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Dynamics of Superior Anti-fractals in a New Orbit

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ABSTRACT. Anti-fractals have interesting features in the complex graphics of dynamical system. The aim of this paper is to visualize Superior tricorns and multicorns using a new iteration introduced by M. Abbas et al.[8] and study the pattern among the anti-fractals in the complex dynamics of anti-polynomial $z \to \bar{z}^m + c$, for $m \geq 2$.

1 Introduction

Fractals are defined as objects that appear to be broken into number of pieces and each piece is a copy of the entire shape. Fractal is the word taken from the Latin word fractus which means broken The term fractal was first used by a young mathematician, Julia [5] in 1918. Julia introduced the concept of iterative function system (IFS) and derived the Julia set in 1919. After that, in 1982, Mandelbrot [2] extended the work of Gaston Julia and introduced the Mandelbrot set, a set of all connected Julia sets. Many researchers have studied Julia and Mandelbrot sets from different aspects.

The polynomials $z \to \bar{z}^m + c$, for $m \geq 2$, have been studied mathematically using one step feedback process. Crowe et. al.[15] considered it as a formal analogy with Mandelbrot sets and named it as Mandelbar set. They also brought their bifurcation features along arcs rather than at points. Multicorns have been found in a real slice of the cubic connectedness locus [15]. Winter[13] showed that the boundary of the tricorn contains arc. The symmetries of tricorn and multicorns have been analyzed by Lau and Schleicher[4]. In 2003, Nakane and Schleicher[14] presented beautiful figures and quoted that multicorns are the generalized tricorns or the tricorns of higher order.

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The dynamics of anti-holomorphic complex polynomials $z \to \bar{z}^m + c$, for $m \geq 2$, was studied and explored to visualize interesting tri-corns and multi-corns anti-fractals with respect to one-step feedback process[12], two step-feedback process [10, 11], three-step feedback process[16] and four step feedback process[1,3]. In this paper we generate a new class of tricorns and multicorns using a new four-step feedback process[8] and analyze them.

2 Preliminaries and notations

Definition 2.1[12]. The multicorns A_c for the quadratic function $A_c(z) = \bar{z}^m + c$ is defined as the collection of all $c \in C$ for which the orbit of the point 0 is bounded, that is

$$A_c = \{c \in C : A_c^n(0) \text{ do not tend to} \infty\} \tag{2.1}$$

where C is a complex space.A_c^n is the n^{th} iterate of the function $A_c(z)$. An equivalent formulation is that the connectedness of loci for higher degree anti-hoomorphic polynomials $A_c(z) = \bar{z}^m + c$ are called multicorns.

Note that at $m = 2$, multicorns reduce to tricorn. Naturally, the tricorns lives in the real slice $d = \bar{c}$ in the two dimensional parameter space of maps $z \to (z^2 + d)^2 + c$. They have $(m+1)$-fold rotational symmetries. Also, by dividing these symmetries, the resulting multicorns are called unicorns [14].

Definition 2.2[6]. The filled in Julia set of the function g is defined as

$$K(g) = \{z \in C : g^k(z) \text{ does not tend to} \infty\},$$

where C is the complex space, $g^k(z)$ is k^{th} iterate of function g and $K(g)$ denotes the filled Julia set. The Julia set of the function g is defined to be the boundary of $K(g)$,
i.e., $J(g) = \partial K(g)$, where $J(g)$ denotes the Julia set.

Definition 2.3[12]. The Mandelbrot set M consists of all parameters c for which the filled Julia set of $\mathcal{Q}_c$ is connected, that is

$$M = \{c \in C : K(\mathcal{Q}_c) \text{ is connected}\}.$$

In fact, M contains an enormous amount of information about the structure of Julia sets. The Mandelbrot set M for the Quadratic $\mathcal{Q}_c(z) = z^2 + c$ is defined as the collection of all $c \in C$ for which the orbit of the point 0 is bounded, that is

$$M = \{c \in C : \{\mathcal{Q}_c^n\}; n = 0, 1, 2, .. \text{is bounded}\}.$$

We choose the initial point 0 as 0 is the only critical point of $\mathcal{Q}_c$.
Now, we give definition of the new orbit, which will be used in the paper to implement four-step feedback process in the dynamics of polynomial $z \to \bar{z}^m + c$.

Definition 2.4[8]. Let us consider a sequence $\{xn\}$ of iterates for initial point $x_0 \in X$ such that

$$\begin{aligned} \{x_{n+1} : x_{n+1} &= (1-\alpha_n)Ty_n + \alpha_n Tz_n; \\ y_n &= (1-\beta_n)Tx_n + \beta_n Tz_n; \\ z_n &= (1-\gamma_n)x_n + \gamma_n Tx_n; n = 0, 1, 2, ..\} \end{aligned} \tag{2.2}$$

where $\alpha_n, \beta_n, \gamma_n \in [0,1]$ and $\{\alpha_n\}, \{\beta_n\}, \{\gamma_n\}$ are sequences of positive numbers. Then the sequence (1) is a function (New Orbit) of five tuples $(T, x_0, \alpha_n, \beta_n, \gamma_n)$.

For visualizing new Superior anti-fractals, the required escape criterion with respect to the new orbit for $z \to \overline{Z}^m + c$ is $max\{|c|, (2/\alpha)^{m-1}, (2/\beta)^{m-1}, (2/\gamma)^{m-1}\}$[7].

3 Multicorns in New Orbit

In this section, we programmed the polynomial $z \to \overline{z}^m + c$ in the software Mathematica 9.0 and generate Superior tricorns and multicorns in a new orbit (see Figs. 1-14).

We find the following observations from generated Superior multicorns:

- The number of branches in the Superior tricorns and multicorns is $m+1$, where m is the power of z. Also, few branches have m subranches (see Figs. 6, 7).
- Superior Multicorns exhibit $(m+1)$-fold rotational symmetries.
- There exist many Superior multicorns for any m.
- We also find that higher degree Superior multicorns become circular saw (Figs.13-14).

Some authors [1,3,11] had also found the similar conclusion while generating multicorns using two-step, three-step, four-step feedback processes. The name circular saw was, first, given by Rani and Kumar to Mandelbrot sets [9].

3.1 Superior Tricorns for $m = 2$:

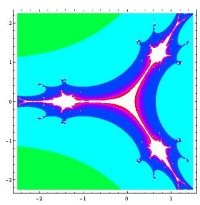

Figure 1 : $\alpha = \beta = 0.3, \gamma = 0.1$

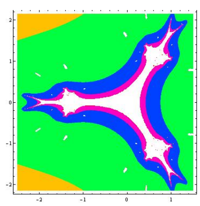

Figure 2 : $\alpha = \beta = 0.3, \gamma = 0.1$

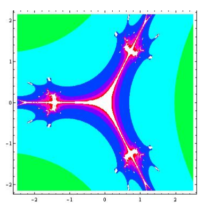

Figure 3 : $\alpha = \beta = \gamma = 0.3$

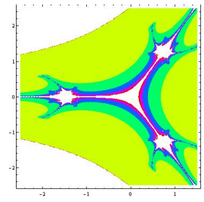

Figure 4 : $\alpha = 0.1, \beta = 0.9, \gamma = 0.1$

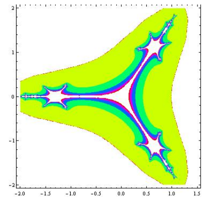

Figure 5 : $\alpha = \beta = 0.9, \gamma = 0.1$

3.2 Superior Multicorns for $m = 3$:

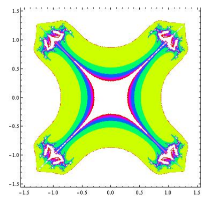

Figure 6 : $\alpha = \beta = 0.6, \gamma = 0.1$

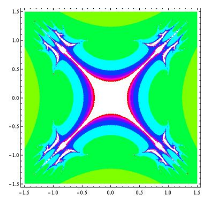

Figure 7 : $\alpha = 0.1, \beta = \gamma = 0.6$

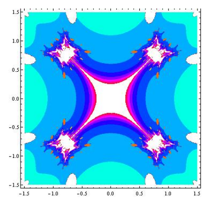

Figure 8 : $\alpha = 0.6, \beta = 0.1, \gamma = 0.6$

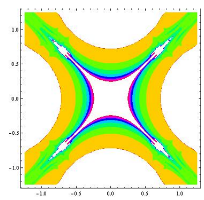

Figure 9 : $\alpha = 0.1, \beta = \gamma = 0.9$

3.3 Superior Multicorns for higher degrees:

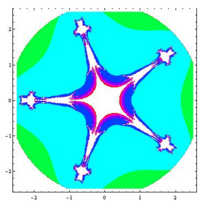

Figure 10 : $m = 4, \alpha = 0.1, \beta = 0.9, \gamma = 0.1$

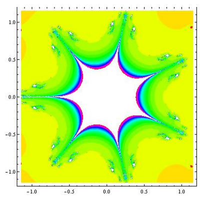

Figure 11 : $m = 6, \alpha = 0.6, \beta = 0.1, \gamma = 0.6$

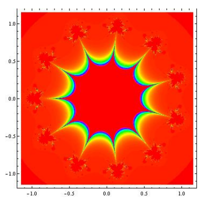

Figure 12 : $m = 10, \alpha = 0.6, \beta = 0.1, \gamma = 0.6$

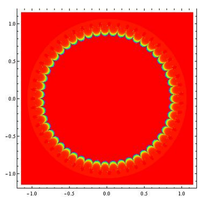

Figure 13 : Circular saw multicorn for $m = 50, \alpha = \beta = 0.6, \gamma = 0.1$

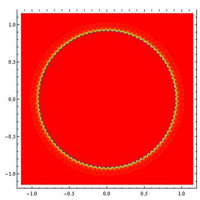

Figure 14 : Circular saw multicorn for $m = 100, \alpha = \beta = 0.6, \gamma = 0.1$

4 New Superior Anti-Julia Sets

Superior Anti Julia sets have been generated for $z \rightarrow \bar{z}^m + c$ in a new orbit. In Figures 15- 17, we can see that the Superior Anti Julia sets look like Superior tricorns or multicorns for $m = 2$. Also, we observed that the higher degree Superior anti Julia sets took different shapes for different values of m, α, β, γ and c.

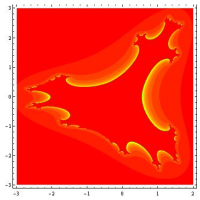

Figure 15 : Superior AntiJulia set for $m = 2, \alpha = 0.4, \beta = 1.0, \gamma = 1.0, c = 0.3 + 0.5I$

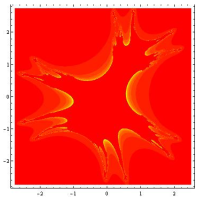

Figure 16 : Superior AntiJulia set for $m = 2, \alpha = \beta = \gamma = 0.5, c = 0.3 + 0.5I$

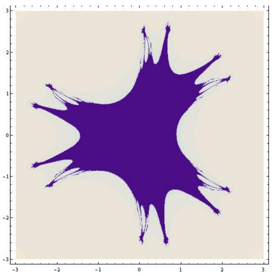

Figure 17 : Superior AntiJulia set for $m = 2, \alpha = \beta = \gamma = 0.5, c = 0.1 + 0.1I$

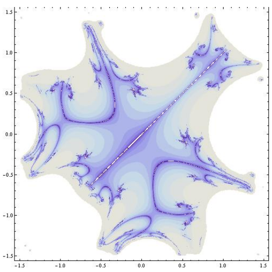

Figure 18 : Superior AntiJulia set for $m = 3, \alpha = \beta = \gamma = 0.4, c = 0.7 + 0.7I$

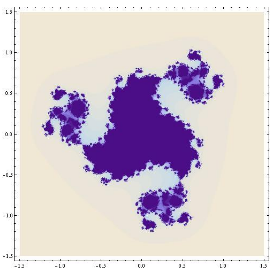

Figure 19 : Superior AntiJulia set for $m = 3, \alpha = \beta = 0.1, \gamma = 0.05, c = 0.6 + 0.5I$

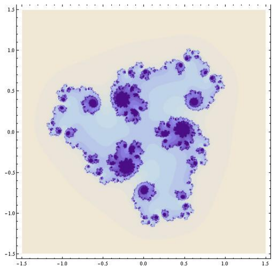

Figure 20 : Superior AntiJulia set for $m = 3, \alpha = \beta = \gamma = 0.05, c = 0.5 + 0.4I$

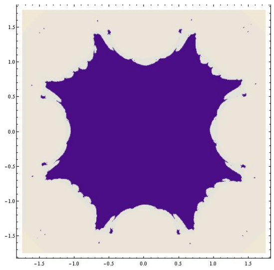

Figure 21 : Superior AntiJulia set for $m = 3, \alpha = \beta = \gamma = 0.4, c = 0.1 + 0.1I$

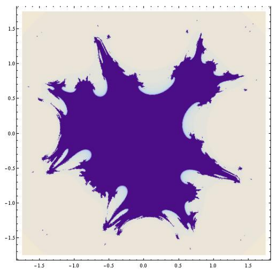

Figure 22 : Superior AntiJulia set for $m = 3, \alpha = \beta = \gamma = 0.4, c = 0.5 + 0.5I$

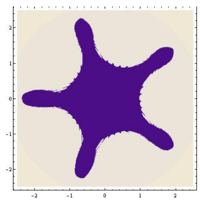

Figure 23 : Superior AntiJulia set for $m = 4, \alpha = 0.1, \beta = 1.0, \gamma = 0.1, c = 0.1 + 0.1I$

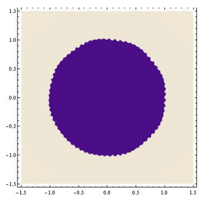

Figure 24 : Superior AntiJulia set for $m = 8, \alpha = \beta = 0.05, \gamma = 0.7, c = 0.05 + 0.05I$

5 Conclusion

In the dynamics of anti-polynomials $z \to \bar{z}^m + c$, where $m \geq 2$, there exist many Superior multicorns for the same value of m in the new orbit. We also generate some Superior Anti- Julia sets in the new orbit. In our results, we found that for higher degrees of the polynomial, all the Superior anti-fractals become circular saw.

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