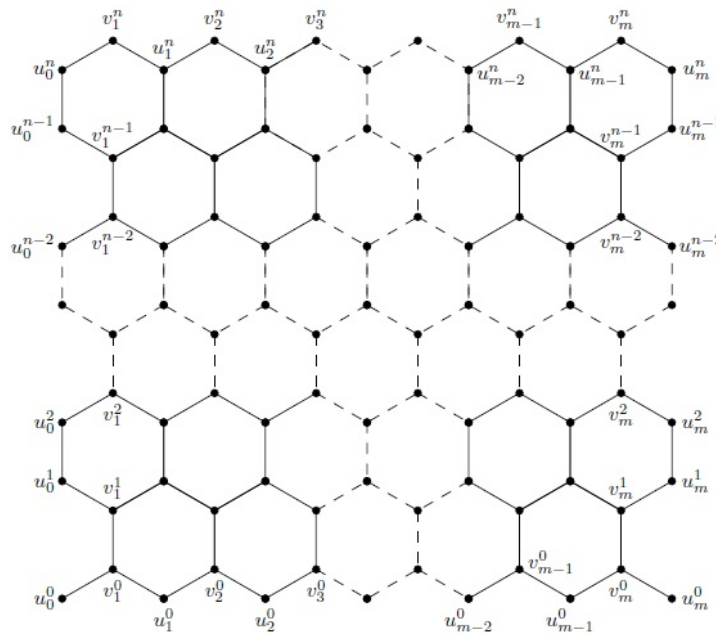


Figure 1:  $NA_m^n$  nanotube.

chemical molecular structures have been obtained in [9]. Fourth atom-bond connectivity index of nanostar dendrimers was considered in [7]. Edge version of atom-bond connectivity index was studied in [6]. Eccentricity version of atom-bond connectivity index of linear polycene parallelogram benzenoid was studied in [10]. The aim of this paper is to compute the eccentricity version of atom-bond connectivity index of  $NA_m^n$  nanotube.

## 2 $NA_m^n$ Nanotube

We consider the  $m \times n$  quadrilateral section  $P_m^n$  with  $m \geq 2$  hexagons on the top and bottom sides and  $n \geq 2$  hexagons on the lateral sides cut from the regular hexagonal lattice  $L$  as shown in Figure 1. If we identify two lateral sides of  $P_m^n$  such that we identify the vertices  $u_0^j$  and  $u_m^j$ , for  $j = 0, 1, 2, \dots, n$  then we obtain the nanotube  $NA_m^n$  [2].

### 3 Eccentricity version of atom-bond connectivity index of $NA_m^n$ nanotube

We consider here  $NA_m^n$  nanotube with  $n = m$  and compute its eccentricity version of atom-bond connectivity index with three cases of  $n$  when  $n \equiv 0(mod2)$ ,  $n \equiv 1(mod4)$  and  $n \equiv 3(mod4)$ .

**Theorem 3.1** For every  $n \equiv 0(mod2)$ , consider the graph of  $G \cong NA_m^n$  nanotube. Then the eccentricity version of atom-bond connectivity index of  $G$  is equal to

$$\begin{aligned}
 ABC_5(G) = & \sum_{i=1}^{\frac{n}{2}} \sum_{p=2n-\frac{n}{2}}^{2n-1} (6i-3) \sqrt{\frac{2p-1}{p^2+p}} + 3n \sqrt{\frac{4p-1}{4p^2+2p}} + 2 \sqrt{\frac{4p}{4p^2+4p+1}} \\
 & + \sum_{i=\frac{n}{2}+3}^{3n-\frac{n}{2}-1} \sum_{p=2n+1}^{\frac{n}{2}-1} \left(\frac{n}{2}-1\right) (6i-16) \sqrt{\frac{2p-1}{p^2+p}} + \sum_{i=3n+6j, 0 \leq j \leq \frac{n}{2}-1} \sum_{p=3n-\frac{n}{2}}^{3n-1} (6n-i) \sqrt{\frac{2p-1}{p^2+p}}.
 \end{aligned}$$

*Proof.* By using the values from Table 1 in Eq. (1.4) we get

$$\begin{aligned}
 ABC_5(G) = & \sum_{i=1}^{\frac{n}{2}} (6i-3) \times \sum_{p=2n-\frac{n}{2}}^{2n-1} \sqrt{\frac{p+p+1-2}{(p)(p+1)}} + 3n \times \sqrt{\frac{2p+2p+1-2}{(2p)(2p+1)}} + 2 \times \sqrt{\frac{2p+1+2p+1-2}{(2p+1)(2p+1)}} \\
 & + \sum_{i=\frac{n}{2}+3} \left(\frac{n}{2}-1\right) \times (6i-16) \times \sum_{p=2n+1}^{3n-\frac{n}{2}-1} \sqrt{\frac{p+p+1-2}{(p)(p+1)}} \\
 & + \sum_{i=3n+6j, 0 \leq j \leq \frac{n}{2}-1} (6n-i) \times \sum_{p=3n-\frac{n}{2}}^{3n-1} \sqrt{\frac{p+p+1-2}{(p)(p+1)}}.
 \end{aligned}$$

By simplifying, we get

$$\begin{aligned}
 ABC_5(G) = & \sum_{i=1}^{\frac{n}{2}} \sum_{p=2n-\frac{n}{2}}^{2n-1} (6i-3) \sqrt{\frac{2p-1}{p^2+p}} + 3n \sqrt{\frac{4p-1}{4p^2+2p}} + 2 \sqrt{\frac{4p}{4p^2+4p+1}} \\
 & + \sum_{i=\frac{n}{2}+3}^{3n-\frac{n}{2}-1} \sum_{p=2n+1}^{\frac{n}{2}-1} \left(\frac{n}{2}-1\right) (6i-16) \sqrt{\frac{2p-1}{p^2+p}} + \sum_{i=3n+6j, 0 \leq j \leq \frac{n}{2}-1} \sum_{p=3n-\frac{n}{2}}^{3n-1} (6n-i) \sqrt{\frac{2p-1}{p^2+p}}. \quad \square
 \end{aligned}$$

**Theorem 3.2** For every  $n \equiv 1(mod4)$ , consider the graph of  $G \cong NA_m^n$  nanotube. Then the eccentricity version of atom-bond connectivity index of  $G$  is equal to

$$\begin{aligned}
 ABC_5(G) = & \sum_{p=\frac{3n+1}{2}} \sqrt{\frac{2p-2}{p^2}} + \sum_{i=(\frac{n-1}{2}) \times 2} \sum_{p=\frac{3n+1}{2}}^{\frac{5n-1}{2}, p \equiv 0 \pmod{2}} i \sqrt{\frac{2p-2}{p^2}} + \sum_{i=1}^{\frac{n-3}{2}, i \equiv 1 \pmod{2}} \sum_{p=\frac{3n+1}{2}}^{2n-2, p \equiv 0 \pmod{2}} 4i \sqrt{\frac{2p-1}{p^2+p}} \\
 & + \sum_{i=2}^{\frac{n-1}{2}, i \equiv 1 \pmod{2}} \sum_{p=\frac{3n+3}{2}}^{2n-1, p \equiv 1 \pmod{2}} (8i-2) \sqrt{\frac{2p-1}{p^2+p}} + \sum_{i=2n+2} \sum_{p=2n}^{\frac{5n-1}{2}, p \equiv 0 \pmod{2}} \left(\frac{n+3}{4}\right) \times i \sqrt{\frac{2p-1}{p^2+p}} \\
 & + \sum_{i=4n} \sum_{p=2n+1}^{\frac{5n-3}{2}, p \equiv 1 \pmod{2}} \left(\frac{n-1}{4}\right) \times i \sqrt{\frac{2p-1}{p^2+p}} + \sum_{i=\frac{n-1}{4}}^1 \sum_{p=\frac{5n+1}{2}}^{3n-2, p \equiv 1 \pmod{2}} 16i \sqrt{\frac{2p-1}{p^2+p}} \\
 & + \sum_{i=\frac{n-3}{2}}^{1, i \equiv 1 \pmod{2}} \sum_{p=\frac{5n+3}{2}}^{3n-1, p \equiv 0 \pmod{2}} 4i \sqrt{\frac{2p-1}{p^2+p}}.
 \end{aligned}$$

Proof. By using the values from Table 2 in Eq. (1.4) we get

$$\begin{aligned}
 ABC_5(G) = & 1 \times \sum_{p=\frac{3n+1}{2}} \sqrt{\frac{p+p-2}{(p)(p)}} + \sum_{i=(\frac{n-1}{2}) \times 2} i \times \sum_{p=\frac{3n+1}{2}}^{\frac{5n-1}{2}, p \equiv 0 \pmod{2}} \sqrt{\frac{p+p-2}{(p)(p)}} \\
 & + \sum_{i=1}^{\frac{n-3}{2}, i \equiv 1 \pmod{2}} 4i \times \sum_{p=\frac{3n+1}{2}}^{2n-2, p \equiv 0 \pmod{2}} \sqrt{\frac{p+p+1-2}{(p)(p+1)}} \\
 & + \sum_{i=2}^{\frac{n-1}{2}, i \equiv 1 \pmod{2}} (8i-2) \times \sum_{p=\frac{3n+3}{2}}^{2n-1, p \equiv 1 \pmod{2}} \sqrt{\frac{p+p+1-2}{(p)(p+1)}} \\
 & + \sum_{i=2n+2} \left(\frac{n+3}{4}\right) \times i \times \sum_{p=2n}^{\frac{5n-1}{2}, p \equiv 0 \pmod{2}} \sqrt{\frac{p+p+1-2}{(p)(p+1)}} \\
 & + \sum_{i=4n} \left(\frac{n-1}{4}\right) \times i \times \sum_{p=2n+1}^{\frac{5n-3}{2}, p \equiv 1 \pmod{2}} \sqrt{\frac{p+p+1-2}{(p)(p+1)}} \\
 & + \sum_{i=\frac{n-1}{4}}^1 16i \times \sum_{p=\frac{5n+1}{2}}^{3n-2, p \equiv 1 \pmod{2}} \sqrt{\frac{p+p+1-2}{(p)(p+1)}} + \sum_{i=\frac{n-3}{2}}^{1, i \equiv 1 \pmod{2}} 4i \times \sum_{p=\frac{5n+3}{2}}^{3n-1, p \equiv 0 \pmod{2}} \sqrt{\frac{p+p+1-2}{(p)(p+1)}}.
 \end{aligned}$$

By simplifying, we get

$$\begin{aligned}
 ABC_5(G) = & \sum_{p=\frac{3n+1}{2}} \sqrt{\frac{2p-2}{p^2}} + \sum_{i=(\frac{n-1}{2}) \times 2} \sum_{p=\frac{3n+1}{2}}^{\frac{5n-1}{2}, p \equiv 0 \pmod{2}} i \sqrt{\frac{2p-2}{p^2}} + \sum_{i=1}^{\frac{n-3}{2}, i \equiv 1 \pmod{2}} \sum_{p=\frac{3n+1}{2}}^{2n-2, p \equiv 0 \pmod{2}} 4i \sqrt{\frac{2p-1}{p^2+p}} \\
 & + \sum_{i=2}^{\frac{n-1}{2}, i \equiv 1 \pmod{2}} \sum_{p=\frac{3n+3}{2}}^{2n-1, p \equiv 1 \pmod{2}} (8i-2) \sqrt{\frac{2p-1}{p^2+p}} + \sum_{i=2n+2} \sum_{p=2n}^{\frac{5n-1}{2}, p \equiv 0 \pmod{2}} \left(\frac{n+3}{4}\right) \times ii \sqrt{\frac{2p-1}{p^2+p}} \\
 & + \sum_{i=4n} \sum_{p=2n+1}^{\frac{5n-3}{2}, p \equiv 1 \pmod{2}} \left(\frac{n-1}{4}\right) \sqrt{\frac{2p-1}{p^2+p}} + \sum_{i=\frac{n-1}{4}}^1 \sum_{p=\frac{5n+1}{2}}^{3n-2, p \equiv 1 \pmod{2}} 16i \sqrt{\frac{2p-1}{p^2+p}} \\
 & + \sum_{i=\frac{n-3}{2}}^{1, i \equiv 1 \pmod{2}} \sum_{p=\frac{5n+3}{2}}^{3n-1, p \equiv 0 \pmod{2}} 4i \sqrt{\frac{2p-1}{p^2+p}}. \quad \square
 \end{aligned}$$

**Theorem 3.3** For every  $n \equiv 3 \pmod{4}$ , consider the graph of  $G \cong NA_m^n$  nanotube. Then the eccentricity version of atom-bond connectivity index of  $G$  is equal to

$$\begin{aligned}
 ABC_5(G) = & \sum_{i=(\frac{n+1}{2}) \times 2}^{\frac{5n+1}{2}, p \equiv 0 \pmod{2}} \sum_{p=\frac{3n+3}{2}} i \sqrt{\frac{2p-2}{p^2}} + \sum_{i=1}^{\frac{n+1}{4}} \sum_{p=\frac{3n+1}{2}}^{2n-1, p \equiv 1 \pmod{2}} (16i-10) \sqrt{\frac{2p-1}{p^2+p}} \\
 & + \sum_{i=1}^{\frac{n+1}{4}} \sum_{p=\frac{3n+3}{2}}^{2n, p \equiv 0 \pmod{2}} 8i \sqrt{\frac{2p-1}{p^2+p}} + \sum_{i=4n}^{\frac{5n-1}{2}, p \equiv 1 \pmod{2}} \sum_{p=2n+1} \left(\frac{n+1}{4}\right) \times i \sqrt{\frac{2p-1}{p^2+p}} \\
 & + \sum_{i=2n+2, n \neq 3}^{\frac{5n-3}{2}, n \neq 3, p \equiv 0 \pmod{2}} \sum_{p=2n+2} \left(\frac{n-3}{4}\right) \times i \sqrt{\frac{2p-1}{p^2+p}} \\
 & + \sum_{i=1}^{\frac{n-3}{4}, n \neq 3} \sum_{p=\frac{3n+3}{2}}^{3n-2, n \neq 3, p \equiv 1 \pmod{2}} 16i \sqrt{\frac{2p-1}{p^2+p}} + \sum_{i=\frac{n+1}{4}}^1 \sum_{p=\frac{3n+1}{2}}^{3n-1, p \equiv 0 \pmod{2}} (8i-4) \sqrt{\frac{2p-1}{p^2+p}}.
 \end{aligned}$$

*Proof.* By using the values from Table 3 in Eq. (1.4) we get

$$\begin{aligned}
 ABC_5(G) = & \sum_{i=(\frac{n+1}{2}) \times 2} i \times \sum_{p=\frac{3n+3}{2}}^{\frac{5n+1}{2}, p \equiv 0 \pmod{2}} \sqrt{\frac{p+p-2}{(p)(p)}} + \sum_{i=1}^{\frac{n+1}{4}} (16i-10) \times \sum_{p=\frac{3n+1}{2}}^{2n-1, p \equiv 1 \pmod{2}} \sqrt{\frac{p+p+1-2}{(p)(p+1)}} \\
 & + \sum_{i=1}^{\frac{n+1}{4}} 8i \times \sum_{p=\frac{3n+3}{2}}^{2n, p \equiv 0 \pmod{2}} \sqrt{\frac{p+p+1-2}{(p)(p+1)}} + \sum_{i=4n} \left(\frac{n+1}{4}\right) \times i \times \sum_{p=2n+1}^{\frac{5n-1}{2}, p \equiv 1 \pmod{2}} \sqrt{\frac{p+p+1-2}{(p)(p+1)}} \\
 & + \sum_{i=2n+2, n \neq 3} \left(\frac{n-3}{4}\right) \times i \times \sum_{p=2n+2}^{\frac{5n-3}{2}, n \neq 3, p \equiv 0 \pmod{2}} \sqrt{\frac{p+p+1-2}{(p)(p+1)}} \\
 & + \sum_{i=1}^{\frac{n-3}{4}, n \neq 3} 16i \times \sum_{p=\frac{3n+3}{2}}^{3n-2, n \neq 3, p \equiv 1 \pmod{2}} \sqrt{\frac{p+p+1-2}{(p)(p+1)}} + \sum_{i=\frac{n+1}{4}}^1 (8i-4) \times \sum_{p=\frac{3n+1}{2}}^{3n-1, p \equiv 0 \pmod{2}} \sqrt{\frac{p+p+1-2}{(p)(p+1)}}.
 \end{aligned}$$

By simplifying, we get

$$\begin{aligned}
 ABC_5(G) = & \sum_{i=(\frac{n+1}{2}) \times 2}^{\frac{5n+1}{2}, p \equiv 0 \pmod{2}} \sum_{p=\frac{3n+3}{2}} i \sqrt{\frac{2p-2}{p^2}} + \sum_{i=1}^{\frac{n+1}{4}} \sum_{p=\frac{3n+1}{2}}^{2n-1, p \equiv 1 \pmod{2}} (16i-10) \sqrt{\frac{2p-1}{p^2+p}} \\
 & + \sum_{i=1}^{\frac{n+1}{4}} \sum_{p=\frac{3n+3}{2}}^{2n, p \equiv 0 \pmod{2}} 8i \sqrt{\frac{2p-1}{p^2+p}} + \sum_{i=4n}^{\frac{5n-1}{2}, p \equiv 1 \pmod{2}} \sum_{p=2n+1} \left(\frac{n+1}{4}\right) \times i \sqrt{\frac{2p-1}{p^2+p}} \\
 & + \sum_{i=2n+2, n \neq 3}^{\frac{5n-3}{2}, n \neq 3, p \equiv 0 \pmod{2}} \sum_{p=2n+2} \left(\frac{n-3}{4}\right) i \sqrt{\frac{2p-1}{p^2+p}} + \sum_{i=1}^{\frac{n-3}{4}, n \neq 3} \sum_{p=\frac{3n+3}{2}}^{3n-2, n \neq 3, p \equiv 1 \pmod{2}} 16i \sqrt{\frac{2p-1}{p^2+p}} \\
 & + \sum_{i=\frac{n+1}{4}}^1 \sum_{p=\frac{3n+1}{2}}^{3n-1, p \equiv 0 \pmod{2}} (8i-4) \sqrt{\frac{2p-1}{p^2+p}}. \quad \square
 \end{aligned}$$

Table 1: The edge partition of  $NA_m^n$  nanotube based on eccentricity of end vertices of each edge and their frequency of occurrence. Here  $n \equiv 0(mod2)$ .

$(\epsilon_u, \epsilon_v)$	Range	Frequency	Range
$(p, p + 1)$	$2n - \frac{n}{2} \leq p \leq 2n - 1$	$3 + 6(i - 1)$	$1 \leq i \leq \frac{n}{2}$
$(2p, 2p + 1)$	$p = n \equiv 0(mod2)$	$3n$	$n \equiv 0(mod2)$
$(2p + 1, 2p + 1)$	$p = n \equiv 0(mod2)$	$2$	$i = \frac{n}{2} + 2$
$(p, p + 1)$	$2n + 1 \leq p \leq 3n - \frac{n}{2} - 1$	$(\frac{n}{2} - 1) [2 + 6(i - 3)]$	$i = \frac{n}{2} + 3$
$(p, p + 1)$	$3n - \frac{n}{2} \leq p \leq 3n - 1$	$6n - i$	$i = 3n + 6j,$ where $0 \leq j \leq \frac{n}{2} - 1.$

Table 2: The edge partition of  $NA_m^n$  nanotube based on eccentricity of end vertices of each edge and their frequency of occurrence. Here  $n \equiv 1(mod4)$ .

$(\epsilon_u, \epsilon_v)$	Range	Frequency	Range
$(p, p)$	$p = \frac{3n+1}{2}$	$1$	
$(p, p)$	$\frac{3n+1}{2} < p \leq \frac{5n-1}{2}, p \equiv 0(mod2)$	$i$	$i = (\frac{n-1}{2})2$
$(p, p + 1)$	$\frac{3n+1}{2} \leq p \leq 2n - 2, p \equiv 0(mod2)$	$4i$	$1 \leq i \leq \frac{n-3}{2}, i \equiv 1(mod2)$
$(p, p + 1)$	$\frac{3n+3}{2} \leq p \leq 2n - 1, p \equiv 0(mod2)$	$8i - 2$	$2 \leq i \leq \frac{n-1}{2}, i \equiv 0(mod2)$
$(p, p + 1)$	$2n \leq p \leq \frac{5n-1}{2}, p \equiv 0(mod2)$	$i = 2n + 2$	$(\frac{n+3}{4})i$
$(p, p + 1)$	$2n + 1 \leq p \leq \frac{5n-3}{2}, p \equiv 1(mod2)$	$i = 4n$	$(\frac{n-1}{4})i$
$(p, p + 1)$	$\frac{5n+1}{2} \leq p \leq 3n - 2, p \equiv 1(mod2)$	$16i$	$\frac{n-1}{4} \leq i \leq 1$
$(p, p + 1)$	$\frac{5n+3}{2} \leq p \leq 3n - 1, p \equiv 0(mod2)$	$4i$	$\frac{n-3}{2} \leq i \leq 1, i \equiv 1(mod2)$

Table 3: The edge partition of  $NA_m^n$  nanotube based on eccentricity of end vertices of each edge and their frequency of occurrence. Here  $n \equiv 3(mod4)$ .

$(\epsilon_u, \epsilon_v)$	Range	Frequency	Range
$(p, p)$	$\frac{3n+3}{2} \leq p \leq \frac{5n+1}{2}$	$i$	$i = (\frac{n+1}{2})2$
$(p, p + 1)$	$\frac{3n+1}{2} \leq p \leq 2n - 1, p \equiv 1(mod2)$	$16i - 10$	$1 \leq i \leq \frac{n+1}{4}$
$(p, p + 1)$	$\frac{3n+3}{2} \leq p \leq 2n, p \equiv 0(mod2)$	$8i$	$1 \leq i \leq \frac{n+1}{4}$
$(p, p + 1)$	$2n + 1 \leq p \leq \frac{5n-1}{2}, p \equiv 1(mod2)$	$i = 4n$	$(\frac{n+1}{4})i$
$(p, p + 1)$	$2n + 2 \leq p \leq \frac{5n-3}{2}, p \equiv 0(mod2), n \neq 3$	$i = 2n + 2, n \neq 3$	$(\frac{n-3}{4})i, n \neq 3$
$(p, p + 1)$	$\frac{5n+3}{2} \leq p \leq 3n - 2, p \equiv 1(mod2), n \neq 3$	$16i$	$\frac{n-3}{4} \leq i \leq 1, n \neq 3$
$(p, p + 1)$	$\frac{5n+1}{2} \leq p \leq 3n - 1, p \equiv 0(mod2)$	$8i - 4$	$\frac{n+1}{4} \leq i \leq 1$

## 4 Conclusion

In this paper, we have considered the molecular graph of  $NA_m^n$  nanotube and computed the eccentricity version of atom-bond connectivity index of  $NA_m^n$  nanotube.

## References

- [1] A. R. Ashrafi, M. Saheli, *The eccentric connectivity index of a new class of nanostar dendrimers*, Optoelectron. Adv. Mater. Rapid Comm., **4**(6) (2010), 898–899.
- [2] M. Baca, J. Horvathova, M. Mokrisova, A. Semanicova, A. Suhanyiova, *On topological indices of carbonnanotube network*, Can. J. Chem., **93**(10) (2015), 1157–1160.
- [3] M. Deza, P. W. Fowler, A. Rassat, K. M. Rogers, *Fullerenes as tiling of surfaces*, J. Chem. Inf. Comput. Sci., **40** (2000), 550–558.
- [4] E. Estrada, L. Torres, L. Rodríguez, I. Gutman, *An atom-bond connectivity index: Modelling the enthalpy of formation of alkanes*, Indian J. Chem., **37A** (1998), 849–855.
- [5] M. R. Farahani, *Eccentricity version of atom-bond connectivity index of benzenoid family  $ABC_5(H_k)$* , World Appl. Sci. J., **21**(9) (2013), 1260–1265.
- [6] M. R. Farahani, *The edge version of atom bond connectivity index of connected graph*, Acta Univ. Apulensis, **36** (2013), 277–284.
- [7] M. R. Farahani, *Fourth atom-bond connectivity index of an infinite class of nanostar dendrimer  $D_3[n]$* , J. Adv. Chem., **4**(1) (2013), 301–305.
- [8] R. Farooq, M. Ali Malik, *Some eccentricity based topological indices of nanostar dendrimers*, Optoelectron. Adv. Mater. Rapid Comm., **9**(5-6) (2015), 842–849.
- [9] W. Gao, W. Wang, *Second atom-bond connectivity index of special chemical molecular structures*, J. Chem., **2014**, Article ID 906254, (2014), 8 pages.
- [10] W. Gao, M. R. Farahani, M. K. Jamil, *The eccentricity version of atom-bond connectivity index of linear polycene parallelogram benzenoid  $ABC_5(P(n, n))$* , Acta Chim. Slov., **63** (2016), 376–379.
- [11] A. Graovac, M. Ghorbani, *A new version of atom-bond connectivity index*, Acta Chimica Slovenica, **57**(3) (2010), 609–612.
- [12] S. Gupta, M. Singh, A. K. Madan, *Application of graph theory: Relationship of eccentric connectivity index and Wiener's index with anti-inflammatory activity*, J. Math. Anal. Appl., **266** (2002), 259–268.
- [13] I. Gutman, O. E. Polansky, *Mathematical Concepts in Organic Chemistry*, Springer-Verlag, New York, 1986.
- [14] N. M. Husin, R. Hasni, N. E. Arif, *Atom-bond connectivity and geometric arithmetic indices of dendrimer nanostars*, Australian J. Basic Appl. Sci., **7**(9) (2013), 10–14.
- [15] J. L. Palacios, *A resistive upper bound for the ABC index*, MATCH Commun. Math. Comput. Chem., **72** (2014), 709–713.
- [16] H. Wiener, *Structural determination of paraffin boiling points*, J. Amer. Chem. Soc., **69** (1947), 17–20.
- [17] B. Zhou, Z. Du, *On eccentric connectivity index*, MATCH Commun. Math. Comput. Chem., **63** (2010), 181–198.