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# Common Fixed Point Theorems for F-Contractions on Generalized Metric Spaces

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ABSTRACT. In this paper we establish two common fixed point theorems for F-Contractions on generalized metric spaces and provide supporting example. An open problem also given.

## 1 Introduction

Metric spaces are very important in Mathematical Science. Many generalization of metric spaces have been introduced by Frechet[1] which is an extension of the distance in real line and were studied with reference to fixed point theory. The concept of F-contraction was introduced by Wardowski[2] and proved a new fixed point theorem about F-contraction,Kumari[14] gives the generalized notion of F-contraction in the view of d-metric spaces. Jleli and Samet[3] obtained a new generalization and they termed it as a generalized metric space as they extended many results available in fixed point theory. Sastry[6] gave the notion of generalized metric space and some the authors have tried to give generalization of metric space in several ways. Gahler and Dhage introduced the concept of 2-metric space and D-metric spaces respectively. Sedghi[15] introduce S-metric space and also proved a fixed point theorem for a self mapping on a complete S-metric space. Mustafa and Sims[16] introduced a new structure of generalized metric spaces which are called G-metric space as a generalization of metric spaces. In this paper we establish two common fixed point theorems for F-Contractions on generalized metric spaces.

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# 2 Main Results

We begin with some known definitions.

**Definition 2.1.** (*Frechet* [1]): A metric on a non empty set $X$ is a function $D : X \times X \to [0, \infty]$ such that for all $x, y, z \in X$ the following conditions are hold:

$(i)$ $D(x, y) = 0 \Longleftrightarrow x = y$;

$(ii)$ $D(x, y) = D(y, x)$;

$(iii)$ $D(x, y) \leq D(x, z) + D(z, y)$. The pair $(X, D)$ is called metric space.

**Definition 2.2.** (*Wardowski* [2]):Let $D(X, D)$ be a generalized metric space. A mapping $T : X \to X$ is said to be a $F-$ contraction if there exist $\tau > 0$ for all $x, y \in X$, $D(Tx, Ty) > 0 \implies \tau + F(D(Tx, Ty)) \leq F(D(x, y))$, where $F : (0, +\infty) \to (-\infty, +\infty)$.

(1) $F$ is strictly increasing. i.e $\forall \alpha, \beta \in (0, +\infty)$ such that $\alpha < \beta, F(\alpha) < (\beta)$.

(2) For any sequence $\{\alpha_n\}$ of positive real numbers, the following holds:

$\lim_{n\to\infty} \alpha_n = 0 \Longleftrightarrow \lim_{n\to\infty} F(\alpha_n) = -\infty$.

(3). $\exists\, k \in (0, 1)$ such that $\lim_{n\to\infty} \alpha^k F(\alpha) = 0$

**Definition 2.3.** (*Jleli and Samet* [3]): Let $X$ be a non empty set and $D : X \times X \to [0, \infty)$ be a function which is satisfies the following conditions:

$(i)$ $D(x, y) = 0 \Longrightarrow x = y$;

$(ii)$ $D(x, y) = D(y, x)$;

$(iii)$ $\exists\, \lambda > 0$ such that if $x, y \in X$ and $\{x_n\} \in \mathcal{C}(D, X, x)$ then $D(x, y) \leq \lambda(\lim_{n\to\infty} supD(x_n, y))$ where $\mathcal{C}(D, X, x) = \{\{x_n\} \in X / D(x_n, y) \to 0\}$.In this case, we say that $D$ is called generalized metric and the pair $(X, D)$ is called generalized metric spaces. we also say that $\lambda$ is a coefficient of $X$. Then we say that $(X, D)$ is a generalized metric space with coefficient $\lambda$.

**Remark 2.4.** Obviously, the set $\mathcal{C}(D, X, x)$ is empty for every $x \in X$ then $(X, D)$ is a generalized metric space if and only if $(i)\&(ii)$ are satisfied.

**Definition 2.5.** Let $(X, D)$ be a generalized metric spaces. Let $\{x_n\}$ be a sequence in $X$ and $x \in X$. We say that $\{x_n\}$ $D-$converges to $x$ if $\{x_n\} \in \mathcal{C}(D, X, x)$.

**Theorem 2.6.** *Let $(X, D)$ be a generalized metric space. Let $\{x_n\}$ be a sequence in Xand $(x, y) \in X \times X$. If $\{x_n\}$ $D-$converges to $x$ and $\{x_n\}D-$converges to $y$ then $x = y$.*

*Proof.* Using the property of $(iii)$, we have $D(x, y) \leq \lambda(\lim_{n\to\infty} supD(x_n, y)) = 0$ which implies from the property $(i)$ that $x = y$. □

**Definition 2.7.** Let $(X, D)$ be a generalized metric space. Let $\{x_n\}$ be a sequence in $x$. We say that $\{x_n\}$ is a $D-$Cauchy sequence if $\lim_{m,n\to\infty} D(x_n, x_{n+m}) = 0$.

**Definition 2.8.**Let $(X, D)$ be a generalized metric space then it is said to be $D-$complete if every $D-$Cauchy sequence in $X$ is converges to some element in $X$.

**Definition 2.9.** Let $f : X \to X$ be a self map and $x \in X$. Write $f^1(x) = f(x)$ and $f^{n+1} = f^1(f(x))$ for $n = 0, 1, 2....$ Then $\{f^n(x)\}$ is called the sequence of iterates $f$ at $x$.

($Jleli\ and\ Samet$ [3]) extended Banach contraction principal to generalized metric space as follows.

**Theorem 2.10.** *($Jleli\ and\ Samet$ [3])(Banach contraction principal for generalized metric space) Let $(X, D)$ be a complete generalized metric space and $f : X \to X$ be such that $D(f(x), f(y)) \leq kD(x, y)$ for some $k \in [0, 1)$ and $\forall x, y \in X$. Suppose $\exists\ x_0 \in X$ such that $\alpha = sup_n D(x_0, f^n(x_0)) < \infty$ then $\{f^n(x_0)\}$ converges to some $w \in X$ and $w$ is a fixed point of $f$. Further $w'$ is another fixed point of $f$ with $D(w, w') < \infty$ then $w = w'$.*

**Definition 2.11.** Let $F : (0, +\infty) \to (-\infty, +\infty)$ be a map satisfying the following conditions.

$(i)$ F *is strictly increasing* and

$(ii)$ $\{t_n\} \to 0 \Longleftrightarrow \{F(t_n)\} \to -\infty$. Then $F$ is called a generalized contraction map, we write $\mathcal{F} = \{F/F is\ a\ generalizedmap\}$.

**Example 2.12.** $F(t) = logt, F(t) = \frac{-1}{t}$ are generalized contraction maps.

**Definition 2.13.** Let $(X, D)$ be a generalized metric space and $f \in \mathcal{F}$. A mapping $T : X \to X$ is said to be $F-$contraction of type(I), if $\tau > 0 \ni \tau + F(D(Tx, Ty)) \leq F(D(x, y)), \forall x, y \in X$where $D(Tx, Ty) > 0$.

**Theorem 2.14.** *Let $(X, D)$ be a complete generalized metric space and $T$ be a generalized $F-$contraction of type(I) whenever $D(Tx, Ty)$ and $D(x, y)$ are non zero and finite. Let $x_0 \in X$. Write $x_n = T^n x_0, n = 1, 2, 3.....$ Then $\{D(x_n, x_{n+1})\}$decreases to zero.If further $\alpha = supD(x_0, T^n x_0) < \infty$ and $D(x_n, x_{n+1}) < \infty for\ n = 0, 1, 2....$,then $\{x_n\}$is a Cauchy sequence. If $x_n \to x$then $x$ is a fixed point of $T$,provided $limsupD(T^n x_0, Tx) < \infty$. Further $T$ has a unique fixed point in this sence that if $y$ is also fixed point of $T$such that $D(x, y) < 0, andD(x, y) < \infty$ then $x = y$.*

*Proof.* Let $x_0 \in X$. A set $Tx_0 = x_1, Tx_1 = x_2\ and\ Tx_n = x_{n+1}$ for $n = 0, 1, 2....$

If $\exists\ n \in N \ni D(x_n, x_{n+1}) = 0 \Longrightarrow x_n = x_{n+1} = Tx_n$.

So that $x_n$is fixed point of $T$.

Hence we suppose that $x_n \neq x_{n+1}$ for $n = 0, 1, 2, ....$

Thus $0 < D(x_n, x_{n+1}) < \infty$ and $D(x_n, x_{n+1}) < \infty$. Now

$$\begin{aligned} &\tau + F(D(Tx_n, Tx_{n+1})) \leq F(D(x_n, x_{n+1})) \\ &\Longrightarrow \tau + F(D(x_{n+1}, x_{n+2})) \leq F(D(x_n, x_{n+1})) \\ &\Longrightarrow F(D(x_{n+1}, x_{n+2})) \leq F(D(x_n, x_{n+1})) - < F(D(x_n, x_{n+1})) \\ &\Longrightarrow F(D(x_{n+1}, x_{n+2})) < F(D(x_n, x_{n+1})) \\ &\Longrightarrow D(x_{n+1}, x_{n+2}) < D(x_n, x_{n+1}) \quad [\ \because F is\ strictly\ increasing\ ] \end{aligned} \tag{2.1}$$

$\{D(x_n, x_{n+1})\}$ is strictly decreasing and hence converges, say, to $r_0 \geq 0$

Write $r_n = D(x_n, x_{n+1})$ then

$F(r_{n+1}) < F(r_n), r_{n+1} < r_n\ \ and\ \ \tau + F(r_{n+1}) \leq F(r_n) - \tau\ \ \forall n$

This implies that

$F(r_{n+1}) \leq F(r_n) - \tau \leq F(r_n) - \tau - \tau.........$

so that $F(r_{n+1}) \leq F(r_0) - (n+1)\tau$, which can be proved by induction.

On letting $n \to \infty$ we get $F(r_{n+1}) \to -\infty \; as \; n \to \infty$ Hence $r_n \to 0 \; as \; n \to \infty$

Therefore $\{D(x_n, x_{n+1})\}$ is decreases to zero.

Now consider$\tau + F(D(x_m, x_n)) \leq F(D(x_{m-1}, x_{n-1})) \Longrightarrow F(D(x_m, x_n)) \leq F(D(x_{m-1}, x_{n-1})) - \tau \leq F(D(x_{m-2}, x_{n-2})) - 2\tau .......F(D(x_{m-n}, x_0)) - n\tau$

On letting $n \to \infty$ we get $F(D(x_m, x_n)) \to -\infty \; as \; m, n \to \infty$ so that $D(x_m, x_n) \to 0 \; as \; n \to \infty$

Thus $\{x_n\}$ is a Cauchy sequence.

Suppose that $x_n \to x$. Hence $D(x_n, x) \to 0 \; as \; n \to \infty$

Suppose $x_n = x$ for infinitely many $n$. Then $x_{n+1} = Tx$ for infinitely many $n$. But $x_{n+1} \to x$

$\therefore x = Tx$

Suppose $x_n \neq x$ on and after certain stage.Then $D(x_n, x) > 0$

$$\begin{aligned}
&Therefore \;\; \tau + F(D(x_{n+1}, Tx)) \leq F(D(x_n, x)) \\
&\Longrightarrow F(D(x_{n+1}, Tx)) \leq F(D(x_n, x)) - \tau \\
&\Longrightarrow F(D(x_{n+1}, Tx)) \to -\infty \\
&\Longrightarrow D(x_{n+1}, Tx)) \to 0 \\
&Therefore \;\; x_{n+1} \to Tx \\
&Therefore \;\; x = Tx
\end{aligned} \tag{2.2}$$

Thus $x$ is a fixed point of T.

Suppose $y$ is also a fixed point of $T$ and $D(x, y) \neq 0$, so that $\tau \leq 0$ which is a contradiction.

Therefore $D(x, y) = 0$

Therefore $x = y$ □

**Suppoting Example**

Let $X = [0, 1]$ and let $D : X \times R_0^+$ defined by $D(x, y) = (max\{x, y\})^2$.

Define $D : X \to X$ as follows

$$Tx = \begin{cases} \frac{x}{4} & \text{if } x \in [0, 1) \\ \frac{1}{8} & \text{if } x = 1 \end{cases}$$

Define $F(t) = \log t \ \ for \ \ t \in R^+$ First we show that it is generalized metric space

$$\begin{aligned}
&\text{i.}D(x,y) = 0 \Longrightarrow (max\{x,y\})^2 = 0 \\
&\qquad \Longrightarrow (max\{x,y\}) = 0 \\
&\qquad \Longrightarrow x = y = 0 \\
&\text{ii.}D(x,y) = (max\{x,y\})^2 = (max\{y,x\})^2 = D(y,x) \\
&\text{iii.}D(x,y) \leq \lambda(\lim_{n\to\infty} sup \ \ D(x_n,y)) \\
&\quad x_n \to x \Longrightarrow D(x_n,y) \to 0 \\
&\Longrightarrow (max\{x,y\})^2 \to 0 \\
&\Longrightarrow x_n \to o \ \ and \ \ x = 0 \\
&D(x,y) = (max\{0,y\})^2 = y^2 \ \ and \ \ D(x_n,y) = (max\{x_n,y\})^2 = y^2 \ \ for \ \ large \ \ n. \\
&Therefore y^2 \leq \lambda y^2 \Longrightarrow \lambda = 1
\end{aligned} \tag{2.3}$$

Now we show that $\tau + F(D(Tx,Ty)) \leq F(D(x,y))$

Suppose $0 \leq x \leq y < 1$ then

$$\begin{aligned}
&\tau + F(D(Tx,Ty)) \leq F(D(x,y)) \\
&\tau + F(D(\frac{x}{4},\frac{y}{4})) \leq F(D(x,y)) \\
&\tau + F(\frac{y}{4}) \leq F(y)^2 \\
&\tau + \log\frac{y^2}{16} \leq \log y^2 \\
&\tau \leq \log y^2 - \log\frac{y^2}{16} = \log 16 = 4\log 2 \\
&\tau \leq 4\log 2
\end{aligned} \tag{2.4}$$

Suppose $x \leq \frac{1}{2} \ \ and \ \ y = 1$ then

$$\begin{aligned}
&\tau + F(D(Tx,Ty)) \leq F(D(x,y)) \\
&\tau + F(D(\frac{x}{4},\frac{1}{8})) \leq F(D(x,1)) \\
&\tau + F(\frac{1}{8})^2 \leq F(1)^2 \\
&\tau + \log\frac{1}{64} \leq \log 1 \\
&\tau \leq \log 64 = 6\log 2 \\
&\tau \leq 6\log 2
\end{aligned} \tag{2.5}$$

Suppose $\frac{1}{2} < x < 1$ *and* $y = 1$ then

$$\begin{aligned}
&\tau + F(D(Tx,Ty)) \leq F(D(x,y))\\
&\tau + F(D(\frac{x}{4},\frac{1}{8})) \leq F(D(x,1))\\
&\tau + F(\frac{1}{8})^2 \leq F(1)^2\\
&\tau + \log\frac{1}{64} \leq \log 1\\
&\tau \leq \log 64 = 6\log 2\\
&\tau \leq 6\log 2
\end{aligned} \tag{2.6}$$

Suppose $x = 1$ *and* $y = 1$ then

$$\begin{aligned}
&\tau + F(D(Tx,Ty)) \leq F(D(x,y))\\
&\tau + F(D(\frac{1}{8},\frac{1}{8})) \leq F(D(x,1))\\
&\tau + F(\frac{1}{8})^2 \leq F(1)^2\\
&\tau + \log\frac{1}{64} \leq \log 1\\
&\tau \leq \log 64 = 6\log 2\\
&\tau \leq 6\log 2
\end{aligned} \tag{2.7}$$

From the above inequalities, $\tau \leq 4\log 2$ Thus $T$ is a $F-$contraction of type(I)

**Definition 2.15.** Let $(X,D)$ be a generalized metric space and $f \in \mathcal{F}$. A mapping $T : X \to X$ is said to be $F-$contraction of type(II), if $\tau > 0 \ni \tau + F(D(Tx,Ty)) \leq max\{F(D(x,y)), F(D(x,Tx)), F(D(y,Ty))\}, \forall x,y \in X$where $D(Tx,Ty) > 0$.

**Theorem 2.16.** *Let $(X,D)$ be a complete generalized metric space and T be a generalized $F-$contraction of type(II) whenever $D(Tx,Ty), D(x,Tx), D(y,Ty)$ and $D(x,y)$ are non zero and finite. Let $x_0 \in X$. Write $x_n = T^n x_0, n = 1,2,3.....$ Then $\{D(x_n,x_{n+1})\}$decreases to zero.If further $P_n = sup_k D(x_n,x_{n+k})$and assume that $P_n < \infty forn = 0,1,2.....$, then $\{x_n\}$ is a Cauchy sequence.*

*Proof.* Let $x_0 \in X$. A set $Tx_0 = x_1, Tx_1 = x_2$ *and* $Tx_n = x_{n+1}$ for $n = 0,1,2....$ If $\exists\, n \in N \ni D(x_n,x_{n+1}) = 0 \Longrightarrow x_n = x_{n+1} = Tx_n$.So that $x_n$is fixed point of $T$.

Write $r_n = D(x_n, x_{n+1})$. Hence we may suppose that $r_n > 0$ for every $n$. Then

$$\tau + F(D(Tx_n, Tx_{n+1})) \leq max\{F(D(x_n, x_{n+1})), F(D(x_n, Tx_n)), F(D(x_{n+1}, Tx_{n+1}))\}$$

$$\tau + F(D(Tx_{n+1}, Tx_{n+2})) \leq max\{F(D(x_n, x_{n+1})), F(D(x_n, x_{n+1})), F(D(x_{n+1}, x_{n+2}))\}$$

$$= \max\{F(D(x_n, x_{n+1})), F(D(x_{n+1}, x_{n+2}))\}$$

$$Therefore \tau + F(D(Tx_{n+1}, Tx_{n+2})) \leq F(D(x_n, x_{n+1}))$$

$$\Longrightarrow F(r_{n+1}) \leq F(r_n) - \tau \tag{2.8}$$

$$\Longrightarrow F(r_{n+1}) \leq F(r_0) - n\tau, which\ can\ be\ proved\ by\ induction.$$

$$\Longrightarrow F(r_{n+1}) \to -\infty\ as\ n \to \infty.$$

$$\Longrightarrow r_n \to 0\ as\ n \to \infty.$$

$$Therefore\ \{r_n\} \downarrow 0.$$

Now consider for $m > n$

$$\tau + F(D(x_m, x_n)) \leq max\{F(D(x_{m-1}, x_{n-1})), F(D(x_{m-1}, x_n)), F(D(x_{n-1}, x_n))\}$$

$$given\ \epsilon > 0, \exists N \ni n > N, we have r_n < \epsilon$$

$$\therefore \tau + F(D(x_m, x_n)) \leq max\{F(D(x_{m-1}, x_{n-1})), F(\epsilon), F(\epsilon)\}$$

$$= max\{F(D(x_{m-1}, x_{n-1})), F(\epsilon)\}$$

$$\therefore F(D(x_m, x_n)) \leq max\{F(D(x_{m-1}, x_{n-1})), F(\epsilon)\} - \tau$$

$$Write A_0 = F(D(x_m, x_n)), A_1 = F(D(x_{m-1}, x_{n-1})), A_2 = F(D(x_{m-2}, x_{n-2}))d\ and\ F(\epsilon) = \gamma$$

$$. \qquad \therefore A_0 \leq max\{A_1, F(\epsilon)\} - \tau \tag{2.9}$$

$$= max\{A_1, \gamma\} - \tau$$

$$\leq max\{\{max\{A_2, \gamma\} - \tau\}, \gamma\} - \tau$$

$$= max\{\{max\{A_2 - \tau, \gamma - \tau\}\}, \gamma\} - \tau$$

$$= max\{\{max\{A_2 - \tau, \gamma - \tau\} - \tau\}, \gamma - \tau\}$$

$$= max\{\{max\{A_2 - 2\tau, \gamma - 2\tau\}\}, \gamma - \tau\}$$

$$= max\{A_2 - 2\tau, \gamma - \tau\}$$

$\therefore A_0 \leq max\{A_{n-N} - (n-N)\tau, \gamma - \tau\}$ by using the fact that for three real numbers $a, b$ and $c$, we have $max\{(a, b) - c\} = max\{a - c, b - c\}$

$\therefore F(D(x_m, x_n)) \leq F(\epsilon) - \tau$ *for large* $m, n$. This is true for $\epsilon > 0$.

$\therefore F(D(x_m, x_n)) \to -\infty\ as\ m, n \to \infty.$

$\therefore D(x_m, x_n) \to 0\ as\ m, n \to \infty.$

$\therefore \{x_n\}$ is a Cauchy sequence. □

**Open Problem**: Let $(X, D)$ be a complete generalized metric space and $T$ be a generalized $F-$contraction of type(II) whenever $D(Tx, Ty), D(x, Tx), D(y, Ty)$ and $D(x, y)$ are non zero and finite. Let $x_0 \in X$. Write $x_n = T^n x_0, n = 1, 2, 3.....$ Then $\{D(x_n, x_{n+1})\}$ decreases to zero. If further $P_n = sup_k D(x_n, x_{n+k})$ and assume that

$P_n < \infty \, for \; n = 0, 1, 2.....$, then $\{x_n\}$ is a Cauchy sequence and Suppose $x_n \to x \; as \; x \to \infty$. Is $x$, a fixed point of $T$?.

**Suppoting Example**

Let $X = [0, 1]$ and a mapping $D : X \times X \to R_0^+$ be defined by

$$D(x, y) = \begin{cases} |x - y| & \text{if } x, y \in (0, 1) \\ y & \text{if } \; x = 0, y \in (0, 1) \\ \infty & \text{if } x = 1, y \in (0, 1) \\ \infty & \text{if } x = 0 \; or \; 1, y = 0 or 1 \end{cases}$$

Define $T : X \to X$ as follows

$$Tx = \begin{cases} \frac{x}{4} & \text{if } x \in (0, 1) \\ 1 & \text{if } \; x = 0 \\ 0 & \text{if } x = 1 \end{cases}$$

Define $F(t) = \log t for t \in R^+$. It is easy to prove that $(X, D)$ is generalized metric space with $\lambda = 1$, since

$(i) \; D(x, y) = o.$

$(ii) \; D(x, y) = (max = \{x, y\})^2 = (max = \{y, x\})^2 = D(y, x).$

$(iii) \; D(x, y) \leq \lambda(\lim_{n\to\infty} \; sup \; D(x_n, y)) \; with \; \lambda = 1.$

Now we show that $\tau + F(D(Tx, Ty)) \leq max\{F(D(x, y)), F(D(x, Tx)), F(D(y, Ty))\}$

If $\quad x = 1 \Longrightarrow T1 = 0$

If $\quad x = 0 \Longrightarrow T0 = 1$

Assume that $x, y \in (0, 1) \; and \; x < y$

$\tau + F(D(\frac{x}{4}, \frac{y}{4})) \leq max\{F(D(x, y)), F(D(x, \frac{x}{4})), F(D(y, \frac{y}{4}))\}$

$\Longrightarrow \tau + F(\frac{y}{4} - \frac{x}{4})) \leq max\{F(y - x), F(\frac{3x}{4}), F(\frac{3y}{4})\}$

$\Longrightarrow \tau + F(\frac{y-x}{4}) \leq F(y - x)$

$\Longrightarrow \tau + \log(\frac{y-x}{4}) \leq \log(y - x)$

$\Longrightarrow \tau \leq \log(y - x) - \log(\frac{y-x}{4})$

$\Longrightarrow \tau \leq 2 \log 2$

$Therefore \;\; \tau = 2 \log 2$

Thus $T$ is a $F-$contraction of type (II) with $\tau = 2 \log 2$.

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