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Supra Open Soft Sets and Associated Soft Separation Axioms

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ABSTRACT. In this paper, we introduce and investigate some weak soft separation axioms by using the notion of supra open soft sets, which is a generalization of the soft (resp. semi soft, pre soft, α-soft and β-soft separation axioms. We study the relationships between these new soft separation axioms and their relationships with some other properties.

1 Introduction

Soft set theory is one of the emerging branches of mathematics that could deal with parameterization inadequacy and vagueness that arises in most of the problem solving methods. It is introduced [26] in 1999 by the Russian mathematician Molodtsov with its rich potential applications in divergent directions such as stability and regularization, game theory, operations research, soft analysis etc [26, 27]. Further research works produced so many definitions, results and practical applications. After presentation of the operations of soft sets, the properties and applications of soft set theory have been studied increasingly [8, 23, 27].

It got some stability only after the introduction of soft topology [28] in 2011. In [14], Kandil et al. introduced some soft operations such as semi open soft, pre open soft, α-open soft and β-open soft and investigated their properties in detail. Kandil et al. [21] introduced the notion of soft semi separation axioms. In particular they

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study the properties of the soft semi regular spaces and soft semi normal spaces. The notion of soft ideal was initiated for the first time by Kandil et al. [17]. They also introduced the concept of soft local function. These concepts are discussed with a view to find new soft topologies from the original one, called soft topological spaces with soft ideal $(X, \tau, E, \tilde{I})$. Applications to various fields were further investigated by Kandil et al. [15, 16, 18, 19, 20, 22]. The notion of supra soft topological spaces was initiated for the first time by [1, 12]. They also introduced new different types of subsets of supra soft topological spaces and study the relations between them in detail. The notion of b-open soft sets was initiated for the first time in [7, 11], which is generalized to the supra soft topological spaces in [3].

The main purpose of this paper, is to introduce and investigate some weak soft separation axioms by using the notion of supra open soft sets, which is a generalization of the soft (resp. semi soft, pre soft, α-soft and β-soft) separation axioms Mentioned in [9, 21, 25, 28, 29, 30, 31, 32]. We study the relationships between these new soft separation axioms and their relationships with some other properties. As a consequence the relations of some supra soft separation axioms are shown in a diagram. Also, we show that the property of being supra soft T_i-spaces $(i = 1, 2)$ is soft topological property under a bijection and supra irresolute open soft mapping. Further, the properties of being supra soft regular and supra soft normal are soft topological properties under a bijection, supra irresolute and supra irresolute open soft functions.

We show that, some classical results in general topology are not true if we consider supra soft topological spaces instead. For instance, if (X, μ, E) is supra soft T_1-space need not every soft point is supra closed soft.

2 Preliminaries

In this section, we present the basic definitions and results of soft set theory which will be needed in the sequel.

Definition 2.1 *[26] Let X be an initial universe and E be a set of parameters. Let $\mathcal{P}(X)$ denote the power set of X and A be a non-empty subset of E. A pair (F, A) is called a soft set over X, where F is a mapping given by $F : A \to \mathcal{P}(X)$.*

Definition 2.2 *[10]A soft set F over X is a set valued function from E to $\mathcal{P}(X)$. It can be written a set of ordered pairs $F = \{(e, F(e)) : e \in E\}$. Note that if $F(e) = \emptyset$, then the element $(e, F(e))$ is not appeared in F. The set of all soft sets over X is denoted by $S_E(X)$.*

Definition 2.3 *[10]Let $F, G \in S_E(X)$. Then,*

i. *If $F(e) = \emptyset$ for each $e \in E$, F is said to be a null soft set, denoted by $\tilde{\Phi}$.*

ii. *If $F(e) = X$ for each $e \in E$, F is said to be absolute soft set, denoted by $\tilde{X}$.*

iii. *F is soft subset of G, denoted by $F \tilde{\subseteq} G$, if $F(e) \subseteq G(e)$ for each $e \in E$.*

iv. *$F = G$, if $F \tilde{\subseteq} G$ and $G \tilde{\subseteq} F$.*

v. *Soft union of F and G, denoted by $F \tilde{\cup} G$, is a soft set over X and defined by $F \tilde{\cup} G : E \to \mathcal{P}(X)$ such that $(F \tilde{\cup} G)(e) = F(e) \cup G(e)$ for each $e \in E$.*

vi. *Soft intersection of F and G, denoted by* $F \tilde{\cap} G$*, is a soft set over X and defined by* $F \tilde{\cap} G : E \to \mathcal{P}(X)$ *such that* $(F \tilde{\cap} G)(e) = F(e) \cap G(e)$ *for each* $e \in E$.

vii. *Soft complement of F is denoted by* $F^{\tilde{c}}$ *and defined by* $F^{\tilde{c}} : E \to \mathcal{P}(X)$ *such that* $F^{\tilde{c}}(e) = X \setminus F(e)$ *for each* $e \in E$.

We will consider Definition[2.2] and Definition[2.3] the rest of paper.

Definition 2.4 *[33] The soft set* $F \in S_E(X)$ *is called a soft point if there exist an* $e \in E$ *such that* $F(e) \neq \emptyset$ *and* $F(e') = \emptyset$ *for each* $e' \in E \setminus \{e\}$*, and the soft point F is denoted by* e_F*. The soft point* e_F *is said to be in the soft set G, denoted by* $e_F \tilde{\in} G$*, if* $F(e) \tilde{\subseteq} G(e)$ *for the element* $e \in E$.

Definition 2.5 *[21, 28]Let* $F \in S_E(X)$*. If* $F(e) \neq \emptyset$ *and* $F(e') = \emptyset$ *for all* $e' \in E \setminus \{e\}$*, then F is called soft point and denoted by* e_F*. Thus* $e_F = (e, F(e))$ *and it can be seen clearly that* $\{e_F\} \in S_E(X)$.

Definition 2.6 *[13] Two soft sets* $F, G \in S_E(X)$ *are said to be disjoint, written* $F \tilde{\cap} G = \tilde{\Phi}$*, if* $F(e) \cap G(e) = \emptyset$*, for all* $e \in E$.

Definition 2.7 *[13] Two soft points* $\{a_F\}, \{b_G\} \in S_E(X)$ *are said to be distinct, written* $a_F \neq b_G$*, if* $F(a) \cap G(b) = \emptyset$.

Definition 2.8 *[6] Let* $S_E(X)$ *and* $S_K(Y)$ *be families of soft sets,* $u : X \to Y$ *and* $p : E \to K$ *be mappings. Therefore* $f_{pu} : S_E(X) \to S_K(Y)$ *is called a soft function.*

i. *If* $F \in S_E(X)$*, then the image of F under* f_{pu}*, written as* $f_{pu}(F)$*, is a soft set in* $S_K(Y)$ *such that*

$$f_{pu}(F)(k) = \begin{cases} \bigcup_{e \in p^{-1}(k)} u(F(e)), & p^{-1}(k) \neq \emptyset \\ \emptyset, & otherwise. \end{cases}$$

for each $k \in Y$.

ii. *If* $G \in S_K(Y)$*, then the inverse image of G under* f_{pu}*, written as* $f_{pu}^{-1}(G)$*, is a soft set in* $S_E(X)$ *such that*

$$f_{pu}^{-1}(G)(e) = \begin{cases} u^{-1}(G(p(e))), & p(e) \in Y \\ \emptyset, & otherwise. \end{cases}$$

for each $e \in E$.

The soft function f_{pu} *is called surjective if p and u are surjective, also is said to be injective if p and u are injective.*

Theorem 2.1 *[6] Let* $S_E(X)$ *and* $S_K(Y)$ *be families of soft sets. For the soft function* $f_{pu} : S_E(X) \to S_K(Y)$*, for each* $F, F_1, F_2 \in S_E(X)$ *and for each* $G, G_1, G_2 \in S_K(Y)$ *the following statements hold,*

i. $f_{pu}^{-1}(G^{\tilde{c}}) = (f_{pu}^{-1}(G))^{\tilde{c}}$.

ii. $f_{pu}(f_{pu}^{-1}(G)) \tilde{\subseteq} G$*. If* f_{pu} *is surjective, then the equality holds.*

iii. $F \tilde{\subseteq} f_{pu}^{-1}(f_{pu}(F))$*. If* f_{pu} *is injective, then the equality holds.*

iv. $f_{pu}(\tilde{X}) \tilde{\subseteq} \tilde{Y}$*. If* f_{pu} *is surjective, then the equality holds.*

v. $f_{pu}^{-1}(\tilde{Y}) = \tilde{X}$ *and* $f_{pu}(\tilde{\Phi}) = \tilde{\Phi}$.

vi. *If* $F_1 \tilde{\subseteq} F_2$, *then* $f_{pu}(F_1) \tilde{\subseteq} f_{pu}(F_2)$.

vii. *If* $G_1 \tilde{\subseteq} G_2$, *then* $f_{pu}^{-1}(G_1) \tilde{\subseteq} f_{pu}^{-1}(G_2)$.

viii. $f_{pu}^{-1}(G_1 \tilde{\cup} G_2) = f_{pu}^{-1}(G_1) \tilde{\cup} f_{pu}^{-1}(G_2)$ *and* $f_{pu}^{-1}(G_1 \tilde{\cap} G_2) = f_{pu}^{-1}(G_1) \tilde{\cap} f_{pu}^{-1}(G_2)$.

ix. $f_{pu}(F_1 \tilde{\cup} F_2) = f_{pu}(F_1) \tilde{\cup} f_{pu}(F_2)$ *and* $f_{pu}(F_1 \tilde{\cap} F_2) \tilde{\subseteq} f_{pu}(F_1) \tilde{\cap} f_{pu}(F_2)$. *If* f_{pu} *is injective, then the equality holds.*

Definition 2.9 *[28] Let* τ *be a collection of soft sets over a universe* X *with a fixed set of parameters* E, *then* $\tau \subseteq S_E(X)$ *is called a soft topology on* X *if*

i. $\tilde{X}, \tilde{\Phi} \in \tau$,

ii. *The soft intersection of any two soft sets in* τ *belongs to* τ,

iii. *The soft union of any number of soft sets in* τ *belongs to* τ.

If τ *is a soft topology over* X, *then triplet* (X, τ, E) *is called a soft topological space over* X. *A soft set* F *over* X *is said to be open soft set in* X *if* $F \in \tau$, *and it is said to be closed soft set in* X, *if its soft complement* $F^{\tilde{c}}$ *is an open soft set.*

Definition 2.10 *[28] Let* (X, τ, E) *be a soft topological space over* X *and* $F \in S_E(X)$. *Then, the soft interior and soft closure of* F, *denoted by* $int(F)$ *and* $cl(F)$, *respectively, are defined as,*

$$
\begin{aligned}
int(F) &= \tilde{\bigcup}\{G : G \text{ is open soft set and } G \tilde{\subseteq} F\} \\
cl(F) &= \tilde{\bigcap}\{H : H \text{ is closed soft set and } F \tilde{\subseteq} H\}.
\end{aligned}
$$

If there exists at least two soft topologies τ and σ over X, then to avoid confusion it can be written $int_\tau(F)$ and $int_\sigma(F)$ for $F \in S_E(X)$.

Definition 2.11 *[33] Let* (X, τ, E) *and* (Y, σ, K) *be soft topological spaces and* $f_{pu} : S_E(X) \to S_K(Y)$ *be a function. Then, the function* f_{pu} *is called,*

i. *Continuous soft if* $f_{pu}^{-1}(G) \in \tau$ *for each* $G \in \sigma$.

ii. *Open soft if* $f_{pu}(F) \in \sigma$ *for each* $F \in \tau$.

Definition 2.12 *[14] Let* (X, τ, E) *be a soft topological space and* $F \in S_E(X)$. *Then* F *is said to be,*

i. *Pre open soft set if* $F \tilde{\subseteq} int(cl(F))$,

ii. *Semi open soft set if* $F \tilde{\subseteq} cl(int(F))$,

iii. α*-open soft set if* $F \tilde{\subseteq} int(cl(int(F)))$,

iv. β*-open soft set if* $F \tilde{\subseteq} cl(int(cl(F)))$.

The set of all pre open (resp. semi open, α*-open,* β*-open) soft sets is denoted by* $POS(X)$ *(resp.* $SOS(X)$, $\alpha OS(X)$, $\beta OS(X)$*) and the set of all pre closed (resp. semi closed,* α*-closed,* β*-closed) soft sets is denoted by* $PCS(X)$ *(resp.* $SCS(X)$, $\alpha CS(X)$, $\beta CS(X)$*).*

Definition 2.13 *[12] Let* μ *be a collection of soft sets over a universe* X *with a fixed set of parameters* E, *then* $\mu \subseteq S_E(X)$ *is called supra soft topology on* X *with a fixed set* E *if* *i.* $\tilde{X}, \tilde{\Phi} \in \mu$,

ii. *The soft union of any number of soft sets in* μ *belongs to* μ.

The triplet (X, μ, E) *is called supra soft topological space (or supra soft spaces) over* X.

Definition 2.14 *[12] Let* (X, τ, E) *be a soft topological space and* (X, μ, E) *be a supra soft topological space. We say that,* μ *is a supra soft topology associated with* τ *if* $\tau \subseteq \mu$.

Definition 2.15 *[12] Let* (X, μ, E) *be a supra soft topological space over* X*, then the members of* μ *are said to be supra open soft sets in* X*. We denote the set of all supra open soft sets over* X *by* $supra\text{-}OS(X, \mu, E)$*, or when there can be no confusion by* $supra\text{-}OS(X)$ *and the set of all supra closed soft sets by* $supra\text{-}CS(X, \mu, E)$ *or* $supra\text{-}CS(X)$.

Definition 2.16 *[12] Let* (X, μ, E) *be a supra soft topological space over* X *and* $F \in S_E(X)$*. Then the supra soft interior of* F*, denoted by* $int^s(F)$ *is the soft union of all supra open soft subsets of* F*. Clearly* $int^s(F)$ *is the largest supra open soft set over* X *which contained in* F *i.e*

$$int^s(F) = \tilde{\bigcup}\{G : G \text{ is a supra open soft set and } G\tilde{\subseteq}F\}.$$

Definition 2.17 *[12] Let* (X, μ, E) *be a supra soft topological space over* X *and* $F \in S_E(X)$*. Then the supra soft closure of* F*, denoted by* $cl^s(F)$ *is the soft intersection of all supra closed super soft sets of* F*. Clearly* $cl^s(F)$ *is the smallest supra closed soft set over* X *which contains* F *i.e*

$$cl^s(F) = \tilde{\bigcap}\{H : H \text{ is a supra closed soft set and } F\tilde{\subseteq}H\}.$$

Definition 2.18 *[12] Let* (X, τ, X) *and* (Y, σ, Y) *be two soft topological spaces,* μ *be an associated supra soft topology with* τ *and* $f_{pu} : S_E(X) \to S_K(Y)$ *be a soft function. Then, the function* f_{pu} *is called supra continuous soft function if* $f_{pu}^{-1}(G) \in \mu$ *for all* $G \in \sigma$.

3 Supra soft separation axioms

In this section, we will introduce soft separation axioms and their some properties on supra-soft topological spaces.

Definition 3.1 *Let* (X, τ, E) *be a soft topological space and* μ *be an associated supra soft topology with* τ*. Let* $\{a_F\}, \{b_G\} \in S_E(X)$ *such that* $a_F \neq b_G$*. Then,* (X, μ, E) *is called*

i. *Supra soft* T_0*-space if there exists a supra open soft set* H *containing one of the soft points* a_F, b_G *but not the other.*

ii. *Supra soft* T_1*-space if there exist supra open soft sets* H_1 *and* H_2 *such that* $a_F \tilde{\in} H_1$, $b_G \tilde{\notin} H_1$ *and* $b_G \tilde{\in} H_2$, $a_F \tilde{\notin} H_2$.

iii. *Supra soft Hausdorff space or supra soft* T_2*-space if there exist supra open soft sets* H_1 *and* H_2 *such that* $a_F \tilde{\in} H_1$, $b_G \tilde{\in} H_2$ *and* $H_1 \tilde{\cap} H_2 = \tilde{\Phi}$.

Proposition 3.1 *Let* (X, τ, E) *be a soft topological space and* $\{a_F\}, \{b_G\} \in S_E(X)$ *such that* $a_F \neq b_G$*. If there exist supra open soft sets* H_1 *and* H_2 *such that either* $a_F \tilde{\in} H_1$ *and* $b_G \tilde{\notin} H_1^{\tilde{c}}$ *or* $b_G \tilde{\in} H_2$ *and* $a_F \tilde{\notin} H_2^{\tilde{c}}$*. Then,* (X, μ, E) *is supra soft* T_0*-space.*

Proof. It is obvious.

Theorem 3.1 *A supra soft topological space (X, μ, E) is supra soft T_0-space if and only if the supra closures of each distinct points a_F and b_G are distinct.*

Proof. $(\Rightarrow)$: Let (X, μ, E) be a supra soft T_0-space and $\{a_F\}, \{b_G\} \in S_E(X)$ such that $a_F \neq b_G$. Then, there exists a supra open soft set H such that $a_F \tilde{\in} H$ and $b_G \tilde{\notin} H$. Hence, $H^{\tilde{c}}$ is supra closed soft set containing b_G but not a_F. It follows that, $cl^s(\{b_G\}) \tilde{\subseteq} H^{\tilde{c}}$. Therefore, $a_F \tilde{\notin} cl^s(\{b_G\})$. Thus, $cl^s(\{a_F\}) \neq cl^s(\{b_G\})$.

$(\Leftarrow)$: Let a_F and b_G be two distinct points in X such that $cl^s(\{a_F\}) \neq cl^s(\{b_G\})$. Then, there exists a soft point c_H belongs to one of the sets $cl^s(\{a_F\}), cl^s(\{b_G\})$ but not the other. Say, $c_H \tilde{\in} cl^s(\{a_F\})$ and $c_H \tilde{\notin} cl^s(\{b_G\})$. Now, if $a_F \tilde{\in} cl^s(\{b_G\})$, then, $cl^s(\{a_F\}) \tilde{\subseteq} cl^s(\{b_G\})$, which is a contradiction with $c_H \tilde{\notin} cl^s(\{b_G\})$. So, $a_F \tilde{\notin} cl^s(\{b_G\})$. Hence, $[cl^s(\{b_G\})]^{\tilde{c}}$ is supra open soft set containing a_F but not b_G. Thus, (X, μ, E) is supra soft T_0-space.

Proposition 3.2 *Let (X, τ, E) be a soft topological space and $\{a_F\}, \{b_G\} \in S_E(X)$ such that $a_F \neq b_G$. If there exist two supra open soft sets H_1 and H_2 such that $a_F \tilde{\in} H_1$ and $b_G \tilde{\notin} H_1^{\tilde{c}}$ and $b_G \tilde{\in} H_2$ and $a_F \tilde{\notin} H_2^{\tilde{c}}$. Then (X, τ, E) is a supra soft T_1-space.*

Proof. It is clear.

Theorem 3.2 *Every supra soft T_i-space is supra soft T_{i-1} for each $i = 1, 2$.*

Proof. Obvious from Definition 3.1.

Remark 3.1 *The converse of Theorem 3.2 is not true in general, as following examples shall show.*

Example 3.1 *Let $X = \{x, y, z\}$, $E = \{a, b\}$ and $\tau = \{\tilde{X}, \tilde{\Phi}, H\}$ where H is a soft sets over X defined as follows:*

$$H = \{(a, \{x\}), (b, X)\}.$$

Then, τ defines a soft topology on X. Consider the associated supra soft topology μ with τ is defined as

$$\mu = \{\tilde{X}, \tilde{\Phi}, H, M\}$$

where H and M are soft sets over X defined as follows:

$$M = \{(a, X), (b, \{y\})\}.$$

Therefore, (X, μ, E) is supra soft T_1-space, but it is not supra soft T_2-space, for $a_F = (a, X)$ and $b_G = (b, X)$, but there are no disjoint supra open soft sets N_1 and N_2 such that $a_F \tilde{\in} N_1$ and $b_G \tilde{\in} N_2$.

Example 3.2 *Let $X = \{x, y, z\}$, $E = \{a, b\}$ and $\tau = \{\tilde{X}, \tilde{\Phi}, H\}$ where H is soft set over X defined as follows by*

$$H = \{(a, \{x, y\}), (b, \{x, y\})\}.$$

Then, τ defines a soft topology on X. The associated supra soft topology μ with τ is defined as $\mu = \{\tilde{X}, \tilde{\Phi}, H, M, N\}$, where M and N are two soft sets over X defined as follows:

$$\begin{aligned} M &= \{(a, \{x\}), (b, \{x\})\} \\ N &= \{(a, \{y, z\}), (b, \{y, z\})\}. \end{aligned}$$

Hence, (X, μ, E) is supra soft T_0-space, but it is not supra soft T_1-space.

Theorem 3.3 *Let (X, μ, E) be a supra soft topological space. If every soft point is supra closed soft set in μ, then (X, μ, E) is supra soft T_1-space.*

Proof. Suppose that every soft point is supra closed soft set in μ. Then, $\{a_F\}^{\tilde{c}}$ is supra open soft set in μ. Let $\{b_G\} \in S_E(X)$ such that $a_F \neq b_G$. Therefore $a_F \tilde{\in} \{b_G\}^{\tilde{c}}$, $b_G \tilde{\notin} \{b_G\}^{\tilde{c}}$ and $b_G \tilde{\in} \{a_F\}^{\tilde{c}}$, $a_F \tilde{\notin} \{a_F\}^{\tilde{c}}$. Thus, (X, μ, E) is supra soft T_1-space over X.

Remark 3.2 *The converse of Theorem 3.3 is not true in general, as following example shall show.*

Example 3.3 *Let $X = \{x, y\}$, $E = \{a, b\}$ and $\tau = \{\tilde{X}, \tilde{\Phi}, H\}$ where H is a soft sets over X defined as follows:*

$$H = \{(a, X), (b, \{y\})\}$$

Then, τ defines a soft topology on X. Consider the associated supra soft topology μ with τ is defined as $\mu = \{\tilde{X}, \tilde{\Phi}, H, M\}$ where M is a soft set over X defined as follows:

$$M = \{(a, \{x\}), (b, X)\}$$

Then, μ defines a supra soft topology on X. Therefore, (X, μ, E) is a supra soft T_1-space. On the other hand, we note that for the soft point $a_F = (a, \{x\})$, the complement $\{a_F\}^{\tilde{c}} = \{(a, \{y\}), (b, X)\}$ is not supra-open soft set. This shows that, the converse of the above theorem does not hold. Also, we have $\mu_a = \{X, \emptyset, \{x\}\}$ and $\mu_b = \{X, \emptyset, \{y\}\}$. Neither (X, μ_a) nor (X, μ_b) is a supra T_1-space [24], at the time that (X, μ, E) is a supra soft T_1-space.

Definition 3.2 *Let (X, τ, E) be a soft topological space and μ be an associated supra soft topology with τ. Let G be a supra closed soft set in X and $\{e_F\} \in S_E(X)$ such that $e_F \tilde{\notin} G$. If there exist supra open soft sets H_1 and H_2 such that $e_F \tilde{\in} H_1$, $G \tilde{\subseteq} H_2$, and $H_1 \tilde{\cap} H_2 = \tilde{\Phi}$, then (X, μ, E) is called a supra soft regular space. A supra soft regular T_1-space is called supra soft T_3-space.*

Proposition 3.3 *Let (X, τ, E) be a soft topological space and μ be an associated supra soft topology with τ. Let G be a supra closed soft set in X and $\{e_F\} \in S_E(X)$ such that $e_F \tilde{\notin} G$. If (X, μ, E) is a supra soft regular space, then there exists a supra open soft set H such that $e_F \tilde{\in} H$ and $H \tilde{\cap} G = \tilde{\Phi}$.*

Proof. Obvious from Definition 3.2.

Proposition 3.4 *Every soft T_i-space is supra soft T_i for each $i = 0, 1, 2, 3$.*

Proof. Obvious.

Proposition 3.5 *Let (X, μ, E) be a supra soft topological space and $G, \{e_F\} \in S_E(X)$. Then:*

i. $e_F \tilde{\in} G$ *if and only if* $\{e_F\} \tilde{\subseteq} G$,

ii. *If* $\{e_F\} \tilde{\cap} (F) = \tilde{\Phi}$, *then* $e_F \tilde{\notin} G$.

Proof. Obvious.

Theorem 3.4 *Let (X, μ, E) be a supra soft topological space and $\{e_F\} \in S_E(X)$. If (X, μ, E) is a supra soft regular space, then*

i. $e_F \tilde{\notin} G$ *if and only if* $\{e_F\} \tilde{\cap} G = \tilde{\Phi}$ *for every supra closed soft set* G.

ii. $e_F \tilde{\notin} H$ *if and only if* $\{e_F\} \tilde{\cap} H = \tilde{\Phi}$ *for every supra open soft set* H.

Proof.

i. Let G be a supra closed soft set such that $e_F \tilde{\notin} G$. Since (X, τ, E) is supra soft regular space. By Theorem 3.3 there exists a supra open soft set H such that $e_F \tilde{\in} H$ and $G \tilde{\cap} H = \tilde{\Phi}$. It follows that, $\{e_F\} \tilde{\subseteq} H$ from Theorem 3.5 *i*. Hence, $\{e_F\} \tilde{\cap} G = \tilde{\Phi}$. Conversely, if $\{e_F\} \tilde{\cap} G = \tilde{\Phi}$, then $e_F \tilde{\notin} G$ from Theorem 3.5 *ii*.

ii. It can be proved similar to *i*.

Corollary 3.1 *Let (X, μ, E) be a supra soft topological space. If (X, μ, E) is supra soft regular space, then the following are equivalent:*

i. (X, τ, E) *is supra soft* T_1*-space.*

ii. For all $\{a_F\}, \{b_G\} \in S_E(X)$ *such that* $a_F \neq b_G$, *there exist supra open soft sets* H_1 *and* H_2 *such that* $\{a_F\} \tilde{\subseteq} H_1$ *and* $\{b_G\} \tilde{\cap} H_1 = \tilde{\Phi}$ *and* $\{b_G\} \tilde{\subseteq} H_2$ *and* $\{a_F\} \tilde{\cap} H_2 = \tilde{\Phi}$.

Proof. It is obvious from Theorem 3.4.

Theorem 3.5 *Let (X, μ, E) be a supra soft topological space and $\{e_F\} \in S_E(X)$. Then, the following are equivalent:*

i. (X, μ, E) *is supra soft regular space.*

ii. For every supra closed soft set G such that $\{e_F\} \tilde{\cap} G = \tilde{\Phi}$, *there exist supra open soft sets* H_1 *and* H_2 *such that* $\{e_F\} \tilde{\subseteq} H_1$, $G \tilde{\subseteq} H_2$ *and* $H_1 \tilde{\cap} H_2 = \tilde{\Phi}$.

Proof.

i. ⇒ *ii.* Let G be a supra closed soft set such that $\{e_F\} \tilde{\cap} G = \tilde{\Phi}$. Then, $e_F \tilde{\notin} G$ from Theorem 3.4 *i*. It follows by *i*., there exist supra open soft sets H_1 and H_2 such that $e_F \tilde{\in} H_1$, $G \tilde{\subseteq} H_2$ and $H_1 \tilde{\cap} H_2 = \tilde{\Phi}$. This means that, $\{e_F\} \tilde{\subseteq} H_1$, $G \tilde{\subseteq} H_2$ and $H_1 \tilde{\cap} H_2 = \tilde{\Phi}$.

ii. ⇒ *i.* Let G be a supra closed soft set such that $e_F \tilde{\notin} G$. Then, $\{e_F\} \tilde{\cap} G = \tilde{\Phi}$ from Theorem 3.4 *i*. It follows by *ii*., there exist supra open soft sets H_1 and H_2 such that $\{e_F\} \tilde{\subseteq} H_1$, $G \tilde{\subseteq} H_2$ and $H_1 \tilde{\cap} H_2 = \tilde{\Phi}$. Hence, $e_F \tilde{\in} H_1$, $G \tilde{\subseteq} H_2$ and $H_1 \tilde{\cap} H_2 = \tilde{\Phi}$. Thus, (X, τ, E) is supra soft regular space.

Theorem 3.6 *Let (X, μ, E) be a supra soft topological space. If (X, μ, E) is a supra soft T_3-space, then each soft point $\{a_F\} \in S_E(X)$ is a supra closed soft set.*

Proof. We want to prove that a_F is supra closed soft set, which is sufficient to prove that $\{a_F\}^{\tilde{c}}$ is a supra open soft set. Let $b_G \tilde{\in} \{a_F\}^{\tilde{c}}$. Since (X, μ, E) is supra soft T_3-space. Then there exist supra open soft sets H_1 and H_2 such that $\{b_G\} \tilde{\subseteq} H_1$, $\{a_F\} \tilde{\cap} H_1 = \tilde{\Phi}$ and $\{a_F\} \tilde{\subseteq} H_2$ and $\{b_G\} \tilde{\cap} H_2 = \tilde{\Phi}$. It follows that,

$$\tilde{\bigcup}_{\substack{\{b_G\} \tilde{\subseteq} H_1 \\ \{a_F\} \tilde{\cap} H_1 = \tilde{\Phi}}} H_1 \tilde{\subseteq} \{a_F\}^{\tilde{c}}. \tag{3.1}$$

If we denote with H the left side of the above equality, the we have

$$H(e)=\bigcup_{\substack{\{b_G\}\tilde{\subseteq}H_1\\ \{a_F\}\tilde{\cap}H_1=\tilde{\Phi}}} H_1(e)$$

for all $e\in E$. Therefore we can write

$$\{a_F\}^{\tilde{c}}\tilde{\subseteq}\tilde{\bigcup}_{\substack{\{b_G\}\tilde{\subseteq}H_1\\ \{a_F\}\tilde{\cap}H_1=\tilde{\Phi}}} H_1. \tag{3.2}$$

From (3.1) and (3.2), we have

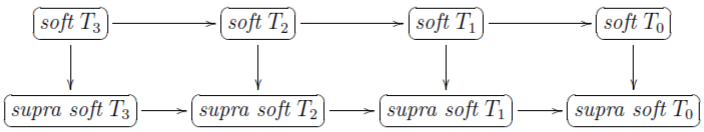
$$\{a_F\}^{\tilde{c}}=\tilde{\bigcup}_{\substack{\{b_G\}\tilde{\subseteq}H_1\\ \{a_F\}\tilde{\cap}H_1=\tilde{\Phi}}} H_1$$

Thus, $\{a_F\}$ is a supra closed soft set.

Theorem 3.7 *Every supra soft T_3-space is supra soft T_2-space.*

Proof. Let (X,μ,E) be a supra soft T_3-space and $\{a_F\},\{b_G\}\in S_E(X)$ such that $a_F\neq b_G$. By Theorem 3.6, $\{b_G\}$ is a supra closed soft set and $a_F\tilde{\notin}\{b_G\}$. It follows from the supra soft regularity, there exist a supra open soft sets H_1 and H_2 such that $a_F\tilde{\in}H_1$, $\{b_G\}\tilde{\subseteq}H_2$ and $H_1\tilde{\cap}H_2=\tilde{\Phi}$. Thus, $a_F\tilde{\in}H_1$, $b_G\tilde{\in}\{b_G\}\tilde{\subseteq}H_2$ and $H_1\tilde{\cap}H_2=\tilde{\Phi}$. Therefore, (X,μ,E) is supra soft T_2-space.

Corollary 3.2 *The following implications hold from Theorem 3.2,Theorem 3.4 and Theorem 3.7 for a supra soft topological space (X,μ,E). These implications are not reversible.*



Definition 3.3 *Let (X,μ,E) be a supra soft topological space, F and G be two supra closed soft sets in X such that $F\tilde{\cap}G=\tilde{\Phi}$. If there exist supra open soft sets H_1 and H_2 such that $F\tilde{\subseteq}H_1$, $G\tilde{\subseteq}H_2$ and $H_1\tilde{\cap}H_2=\tilde{\Phi}$, then (X,μ,E) is called supra soft normal space. A supra soft normal T_1-space is called a supra soft T_4-space.*

Theorem 3.8 *Let (X,μ,E) be a supra soft topological space and $\{e_F\}\in S_E(X)$. Then, the following are equivalent:*

i. *(X,μ,E) is supra soft normal space.*

ii. *For every supra closed soft set G and supra open soft set H such that $G\tilde{\subseteq}H$, there exists a supra open soft set H such that $G\tilde{\subseteq}H$ and $cl^s(H)\tilde{\subseteq}G$.*

Proof.

i. $\Rightarrow$ *ii.* Let F be a supra closed soft set and G be a supra open soft set such that $F\tilde{\subseteq}G$. Then, $F, G^{\tilde{c}}$ are supra closed soft sets such that $F\tilde{\cap}G^{\tilde{c}}=\tilde{\Phi}$. It follows by *i.*, there exist supra open soft sets H_1 and H_2 such that $F\tilde{\subseteq}H_1$, $G^{\tilde{c}}\tilde{\subseteq}H_2$ and $H_1\tilde{\cap}H_2=\tilde{\Phi}$. Now, $H_1\tilde{\subseteq}H_2^{\tilde{c}}$, so $cl^s(H_1)\tilde{\subseteq}cl^s(H_2^{\tilde{c}})=H_2^{\tilde{c}}$.

i. $\Rightarrow$ *ii.* Let G_1,G_2 be supra closed soft sets such that $G_1\tilde{\cap}G_2=\tilde{\Phi}$. Then $G_1\tilde{\subseteq}G_2^{\tilde{c}}$, then by hypothesis, there exists a supra open soft set H such that $G_1\tilde{\subseteq}H$, $cl^s(H)\tilde{\subseteq}G_2^{\tilde{c}}$. So, $G_2\tilde{\subseteq}[cl^s(H)]^{\tilde{c}}$, $G_1\tilde{\subseteq}H$ and $[cl^s(H)]^{\tilde{c}}\tilde{\cap}H=\tilde{\Phi}$, where H and $[cl^s(H)]^{\tilde{c}}$ are supra open soft sets. Thus, (X,μ,E) is supra soft normal space.

Theorem 3.9 *Let (X, μ, E) be a supra soft topological space. If (X, μ, E) is supra soft normal space and a_F is a supra closed soft set, then (X, τ, E) is supra soft T_3-space.*

Proof. Since a_F is a supra closed soft set, then (X, μ, E) is supra soft T_1-space from Theorem 3.3. Also, (X, τ, E) is supra soft regular space from Theorem 3.5 and Definition 3.3. Hence, (X, μ, E) is supra soft T_3-space.

Proposition 3.6 *Not every open (supra open) soft subspace of supra soft T_i-space is supra soft T_i-space for each $i = 0, 1, 2, 3, 4$.*

Proof. Obvious from the fact that, the soft intersection of an open soft set and a supra open soft set need not to be supra open soft.

Remark 3.3 *Many properties of soft (resp. semi soft, pre soft, α-soft and β-soft) separation axioms Mentioned in [9, 21, 25, 28, 29, 30, 31, 32] can easily obtained here from the properties of supra soft septation axioms by setting $\mu = \tau$ (resp. $SOS(X)$, $POS(X)$, $\alpha OS(X)$ and $\beta OS(X)$).*

4 Supra irresolute soft functions

Definition 4.1 *Let (X, τ, E) and (Y, σ, K) be soft topological spaces, μ be an associated supra soft topology with τ and $f_{pu} : S_E(X) \to S_K(Y)$ be a soft function. Then, the soft function f_{pu} is called supra open soft if $f_{pu}(F) \in \mu$ for each $F \in \tau$.*

Definition 4.2 *Let (X, τ, E) and (Y, σ, K) be soft topological spaces, μ and ν be associated supra soft topologies with τ and σ, respectively. The soft function $f_{pu} : S_E(X) \to S_K(Y)$ is called*

i. Supra irresolute soft if $f_{pu}^{-1}(G) \in \mu$ for each $G \in \nu$.

ii. Supra irresolute open soft if $f_{pu}(F) \in \nu$ for each $F \in \mu$.

Theorem 4.1 *Let (X, τ, E) and (Y, σ, K) be soft topological spaces, μ and ν be associated supra soft topologies with τ and σ, respectively and $f_{pu} : S_E(X) \to S_K(Y)$ be a soft function which is bijective and supra irresolute open soft. If (X, τ, E) is supra soft T_0-space, then (Y, σ, K) is also a supra soft T_0-space.*

Proof. Let $b_{G_1}, b'_{G_2} \in S_K(Y)$ such that $b_{G_1} \neq b'_{G_2}$. Since f_{pu} is surjective, then there exist $a_{F_1}, a'_{F_2} \in S_E(X)$ such that $f_{pu}(a_{F_1}) = b_{G_1}$, $f_{pu}(a'_{F_2}) = b'_{G_2}$ and $a_{F_1} \neq a'_{F_2}$. By hypothesis, there exist supra open soft sets H_1 and H_2 in X such that either $a_{F_1} \tilde{\in} H_1$ and $a'_{F_2} \tilde{\notin} H_1$, or $a'_{F_2} \tilde{\in} H_2$ and $a_{F_1} \tilde{\notin} H_1$. Hence, either $b_{G_1} \tilde{\in} f_{pu}(H_1)$ and $b'_{G_2} \tilde{\notin} f_{pu}(H_1)$ or $b'_{G_2} \tilde{\in} f_{pu}(H_2)$ and $b_{G_1} \tilde{\notin} f_{pu}(H_2)$. Since f_{pu} is supra irresolute open soft function, then $f_{pu}(H_1)$ and $f_{pu}(H_2)$ are supra open soft sets in Y. Hence, (Y, σ, K) is also a supra soft T_0-space.

Theorem 4.2 *Let (X, τ, E) and (Y, σ, K) be soft topological spaces, μ and ν be associated supra soft topologies with τ and σ, respectively. Let $f_{pu} : S_E(X) \to S_K(Y)$ be a soft function which is bijective and supra irresolute open soft. If (X, τ, E) is supra soft T_1-space, then (Y, σ, K) is also a supra soft T_1-space.*

Proof. It is similar to the proof of Theorem 4.1.

Theorem 4.3 *Let (X,τ,E) and (Y,σ,K) be soft topological spaces, μ and ν be associated supra soft topologies with τ and σ, respectively. Let $f_{pu}: S_E(X) \to S_K(Y)$ be a soft function which is bijective and supra irresolute open soft. If (X,τ,E) is supra soft T_2-space, then (Y,σ,K) is also a supra soft T_2-space.*

Proof. $b_{G_1}, b'_{G_2} \in S_K(Y)$ such that $b_{G_1} \neq b'_{G_2}$. Since f_{pu} is surjective, then there exist $a_{F_1}, a'_{F_2} \in S_E(X)$ such that $f_{pu}(a_{F_1}) = b_{G_1}$, $f_{pu}(a'_{F_2}) = b'_{G_2}$ and $a_{F_1} \neq a'_{F_2}$. By hypothesis, there exist supra open soft sets H_1 and H_2 in X such that $a_{F_1} \tilde{\in} H_1$, $a'_{F_2} \tilde{\in} H_2$ and $H_1 \tilde{\cap} H_2 = \tilde{\Phi}$. Hence, $b_{G_1} \tilde{\in} f_{pu}(H_1)$, $b'_{G_2} \tilde{\in} f_{pu}(H_2)$ and

$$f_{pu}(H_1)\tilde{\cap} f_{pu}(H_2) = f_{pu}(H_1 \tilde{\cap} H_2) = f_{pu}(\tilde{\Phi}) = \tilde{\Phi}$$

from Theorem 2.1. Since f_{pu} is supra irresolute open soft function, then $f_{pu}(H_1)$ and $f_{pu}(H_2)$ are supra open soft sets in Y. Thus, (Y,σ,K) is also a supra soft T_2-space.

Theorem 4.4 *Let (X,τ,E) and (Y,σ,K) be soft topological spaces, μ and ν be associated supra soft topologies with τ and σ, respectively. Let $f_{pu}: S_E(X) \to S_K(Y)$ be a soft function which is bijective, supra irresolute soft and supra irresolute open soft. If (X,τ,E) is supra soft regular space, then (Y,σ,K) is also a supra soft regular space.*

Proof. Let M be a supra closed soft set in Y and $\{b_G\} \in S_K(Y)$ such that $b_G \tilde{\notin} M$. Since f_{pu} is surjective and supra irresolute soft, then there exists $\{a_F\} \in S_E(X)$ such that $f_{pu}(a_F) = b_G$ and $f_{pu}^{-1}(M)$ is supra closed soft set in X such that $a_F \tilde{\notin} f_{pu}^{-1}(M)$. By hypothesis, there exist supra open soft sets H_1 and H_2 in X such that $a_F \tilde{\in} H_1$, $f_{pu}^{-1}(M) \tilde{\subseteq} H_2$ and $H_1 \tilde{\cap} H_2 = \tilde{\Phi}$. It follows that, $M = f_{pu}(f_{pu}^{-1}(M)) \tilde{\subseteq} f_{pu}(H_2)$ from Theorem 2.1. Hence, $b_G \tilde{\in} f_{pu}(H_1)$, $M \tilde{\subseteq} f_{pu}(H_2)$ and

$$f_{pu}(H_1)\tilde{\cap} f_{pu}(H_2) = f_{pu}(H_1 \tilde{\cap} H_2) = f_{pu}(\tilde{\Phi}) = \tilde{\Phi}$$

from Theorem 2.1. Since f_{pu} is supra irresolute open soft function. Then, $f_{pu}(H_1)$ and $f_{pu}(H_2)$ are supra open soft sets in Y. Thus, (Y,σ,K) is also a supra soft regular space.

Theorem 4.5 *Let (X,τ,E) and (Y,σ,K) be soft topological spaces, μ and ν be associated supra soft topologies with τ and σ, respectively. Let $f_{pu}: S_E(X) \to S_K(Y)$ be a soft function which is bijective, supra irresolute soft and supra irresolute open soft. If (X,τ,E) is supra soft T_3-space, then (Y,σ,K) is also a supra soft T_3-space.*

Proof. Since (X,τ,E) is supra soft T_3-space, then (X,τ,E) is supra soft regular T_1-space. It follows that, (Y,σ,K) is also a supra soft T_1-space from Theorem 4.2 and supra soft regular space from Theorem 4.4. Hence, (Y,σ,K) is also a supra soft T_3-space.

Theorem 4.6 *Let (X,τ,E) and (Y,σ,K) be soft topological spaces, μ and ν be associated supra soft topologies with τ and σ, respectively. Let $f_{pu}: S_E(X) \to S_K(Y)$ be a soft function which is bijective, supra irresolute soft and supra irresolute open soft. If (X,τ,E) is supra soft normal space, then (Y,σ,K) is also a supra soft normal space.*

Proof. Let M_1, M_2 be supra closed soft sets in Y such that $M_1 \tilde{\cap} M_2 = \tilde{\Phi}$. Since f_{pu} is supra irresolute soft, then $f_{pu}^{-1}(M_1)$ and $f_{pu}^{-1}(M_2)$ are supra closed soft set in X such that

$$f_{pu}^{-1}(M_1)\tilde{\cap} f_{pu}^{-1}(M_2) = f_{pu}^{-1}(M_1 \tilde{\cap} M_2) = f_{pu}^{-1}(\tilde{\Phi}) = \tilde{\Phi}$$

from Theorem 2.1. By hypothesis, there exist supra open soft sets H_1 and H_2 in X such that $f_{pu}^{-1}(M_1)\tilde{\subseteq}H_1$, $f_{pu}^{-1}(M_2)\tilde{\subseteq}H_2$ and $H_1\tilde{\cap}H_2=\tilde{\Phi}$. It follows that,

$$M_1=f_{pu}(f_{pu}^{-1}(M_1))\tilde{\subseteq}f_{pu}(H_1) \text{ and } M_2=f_{pu}(f_{pu}^{-1}(M_2))\tilde{\subseteq}f_{pu}(H_2)$$

from Theorem 2.1 and

$$f_{pu}(H_1)\tilde{\cap}f_{pu}(H_2)=f_{pu}(H_1\tilde{\cap}H_2)=f_{pu}(\tilde{\Phi})=\tilde{\Phi}$$

from Theorem 2.1. Since f_{pu} is supra irresolute open soft function. Then, $f_{pu}(H_1)$ and $f_{pu}(H_2)$ are supra open soft sets in Y. Thus, (Y,σ,K) is also a supra soft normal space.

Corollary 4.1 *Let (X,τ,E) and (Y,σ,K) be soft topological spaces, μ and ν be associated supra soft topologies with τ and σ, respectively. Let $f_{pu}:S_E(X)\rightarrow S_K(Y)$ be a soft function which is bijective, supra irresolute soft and supra irresolute open soft. If (X,τ,E) is supra soft T_4-space, then (Y,σ,K) is also a supra soft T_4-space.*

Proof. It is obvious from Theorem 4.2 and Theorem 4.6.

5 Conclusion

In this paper, we introduce and investigate some weak soft separation axioms by using the notion of supra open soft sets, which is a generalization of the soft (resp. semi soft, pre soft, α-soft and β-soft) separation axioms Mentioned in [9, 21, 25, 28, 29, 30, 31, 32]. In particular, we study the properties of the supra soft regular spaces and supra soft normal spaces. We show that, some classical results in general topology are not true if we consider soft topological spaces instead. For instance, if (X,μ,E) is supra soft T_1-space need not every soft singleton a_F is supra closed soft. As a direct generalization of the results which have been obtained here, by using the the notion of β-open soft sets [14] and b-open soft sets [11], can be found in [2, 5]. We hope that, the results in this paper will help researcher enhance and promote the further study on supra soft topology.

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