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On Multi Weak Structures

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ABSTRACT. In this paper, the concept of multi weak structures is introduced and its several results are presented in details. It is worth mentioning that multi weak structures are a generalization of weak structures and multi topological spaces. Moreover, we give the deviations between the previous work and the current work. Also, the concepts of ω-dense, ω-nowhere dense and generalized ω-closed multisets are introduced in multi weak structures. In addition, its properties and some important results are discussed.

1 Introduction

Multiset theory was introduced in 1986 by Yager [25]. A multiset is considered to be the generalization of a classical set. In classical set theory, a set is a well-defined collection of distinct objects. It states that a given element can appear only once in a set without repetition. So, the only possible relation between two mathematical objects is either they are equal or they are different. The situation in science and in ordinary life is not like this. If the repetitions of any object is allowed in a set, then a mathematical structure, that is known as multiset (mset [2] or bag [25], for short), is obtained in [4, 16, 23, 24]. For the sake of convenience an mset is written as $\{k_1/x_1, k_2/x_2, ..., k_n/x_n\}$ in which the element x_i occurs k_i times. The number of occurrences of an object x in an mset A, which is finite in most of the studies that involve msets, is called its multiplicity or characteristic value, usually denoted by $m_A(x)$

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or $C_A(x)$ or simply by $A(x)$. Noted that each multiplicity k_i is a positive integer. Mathematics, the equation $x^2 - 4x + 4 = 0$ has a solution $x = 2, 2$ which gives the mset $S = \{2/2\}$. Additionally, One of the simplest examples is the mset of prime factors of a positive integer n. The number 504 has the factorization $504 = 2^3 3^2 7^1$ which gives the mset $X = \{3/2, 2/3, 1/7\}$ where $C_X(2) = 3$, $C_X(3) = 2$, $C_X(7) = 1$. In Chemistry, a water molecule H_2O is represented by the mset $M = \{2/H, 1/O\}$ and without one of the two hydrogen atoms, the water molecule is not created.

the physical world it is observed that there is enormous repetition. For instance, there are many hydrogen atoms, many water molecules, many strands of DNA, etc. This leads to three possible relations between any two physical objects; they are different, they are the same but separate or they coincide and are identical. Many conclusive results were established by these authors and further study was carried on by Jena et al. [20] and many others [5, 12, 13]. The notion of an mset is well established both in mathematics and computer science [2, 3, 15]. Additionally, the basic definitions of relations in mset context are introduced by Girish et al. [15, 17]. An application in medical field is presented by M. Hosny et al. [19].

In 2011, *Császár* [11] presented the concept of generalized topology as a generalization of the concept of general topology. The properties of the generalized topology are investigated by *Császár* [6, 7, 8, 10] and many other authors [14]. In 2012, a new type of sets called generalized ω-closed (briefly $g\omega$-closed) sets is introduced in the concept of weak structures by Al-Omari et al. [1]. The class of all $g\omega$-closed sets is a generalization of the class of all ω-closed sets. Some of their properties are discussed. Additionally, they presented the notions of ω-regular and ω-normal spaces. Navaneethakrishnan and Thamaraiselvi [21] extended the study of weak structures and m-structures defined on a set X and proved that an m-structure generates a finer topology. Moreover, they defined and studied the properties of some subsets of X with respect to a weak structure on X (see [22]). Further, Zahran et al. [26] modified the weak structure by the hereditary classes and proved that the structures resulting from a generalized topology and a hereditary class introduced by *Császár* [9] still valid. Recently, Zahran et al. [27] introduced the concept of generalized closed sets and some separation axioms on weak structures.

In this paper, we introduce the concept of multi weak structure which is a generalization of weak structures and multi topological spaces. Moreover, we discuss its properties and some important results. Finally, we present the notions of ω-dense, ω-nowhere dense and generalized ω-closed msets in multi weak structures.

2 Preliminaries

The aim of this section is to present the basic concepts and properties of mset theory which will be needed in the sequel.

Definition 2.1 *[20] An mset M drawn from the set X is represented by a count function C_M defined as, C_M: $X \to N$, where N represents the set of non-negative integers.*

Here $C_M(x)$ is the number of occurrences of the element x in the mset M. The mset M is drawn from the set $X = \{x_1, x_2, x_3, ..., x_n\}$ and it is written as $M = \{m_1/x_1, m_2/x_2, m_3/x_3, ...,$
$m_n/x_n\}$ where m_i is the number of occurrences of the element x_i, $i = 1, 2, 3, ..., n$ in the mset M.

Definition 2.2 *[20] A domain X, is defined as a set of elements from which msets are constructed. The mset space $[X]^w$ is the set of all msets whose elements are in X such that no element in the mset occurs more than w times.*

The mset space $[X]^\infty$ is the set of all msets over a domain X such that there is no limit on the number of occurrences of an element in an mset. If $X = \{x_1, x_2, ..., x_k\}$, then $[X]^w = \{\{m_1/x_1, m_2/x_2, ..., m_k/x_k\} : \ m_i \in \{0, 1, 2, ..., w\}, \ i = 1, 2, ..., k\}$.

Definition 2.3 *[20] Let M and N be two msets drawn from a set X. Then,*

1. *$M = N$ if $C_M(x) = C_N(x)$ for all $x \in X$,*
2. *$M \subseteq N$ if $C_M(x) \leq C_N(x)$ for all $x \in X$,*
3. *$P = M \cup N$ if $C_P(x) = max\{C_M(x), C_N(x)\}$ for all $x \in X$,*
4. *$P = M \cap N$ if $C_P(x) = min\{C_M(x), C_N(x)\}$ for all $x \in X$,*
5. *$P = M \oplus N$ if $C_P(x) = min\{C_M(x) + C_N(x), w\}$ for all $x \in X$,*
6. *$P = M \ominus N$ if $C_P(x) = max\{C_M(x) - C_N(x), 0\}$ for all $x \in X$, where $\oplus$ and $\ominus$ represent mset addition and mset subtraction respectively.*

Let M be an mset drawn from a set X. The support set of M denoted by M^* is a subset of X and $M^* = \{x \in X : C_M(x) > 0\}$, i.e., M^* is an ordinary set.

Definition 2.4 *[20] Let M be an mset drawn from the set X. If $C_M(x) = 0 \ for \ all \ x \in X$, then M is called an empty mset and denoted by ϕ, i.e., $\phi(x) = 0 \ for \ all \ x \in X$.*

Definition 2.5 *[20] Let M be an mset drawn from the set X and $[X]^w$ be the mset space defined over X. Then for any mset $M \in [X]^w$, the complement M^c of M in $[X]^w$ is an element of $[X]^w$ such that $C_{M^c}(x) = w - C_M(x)$ for every $x \in X$.*

Definition 2.6 *[2] Let $X \in [U]^w$ be an mset. Then, the power mset $P(X)$ of X is the set of all submsets of X. We have $Y \in P(X)$ if and only if $Y \subseteq X$. If $Y = \phi$, then $Y \in^1 P(X)$. If $Y \neq \phi$, then $Y \in^k P(X)$ where $k = \prod_z \begin{pmatrix} \mid [X]_z \mid \\ \mid [Y]_z \mid \end{pmatrix}$, the product $\prod_z$ is taken over by distinct elements of z of the mset Y and $\mid [X]_z \mid = m$ iff $z \in^m X$, $\mid [Y]_z \mid = n$ iff $z \in^n Y$, then $\begin{pmatrix} \mid [X]_z \mid \\ \mid [Y]_z \mid \end{pmatrix} = \begin{pmatrix} m \\ n \end{pmatrix} = \frac{m!}{n!(m-n)!}$.*

The power set of an mset is the support set of the power mset and is denoted by $P^*(X)$. The following theorem shows the cardinality of the power set of an mset.

Definition 2.7 *[17] Let $M \in [X]^w$ and $\tau \subseteq P^*(M)$. Then, τ is called an M-topology on M if τ satisfies the following properties:*

1. *$M, \phi \in \tau$,*
2. *the union of the elements of any subcollection of τ is in τ,*
3. *the intersection of the elements of any finite subcollection of τ is in τ.*

Hence, (M, τ) is called an M-topological space. Each element in τ is called an open mset.

Definition 2.8 *[18] The complement of any submset N in an M-topological space* (M,τ) *is defined by:* $N^c = M \ominus N$, *i.e.,* $C_{N^c}(x) = C_M(x) - C_N(x)\ \forall x \in M^*$.

Definition 2.9 *[18] A submset N of an M-topological space* (M,τ) *is said to be closed if the mset* $M \ominus N$ *is open.*

Definition 2.10 *[18] Let A be a submset of an M-topological space* (M,τ). *Then,*

1. *the interior of A is defined as the union of all open msets contained in A and denoted by* $int(A)$, *i.e.,* $int(A) = \cup\{G \subseteq M : G \text{ is an open mset and } G \subseteq A\}$ *and* $C_{int(A)}(x) = max\{C_G(x) : G \in \tau, G \subseteq A\}$,
2. *the closure of A is defined as the intersection of all closed msets containing A and denoted by* $cl(A)$, *i.e.,* $cl(A) = \cap\{K \subseteq M : K \text{ is a closed mset and } A \subseteq K\}$ *and* $C_{cl(A)}(x) = min\{C_K(x) : K \in \tau^c, A \subseteq K\}$.

3 Multi weak structures

The purpose of this section is to present for the first time the concept of multi weak structures and its important results in details. Moreover, the notions of ω-dense and ω-nowhere dense are introduced and some of their properties are presented.

Definition 3.1 *Let X be a non-empty multiset and* $\omega \subseteq P(X)$. *If* $\phi \in \omega$, *then* ω *is called a multi weak structure (briefly MWS) on X. Also, a submset N is called* ω*-open if* $N \in \omega$ *and called* ω*-closed if* $N^c \in \omega$. *Moreover, a multi weak structure* ω *is called multi minimal structure if* $X \in \omega$. *That's mean a multi minimal structure is a special case of a multi weak structure.*

It's clear that every multi topology is a multi weak structure and anyone can give an example to show that the converse is not true in general.

It should be noted that the union and the intersection of two multi weak (minimal) structures will be a multi weak (minimal) structure which is not satisfied in the multi topology in general.

Definition 3.2 *Let* ω *be a multi weak structure on a non-empty mset X and* $Y \subseteq X$. *Then, we define:*

1. $i_\omega(Y) = \cup\{N \subseteq X :\ N \in \omega,\ N \subseteq Y\}$,
2. $c_\omega(Y) = \cap\{F \subseteq X :\ F \in \omega^c,\ Y \subseteq F\}$.

Proposition 3.3 *Let* ω *be a multi weak structure on a non-empty mset X and* $F, G \subseteq X$, *then*

1. $i_\omega(\phi) = \phi$ *and* $c_\omega(X) = X$.
2. $i_\omega(F) \subseteq F \subseteq c_\omega(F)$.
3. *If* $F \in \omega$, *then* $i_\omega(F) = F$.
4. *If* $F \in \omega^c$, *then* $c_\omega(F) = F$.
5. *If* $F \subseteq G$, *then* $i_\omega(F) \subseteq i_\omega(G)$ *and* $c_\omega(F) \subseteq c_\omega(G)$.
6. $i_\omega(F \cap G) \subseteq i_\omega(F) \cap i_\omega(G)$ *and* $c_\omega(F) \cup c_\omega(G) \subseteq c_\omega(F \cup G)$.

7. $i_\omega(i_\omega(F)) = i_\omega(F)$ *and* $c_\omega(c_\omega(F)) = c_\omega(F)$.

8. $i_\omega(F^c) = (c_\omega(F))^c$ *and* $c_\omega(F^c) = (i_\omega(F))^c$.

9. $i_\omega(c_\omega(i_\omega(c_\omega(F)))) = i_\omega(c_\omega(F))$ *and* $c_\omega(i_\omega(c_\omega(i_\omega(F)))) = c_\omega(i_\omega(F))$.

Proof. The proof of parts $(1-4)$ is straightforward by the definition.

5.

$$\begin{aligned} i_\omega(F) &= \cup\{N \subseteq X : N \in \omega, N \subseteq F\} \\ &\subseteq \cup\{N \subseteq X : N \in \omega, N \subseteq G\}, \text{ as } F \subseteq G \\ &= i_\omega(G). \end{aligned}$$

By the similar way, $c_\omega(F) \subseteq c_\omega(G)$.

6. Immediate from part (5).

7. Since, $i_\omega(F) \in \omega$, then $i_\omega(i_\omega(F)) = i_\omega(F)$ by using part (3). By the similar way, $c_\omega(c_\omega(F)) = c_\omega(F)$.

8.

$$\begin{aligned} (c_\omega(F))^c &= [\cap\{N \subseteq X : N \in \omega^c, F \subseteq N\}]^c \\ &= \cup\{N^c \subseteq X : N^c \in \omega, N^c \subseteq F^c\} \\ &= i_\omega(F^c). \end{aligned}$$

Similarly, $c_\omega(F^c) = (i_\omega(F))^c$.

9. Immediately.

It should be noted that the following statements are not satisfied in general in multi weak structures, $i_\omega(X) = X$ and $c_\omega(\phi) = \phi$, as shown by the following example.

Let $X = \{3/a, 4/b, 6/c\}$ be an mset and $\omega = \{\phi, \{3/a\}\}$ be a MWS on X, then $i_\omega(X) = \{3/a\} \neq X$ and $c_\omega(\phi) = \{4/b, 6/c\} \neq \phi$.

The equality in part (2) of Proposition 3.3 is not true in general as shown by Example 3. The converse of parts (3) and (4) in Proposition 3.3 are not true in general as shown by the following example.

Let $X = \{2/a, 3/b, 1/c, 4/d\}$ be an mset and $\omega = \{\phi, \{2/a\}, \{1/c\}\}$ be a MWS on X. If $F = \{2/a, 1/c\}$, then $i_\omega(F) = \{2/a, 1/c\} = F$ but F is not a ω-open mset. Also, if $G = \{3/b, 4/d\}$, then $c_\omega(G) = G$ but G is not a ω-closed mset.

The equality in part (6) of Proposition 3.3 is not true in general as shown by the following example.

Let $X = \{2/a, 3/b, 1/c, 5/d\}$ be an mset and $\omega = \{\phi, \{2/a, 2/b\}, \{2/b, 1/c\}\}$ be a MWS on X.

1. If $F = \{2/a, 3/b\}$ and $G = \{3/b, 1/c\}$, then $i_\omega(F) = \{2/a, 2/b\}$ and $i_\omega(G) = \{2/b, 1/c\}$ but $i_\omega(F \cap G) = \phi$.
2. If $F = \{1/c, 5/d\}$ and $G = \{2/a, 5/d\}$, then $c_\omega(F) = \{1/b, 1/c, 5/d\}$ and $c_\omega(G) = \{2/a, 1/b, 5/d\}$ but $c_\omega(F \cup G) = X$.

Definition 3.4 *Let ω be a multi weak structure on a non-empty mset X and $F \subseteq X$. If $c_\omega(F) = X$, then F is called a ω-dense mset.*

Let $X = \{2/a, 3/b, 1/c\}$ be an mset and $\omega = \{\phi, \{2/a, 1/c\}\}$ be a MWS on X. If $F = \{2/a, 3/b\}$, then $c_\omega(F) = X$, i.e., F is a ω-dense mset.

Theorem 3.5 *Let ω be a multi weak structure defined over a non-empty mset X and $F \subseteq X$. Then, F is a ω-dense mset if and only if $i_\omega(F^c) = \phi$.*

Proof. Clear.

Theorem 3.6 *Let ω be a multi weak structure defined over a non-empty mset X and $F \subseteq X$. If F is a ω-dense mset, then $F \cap G \neq \phi$ whenever $\phi \neq G \in \omega$.*

Proof. Assume that F is a ω-dense mset and there exists G such that $\phi \neq G \in \omega$ and $F \cap G = \phi$. Then, $F \subseteq G^c$. Therefore, $c_\omega(F) \subseteq c_\omega(G^c) = G^c$. Since, $c_\omega(F) = X$. Thus, $X \subseteq G^c$ that's mean $G^c = X$. Hence, $G = \phi$ which is a contradiction.

Theorem 3.7 *The union of two ω-dense msets is a ω-dense mset.*

Proof. Let F and G be two ω-dense msets, then $c_\omega(F) = X$ and $c_\omega(G) = X$. Since, $F \subseteq F \cup G$, then $c_\omega \subseteq c_\omega(F \cup G)$. Therefore, $X \subseteq c_\omega(F \cup G)$. Hence, $c_\omega(F \cup G) = X$.

The intersection of two ω-dense msets is not a ω-dense mset in general as shown by the following example.

Let $X = \{2/a, 3/b, 1/c\}$ be an mset and $\omega = \{\phi, \{3/b, 1/c\}\}$ be a MWS on X. If $F = \{2/a, 2/b\}$ and $G = \{2/a, 1/c\}$, then F and G are two ω-dense msets and $F \cap G = \{2/a\}$. Thus, $c_\omega(F \cap G) = \{2/a\}$. Hence, $F \cap G$ is not a ω-dense mset.

Definition 3.8 *Let ω be a multi weak structure defined over a non-empty mset X and $Y \subseteq X$. If $i_\omega(c_\omega(Y)) = \phi$, then Y is called a ω-nowhere dense mset.*

The following example shows that the family of all ω-dense (ω-nowhere dense respectively) msets is not a multi weak structure in general.

Let $X = \{2/a, 4/b, 6/d\}$ be an mset and $\omega = \{\phi, \{1/a, 2/b, 3/d\}\}$ be a MWS on X. Then, $c_\omega(\phi) = \{1/a, 2/b, 3/d\} \neq X$ and $i_\omega(c_\omega(\phi)) = \{1/a, 2/b, 3/d\} \neq \phi$. Hence, ϕ does not belong to the family of all ω-dense (ω-nowhere dense respectively) msets. The following example shows that the union of two ω-nowhere dense msets is not a ω-nowhere dense mset in general.

Let $X = \{3/a, 4/b, 6/c\}$ be an mset and $\omega = \{\phi, \{3/a, 4/b\}, \{4/b, 6/c\}\}$ be a MWS on X. If $F = \{3/a\}$ and $G = \{6/c\}$, then F and G are ω-nowhere dense msets but $i_\omega(c_\omega(F \cup G)) = \{3/a, 4/b\} \neq \phi$ i.e., $F \cup G$ is not ω-nowhere dense.

Theorem 3.9 *Let ω be a multi weak structure defined over a non-empty mset X and $Y \subseteq X$. Y is a ω-nowhere dense mset if and only if $(c_\omega(Y))^c$ is a ω-dense mset.*

Proof.

($\Rightarrow$) Let Y be a ω-nowhere dense mset, i.e., $i_\omega(c_\omega(Y)) = \phi$. Then, $c_\omega((c_\omega(Y))^c) = (i_\omega(c_\omega(Y)))^c = \phi^c = X$.

($\Leftarrow$) Let $(c_\omega(Y))^c$ is a ω-dense mset, i.e., $c_\omega((c_\omega(Y))^c) = X$. Then, $(i_\omega(c_\omega(Y)))^c = X$. Thus, $i_\omega(c_\omega(Y)) = \phi$. Hence, Y is a ω-nowhere dense mset.

4 Generalized ω-closed multisets

The aim of this section is to introduce generalized ω-closed msets in multi weak structures and discuss its properties and results.

Definition 4.1 *Let ω be a multi weak structure defined over a non-empty mset X. A submset Y is said to be generalized ω-closed ($g\omega$-closed, for short) if $c_\omega(Y) \subseteq G$ whenever $Y \subseteq G$ and G is a ω-open mset. Moreover, the complement of a generalized ω-closed mset is said to be generalized ω-open ($g\omega$-open, for short).*

Theorem 4.2 *Let ω be a multi weak structure defined over a non-empty mset X. A submset Y is $g\omega$-open if and only if $H \subseteq i_\omega(Y)$ whenever $H \subseteq Y$ and H is a ω-closed mset.*

Proof. Clear.

Theorem 4.3 *Let ω be a multi weak structure defined over a non-empty mset X and $F \subseteq X$. If F is a ω-closed mset, then F is $g\omega$-closed.*

Proof. Immediate from Definition 4.1.

Corollary 4.4 *Let ω be a multi weak structure defined over a non-empty mset X and $F \subseteq X$. If F is a ω-open mset, then F is $g\omega$-open.*

The converse of Theorem 4.3 is not true in general as shown by the following example.

From Example 3, if $F = \{3/a, 4/b\}$, then F is a $g\omega$-closed mset and it is not ω-closed. Also, it's easy to get another example to show that the converse of Corollary 4.4 is not true in general. The intersection and union of two $g\omega$-closed mset is not $g\omega$-closed in general as shown by the following examples.

1. From Example 3, if $F = \{3/a, 4/b\}$ and $G = \{3/a, 6/c\}$, then F and G are $g\omega$-closed msets but $F \cap G = \{3/a\}$ is not a $g\omega$-closed mset.
2. Let $X = \{4/a, 3/b, 2/c, 5/d\}$ be an mset and $\omega = \{\phi, \{4/a, 3/b, 2/c\}, \{4/a, 3/b, 5/d\}, \{4/a, 2/c, 5/d\}, \{3/b, 2/c, 5/d\}, \{4/a, 3/b\}\}$ be a MWS on X. If $F = \{4/a\}$ and $G = \{2/c, 5/d\}$, then F and G are $g\omega$-closed msets but $F \cup G = \{4/a, 2/c, 5/d\}$ is not a $g\omega$-closed mset.

Definition 4.5 *Let ω be a multi weak structure on a non-empty mset X. A collection $\{F_i : F_i \subseteq X, i \in I\}$ is said to be ω-locally finite if $c_\omega(\cup_{i\in I} F_i) = \cup_{i\in I} c_\omega(F_i)$.*

Theorem 4.6 *Let ω be a multi weak structure on a non-empty mset X. The arbitrary union of $g\omega$-closed msets F_i, $i \in I$ in X is a $g\omega$-closed mset if the collection $\{F_i : F_i \subseteq X, i \in I\}$ is ω-locally finite.*

Proof. Let G be a ω-open mset such that $\cup_{i\in I} A_i \subseteq G$, then $A_i \subseteq G$ for all $i \in I$. Since, A_i are $g\omega$-closed msets. Then, $c_\omega(A_i) \subseteq G$ for all $i \in I$. Therefore, $\cup_{i\in I} c_\omega(A_i) \subseteq G$. Hence, $c_\omega(\cup_{i\in I} A_i) \subseteq G$. This completes the proof.

Corollary 4.7 *Let ω be a multi weak structure on a non-empty mset X. The arbitrary intersection of $g\omega$-open msets F_i, $i \in I$ in X is a $g\omega$-open mset if the collection $\{F_i : F_i \subseteq X, i \in I\}$ is ω-locally finite.*

There exists a deviation between the previous work [27] and the current work which represented by $c_\omega(F) \ominus F$ may be containing non-empty ω-closed mset if F is $g\omega$-closed, as shown by the following example.

Let $X = \{3/a, 4/b, 6/c\}$ be an mset and $\omega = \{\phi, \{2/b\}, \{3/c\}, \{3/a, 4/b, 3/c\}\}$ be a MWS on X. If $F = \{3/a, 4/b\}$, then F is a $g\omega$-closed mset but $c_\omega(F) \ominus F = \{3/a, 4/b, 3/c\}$ is a ω-closed mset.

Theorem 4.8 *Let ω be a multi weak structure on a non-empty mset X and F be a $g\omega$-closed mset with $F \subseteq G \subseteq c_\omega(F)$, then G is $g\omega$-closed.*

Proof. Clear.

Corollary 4.9 *Let ω be a multi weak structure on a non-empty mset X and F be a $g\omega$-open mset with $i_\omega(F) \subseteq G \subseteq F$, then G is $g\omega$-open.*

Theorem 4.10 *Let ω be a multi weak structure on a non-empty mset X. If each ω-open mset is ω-closed, then each submset of X is $g\omega$-closed.*

Proof. Clear.

The converse of Theorem 4.10 is not true in general as shown by the following example.

Let $X = \{2/a, 3/b, 5/c\}$ be an mset and $\omega = \{\phi, X, \{2/a\}, \{3/b\}, \{5/c\}, \{2/a, 5/c\}, \{3/b, 5/c\}\}$ be a MWS on X. Then, it's easy to show that every submset of X is a $g\omega$-closed mset, but $F = \{5/c\}$ is ω-open and not ω-closed.

5 Conclusions

The main aim of multi weak structures is give a generalization of weak structures and multi topologies. Further, we discussed its properties and some important results. In addition, we presented the notions of ω-dense, ω-nowhere dense and generalized ω-closed msets in multi weak structures. Also, we established several interesting theorems and examples about these notions. Finally, we hope that our findings help the researchers to enhance and promote their studies on multi topology to carry out a general framework for their applications in life.

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