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Portfolio Strategy for an Investor with Stochastic Premium Under Exponential Utility via Legendre Transform and Dual Theory

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ABSTRACT. We investigate the optimal investment strategies of an insurance company. We assume that the rates at which premiums are paid to insurance companies are stochastic, the total claims are modeled by a compound Poisson process, we assume that surplus of the insurance company is invested in risk free asset and in a risky asset such as stocks. We applied the Jacobi Hamilton-Jacobi-Bellman equation to obtain an optimized problem and used the Legendre transform and dual theory to obtain the optimal investment strategy for constant absolute risk aversion (CARA) utility function. Our result will enables insurance companies to determine the proportion of their wealth to be invested in risk-free asset and a risky asset in order to optimize profit knowing full well it has responsibility to pay the policy holders whenever there is claims occurrence.

1 Introduction

The optimal investment strategy is very crucial in the study of financial dynamics of any institution. It enable the institution to determine the best way the institutions wealth can be spread among assets such as risk free asset (cash) and risky assets (stocks), with an intention of financing future expenses over a given period of time. This strategy is not static; it requires continuous evaluation to decide at any given time the most viable approach to

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invest with minimal risk and optimized profit. In our present days insurance companies do not invest in banks alone but are heavily involve in stocks investment which in contrary to the earlier investment is very risky, but also. Since stock market is highly risky, the need for proper investment strategies and risk management has really developed over the years. The investment strategies for asset with risky asset for an insurance portfolio is different from Markowitzs mean-variance model [11], Samuelson[14] obtained optimal decision for a consumption investment model using dynamic stochastic programming method, Merton[12] studied optimal portfolio strategy under specific assumptions on asset returns and investor preferences and obtained a closed-form solution of optimal portfolio strategy using stochastic optimal control approach in continuous finance and others such as Gerber and Shiu[6], and Korn[10]. Browne[1] studied a model with; total claims modeled by a Brownian motion with drift, and considered that the risky asset is modeled by a GBM. Hipp and Taksar[8] use the compound Poisson model to model the insurance business and consider the problem of optimal choice of new business to minimize the infinite time ruin probability. Hipp and Plum[7] obtained explicit solutions for claim size which was exponentially distributed by modeling the risk process of an insurance company using the Cramer-Lundberg model and assume that the surplus of the insurance company can also be invested in a risky asset that follows a geometric Brownian motion. Taksar[16] shows that the optimal dividend distribution scheme is a barrier type, and the risk control policy depends substantially on the nature of reinsurance available using stochastic models of risk control and dividend optimization techniques. Osu[13] studied optimization of time varying investment returns of insurance company under power utility function, they observed that the optimized investment in the assets and the optimal value function are dependent on horizon and the wealth, Chi Sang and Hailiang[2], studied the causes of disparity in probability of survival and stock volatility on optimal investment strategies, and computed the optimal policy numerically for various claim size. This paper studies the wealth investment strategies for an insurance company and we are interested in determining the optimal investment strategy for an insurance company whose premium is stochastic in nature: that is, to decides the proportion of the companys wealth to be invested in a risk-free asset and a risky asset (stock) knowing full well its responsibility to pay claims to policy holders when it occur. Continuous-time model is used to facilitate the mathematical treatment as the method of dynamic programming can be applied more efficiently. This study offers theoretical support to the optimal portfolio insurance companies should have and may be very useful in deciding investment strategies in real life. The next section describes the mathematical model then followed by the Hamilton-Jacobi-Bellman (HJB) equation, next the Legendre transformation of the optimized problem and finally obtaining the solution for the CARA utility function.

2 Preliminaries

In insurance scheme, contributions are made by the policy holders monthly into the insurance company account and the surplus is being invested into both risk free asset and risky asset. We assume that payment is stochastic and can be described by the Brownian with drift as follows

$$dL(t) = pdt + adW_c(t), \tag{1}$$

Where p and a are positive constants, W_c is a standard Brownian motion.

Assume $\sum_{j=1}^{N_t} C_j$, is the accumulated claims at time t where N_t shows the number of claims occurred up to time t. When the process $N_t, t \geq 0$ is poison we have a jump diffusion process. In such cases $V(t), t \geq 0$ is continuous on the time interval $[T_j, T_{j-1}]$ and right continuous with left hand limit at time $T_j,\ j = 1, 2, 3...j$.

From the work of Sultan (2017), claim size $C_j, j = 1, 2, 3...$ occurs at random times T_j. When T_j and T_{j-1} are independent and identically distributed, then N_t is a poison process and $\sum_{j=1}^{N_t} C_j$, is a compound poisson process where N_t and C_j are independent of each other. Let $\sum_{j=1}^{N_t} C_j$,

Financial Market: We start with a complete and frictionless financial market that is continuously open over the fixed time interval $[0, 1]$, for $T > 0$ representing the retirement time of a given shareholder.

We assume a financial market which is composed of two assets namely the risk free asset and risky asset. Consider a complete probability space (Ω, F, P) with Ω as real space P and as probability measure, the filtration F denotes the possible information generated by the standard two dimensional Brownian motion $\{W_c(t), W_s(t)\}$. Let $S_0(t)$ be the price of the risk free asset, It dynamics is as follows

$$\frac{dS_0(t)}{S_0(t)} = rdt \tag{2}$$

Where r is a constant interest rate generated by the risk free asset and let $S_t(t)$ be the price of the risk free asset, such that it dynamics is as follows

$$\frac{dS_t(t)}{S_t(t)} = \alpha dt + kS_t^{\gamma} dW_s(t),\ t \geq 0 \tag{3}$$

Where α an expected instantaneous rate of return of the risky asset is kS_t^{γ} is the instantaneous volatility and the process $W_s(t)$ is a standard Brownian motion.

3 Methodology

3.1 Hamilton-Jacobi-Bellman (HJB) equation:

Let u_s represent the strategy and the utility attained at any given state y and time t is given as

$$G_{u_s}(t, s, y) = E_{u_s}[V(Y(T))|S_t(t) = s, Y(t) = y]. \tag{4}$$

Our aim is to obtain the optimal value function

$$G(t, s, y) = sup_{u_s} G_{u_s}(t, s, y) \tag{5}$$

and the optimal strategy u_s such that

$$G(t, s, y) = G_{u_s}(t, s, y) \tag{6}$$

3.2. Legendre Transformation:

The Legendre transform and dual theory help to transform non linear partial differential equation to a linear partial differential equation.

Theorem 3.1. Let $f : \mathbb{R}^{n \to \mathbb{R}}$ be a convex function for $z > 0$, define the Legendre transform

$$M(z) = max_y\{f(y) - zy\}, \tag{7}$$

where $M(z)$ is the Legendre dual of $f(y)$ Jonsson and Sircar[9]. Since $f(y)$ is convex, and using theorem 3.1 the Legendre transform for $G(t,s,y)$ is defined as

$$\widehat{G}(t,s,z) = sup\{G(t,s,h) - zy \ /0 < y < \infty\}, t \in (0,T). \tag{8}$$

$\widehat{G}$ is the dual of G and $z > 0$ is the dual variable of y. The value of y where this optimum is attained is denoted by $h(t,s,z)$, so that

$$h(t,s,z) = inf\{y \ /G(t,s,h) \geq zy + \widehat{G}(t,s,z),\}t \in (0,T). \tag{9}$$

The function h and $\widehat{G}$ are very related and are refers to as the dual of G They are related as follows

$$\widehat{G}(t,s,z) = G(t,s,h) - zy \text{ with } h(t,s,z) = y, G_y = z, h = -\widehat{G}_z. \tag{10}$$

At terminal time, we denote $\widehat{V}(z) = sup\{V(y) - zy/0 < y < \infty\}$, and $F(z) = sup\{y/V(y) \geq zy + \widehat{V}(z)\}$.
As a result

$$F(z) = (V')^{-1}(z), \tag{11}$$

F is the inverse of the marginal utility V and note that $G(T,s,y) = V(y)$ At terminal time T, we can define

$$h(T,s,z) = inf_{y>0}\{y/V(y) \geq zy + \widehat{G}(t,s,z) and \widehat{G}(t,s,z) = sup_{y>0}\{V(y) - zy\}\}$$

so that

$$h(T,s,z) = (V')^{-1}(z) \tag{12}$$

4 Model Formulation

Let $R_t(t)$ denote the surplus of the insurance company after claims at time t has been paid to the insurers, and then the dynamics of the surplus is given by

$$dR_t(t) = dL - Z \tag{13}$$

$$dR_t(t) = pdt + adW_ct - Z. \tag{14}$$

Suppose $Y(t)$ is the accumulated wealth of the insurance company at time t such that $0 \leq t \leq T$ let u_s represent the fraction of the fund invested in the risky asset and $1 - u_s$, the fraction invested in risk free asset. The insurance wealth is represented as

$$dY(t) = u_sY(t)\frac{dS_t(t)}{S_t(t)} + (1-u_s)Y(t)\frac{dS_0(t)}{S_0(t)} + dR_t \tag{15}$$

Substituting (2) and (3) into (14) into (15) we have

$$dY(t) = [Y(t)(u_s(\alpha - r) + r) + p - Z]dt + adW_c(t) + u_sY(t)kS_t^{\gamma}dW_s(t) \tag{16}$$

Next we follow the method of solution in Chubing and Ximing[3] and Gao[5] The Hamilton-Jacobi-Bellman (HJB) equation associated with (16) is

$$\begin{gathered} G_t + \alpha sG_s + (ry + p - Z)G_y + \frac{1}{2}k^2s^{2\gamma+2}G_{ss} + \frac{1}{2}a^2G_{yy} \\ +sup\{\frac{1}{2}u_s^2k^2s^{2\gamma}y^2G_{yy} + u_s[(\alpha - r)yG_y + k^2s^{2\gamma+1}yG_{ys}]\} = 0. \end{gathered} \tag{17}$$

To obtain the first order maximizing condition for u_s^*, we differentiate (17) with respect to u_s and equate it to zero

$$u_s k^2 s^{2\gamma} y^2 G_{yy} + [(\alpha - r)yG_y + k^2 s^{2\gamma+1} yG_{ys}] = 0, \tag{18}$$

so that

$$u_s^* = -\frac{[(\alpha - r)yG_y + k^2 s^{2\gamma+1} yG_{ys}]}{k^2 s^{2\gamma} yG_{yy}}. \tag{19}$$

Substituting (19) into (17), we have

$$G_t + \alpha s G_s + (ry + p - Z)G_y + \frac{1}{2}k^2 s^{2\gamma+2} G_{ss} + \frac{1}{2}a^2 G_{yy} - \Big[\frac{((\alpha - r)G_y + k^2 s^{2\gamma+1} G_{ys})^2}{2k^2 s^{2\gamma G_{yy}}}\Big] = 0. \tag{20}$$

So that

$$\begin{aligned} G_t + \alpha s G_s + (ry + p - Z)G_y + \frac{1}{2}k^2 s^{2\gamma+2}\Big[G_{ss} - \frac{G_{ys}^2}{G_{yy}}\Big] \\ + \frac{1}{2}a^2 G_{yy} - \frac{z^2(\alpha - r)^2}{2k^2 s^{2\gamma}}\frac{G_y^2}{G_{yy}} - (\alpha - r)s\frac{G_y G_{ys}}{G_{yy}} = 0 \end{aligned} \tag{21}$$

Differentiating (10) with respect to t, s and y we have

$$G_t = \widehat{G}_t, G_s = \widehat{G}_s, G_y = z, G_{sy} = \frac{-\widehat{G}_{sz}}{\widehat{G}_{zz}}, G_{yy} = \frac{-1}{\widehat{G}_{zz}}, G_{sy} = \widehat{G}_{ss} - \frac{-\widehat{G}_{sz}^2}{\widehat{G}_{zz}}. \tag{22}$$

Substituting (22) in (21)

$$\widehat{G}_t + \alpha s\widehat{G}_s + (ry + p - Z)z + \frac{1}{2}k^2 s^{2\gamma+2}\widehat{G}_{ss} - \frac{1}{2}a^2\frac{1}{\widehat{G}_{zz}} - \frac{z^2(\alpha - r)^2}{2k^2 s^{2\gamma}}\widehat{G}_{zz}^2 - (\alpha - r)sz\widehat{G}_{zz} = 0 \tag{23}$$

and

$$u_s^* = -\frac{[(\alpha - r)z\widehat{G}_{zz} - k^2 s^{2\gamma+1}\widehat{G}_{sz}]}{k^2 s^{2\gamma} y}. \tag{24}$$

Differentiating (23) and (24) with respect to z and $y = h = -\widehat{G}_z$, we have

$$\begin{aligned} h_t + rsh_s - (rh + p - Z) + \frac{1}{2}k^2 s^{2\gamma+2} h_{ss} + \Big(\frac{(\alpha - r)^2}{k^2 s^{2\gamma}} - r\Big)zh_z \\ + \frac{1}{2}a^2\frac{h_{zz}}{h_z^2} + \frac{z^2(\alpha - r)^2 h_{zz}}{2k^2 s^{2\gamma}} - (\alpha - r)szh_{sz} = 0 \end{aligned} \tag{25}$$

and

$$u_s^* = -\frac{[(\alpha - r)zh_z - k^2 s^{2\gamma+1} h_s}{hk^2 s^{2\gamma}}. \tag{26}$$

5 Optimal investment strategy for CARA utility function

Assume that the insurance company takes an exponential utility

$$V(y) = -\frac{1}{u}e^{-uy},\ y > 0. \tag{27}$$

The absolute risk aversion of a decision maker with the utility described in (27) is constant and is a CARA utility. Since $h(T, s, z) = (V')^{-1}(z)$ with the CARA utility function we obtain

$$h(T, s, z) = -\frac{1}{u}lnz. \tag{28}$$

Hence we formulate a solution to (25) as follows

$$h(T,s,z) = -\frac{1}{u}[q(t)(lnz + m(t,s))] + n(t), \tag{29}$$

with boundary conditions $q(T) = 1, n(T) = 0, m(T,s) = 0$

$$h_t = -\frac{1}{u}[q'(t)(lnz + m(t,s)) + qm_t] + n'(t),$$

$$h_s = -\frac{1}{y}qm_s, h_z = -\frac{q}{yz}, h_{zz} = -\frac{q}{yz^2}, h_{ss} = -\frac{1}{y}qm_{ss}, h_{sz} = 0. \tag{30}$$

Substituting (30) into (25), we have

$[q'(t) - rq(t)]lnz + [-n'(t) + rn(t) + p - Z]u + [m_t + rsm_s + \frac{1}{2}k^2s^{2\gamma+2}m_{ss} + \frac{(\alpha-r)^2}{2k^2s^{2\gamma}} - rm + \frac{q'}{q}m - r - \frac{1}{2}a^2]q = 0.$

Such that

$$q'(t) - rq(t) = 0 \tag{31}$$

and

$$m_t + rsm_s + \frac{1}{2}k^2s^{2\gamma+2}m_{ss} + \frac{(\alpha-r)^2}{2k^2s^{2\gamma}} - r - \frac{1}{2}a^2 = 0. \tag{32}$$

So that

$$-n'(t) + rn(t) + p - Z = 0 \tag{33}$$

Solving (31) and (33)

$$q(t) = e^{-r(t-T)} \tag{34}$$

and

$$n(t) = -\frac{p-z}{r}(1 - e^{-r(t-T)}). \tag{35}$$

We next formulate a solution for (32) in the following form

$$\begin{aligned} m(t,s) &= A(t) + B(t)s^{-2\gamma}, A(T) = 0, B(T) = 0, v_t = A' + B's^{-2\gamma}, v_t = A' + B's^{-2\gamma}, \\ v_s &= -2\gamma Bs^{-2\gamma-1}, v_{ss} = 2\gamma(2\gamma+1)Bs^{-2\gamma-2}. \end{aligned} \tag{36}$$

Substituting (36) into (32) we have

$$A' + \gamma(2\gamma+1)k^2B - r - \frac{1}{2}a^2 + s^{-2\gamma}[B' - 2r\gamma B + \frac{(\alpha-r)^2}{2k^2}] = 0, \tag{37}$$

so that

$$A' + \gamma(2\gamma+1)k^2B - r - \frac{1}{2}a^2 = 0, \tag{38}$$

and

$$B' - 2r\gamma B + \frac{(\alpha-r)^2}{2k^2} = 0. \tag{39}$$

Solving (39) with the given condition gives;

$$B(t) = \frac{(\mu-r)^2}{4k^2r\gamma}[1 - e^{2r\gamma(t-T)}]. \tag{40}$$

Next substituting (40) into (38) and solving (38) with the given condition we have

$$A(t) = [\frac{(2\gamma+1)(\alpha-r)^2}{4r} - r - \frac{1}{2}a^2](T-t) - [\frac{(2\gamma+1)(\alpha-r)^2}{8r^2\gamma}(1 - e^{2r\gamma(t-T)})]. \tag{41}$$

Since

$$q(t) = e^{r}(t-T),$$
$$n(t) = -\frac{(p-Z)[1-e^{-r(T-t)}]}{r},$$

and

$$\begin{aligned} m(t,s) =& [\frac{(2\gamma+1)(\alpha-r)^2}{4r} - r - \frac{1}{2}a^2](T-t) - [\frac{(2\gamma+1)(\alpha-r)^2}{8r^2\gamma}(1-e^{2r\gamma(t-T)})] \\ &+ [\frac{(s^{-2\gamma})(\alpha-r)^2}{4k^2 r\gamma}(1-e^{2r\gamma(t-T)})]. \end{aligned} \tag{42}$$

The optimal investment strategy is given as

$$u_s^* = \frac{1}{k}\frac{(\alpha-r)}{k^2 s^{2\gamma} h} e^{r(t-T)}[1+\frac{(\alpha-r)}{2r}(1-e^{2r\gamma(t-T)})]$$

with $h_z = -\frac{1}{yz}e^{r(t-T)}$ and $h_s = \frac{1}{y}e^{r(t-T)}\frac{(s^{-2\gamma-1})(\alpha-r)^2}{2k^2 r}(1-e^{2r\gamma(t-T)})$, If $ks^{\gamma} = \sigma_s J(\sigma_s) = \frac{(\alpha-r)}{y\sigma_s^2}, H(t) = 1 + \frac{(\alpha-r)}{2r}(1-e^{-2r\gamma(t-T)})$ and $h = y$, then

$$u_s^* = h^{-1}J(\sigma_s)H(t). \tag{43}$$

Corollary 5.1. $H(t)$ is monotonic increasing with respect to time and satisfies the condition

$$1 + \frac{(\alpha-r)}{2r}(1-e^{2r\gamma(t-T)}) \leq H(t) \leq 1.$$

Proof: Let $H(t) = 1 + \frac{(\alpha-r)}{2r}(1-e^{2r\gamma(t-T)})$, $\alpha - r > 0, \gamma < 0$, then $H(t) = -\gamma(\alpha-r)(1-e^{2r\gamma(t-T)}$

This implies that $H(t)$ is monotonic increasing function.

When $t = 0$,

$$H(t) = 1 + \frac{(\alpha-r)}{2r}(1-e^{-2r\gamma T}).$$

When $t = 1$ and $T = 0$,

$$H(t) = 1.$$

Therefore $1 + \frac{(\alpha-r)}{2r}(1-e^{-2r\gamma T}) \leq H(t) \leq 1$.

Proposition 5.2. For an investor with CARA utility function, the optimal investment strategy is given as

$$u_s^* = \begin{cases} h^{-1}J(\sigma_s)H(t); \;\; if \;\; 0 \leq \pi_s^* \leq 1 \\ 1; \;\; if \;\; \pi_s^* > 1. \end{cases}$$

Let Y denote the invested capital, we assume that for any $D > 0$ that $Y > D$, also

$$J(\sigma_s) = \frac{(\alpha-r)}{k\sigma_s^2} > 0.$$

6 Discussion and Conclusion

6.1 Discussion:

We consider the following cases:

Case.1:

If

$$\frac{(\alpha-r)}{2r}(1-e^{-2r\gamma T}) \geq -1 \text{and } Dk\sigma_s^2 < (\alpha-r) \leq \frac{Dk\sigma_s^2 e^{rt}}{H(0)}.$$

and

$$t > T - (ln(\alpha - r) + ln(H(0))) - ln(Dk\sigma_s^2),$$

then $u_s^* > 1$.

From proposition 5.2, $u_s^* = 1$. The implication of this is that the investor invest in only risky asset.

Case.2:

If

$$\frac{(\alpha-r)}{2r}(1-e^{-2r\gamma T}) < -1, t_0 \leq t \leq T \text{and } Dk\sigma_s^2 < (\alpha-r) \leq \frac{Dk\sigma_s^2 e^{r_0 T}}{H(0)}.$$

and

$$t > T - \frac{ln(\alpha - r) + ln(H(0)) - ln(Dk\sigma_s^2)}{\tau},$$

Then $0 \leq u_s^* \leq 1$. From the proposition 5.2, $u_s^* = h^{-1}J(\sigma_s)H(t)$. The implication is that the investor invests the proportion equal to $h^{-1}J(\sigma_s)H(t)$ in the risky asset.

6.2 Conclusion:

We considered optimal investment strategy for an insurance company whose premium is stochastic, we obtained an optimized problem which is in the form of a non linear partial differential equation and applied Legendre transformation and dual theory to reduce the non linear partial differential equation to linear partial differential and then solved for the optimal strategy for an exponential utility function CARA and obtain an explicit solution. The result provides insights for managers of insurance companies on how their companies can invest between a risk-free asset and a risky asset subject to its obligation to pay the policy holders when claims occur.

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