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$(\in, \in \vee q)$-Interval-Valued Fuzzy Prime Ideals of BCK-Algebras

S. R. Barbhuiya*
Department of Mathematics, Srikishan Sarda College, Hailakandi-788151,Assam, India,
E-mail:saidurbarbhuiya@gmail.com

ABSTRACT. The notion of $(\in, \in \vee q)$-interval-valued fuzzy prime ideals in commutative BCK-algebras is introduced. Some characterization theorems in $(\in, \in \vee q)$-interval-valued fuzzy prime ideals in commutative BCK-algebras are derived. Also we study relation between $(\in, \in \vee q)$-interval-valued fuzzy prime ideal and $(\in \vee q)$-interval-valued fuzzy level prime ideals. Characterization of interval-valued fuzzy set to be an $(\in, \in \vee q)$-interval-valued fuzzy prime ideals are provided.

1 Introduction

Zadeh [28] defined a fuzzy set as a generalisation of the characteristic function of a subset. A fuzzy set can be defined mathematically by assigning to each possible individual in the universe of discourse, a value representing its grade of membership in the set. The membership grades are very often represented by real numbers in the closed interval$[0, 1]$. The nearer the value of an element to unity, the higher the grade of its membership. Rosenfeld ([27]) was the first who consider the case of a groupoid in terms of fuzzy sets. Since then these ideas have been applied to other algebraic structures such as group, semigroup, ring,vector spaces etc. Imai and Iseki ([14])introduced two classes of abstract algebras: BCK-algebras and BCI-algebras. It is known that the class of BCK-algebra is a proper sub class of the class of BCI-algebras. Iseki([16]) introduced the concept of prime ideal in commutative BCK-algebras. In ([4]) Ahsan, Deeba and Thaheem have studied the theory of ideals, in particular, prime ideals of a commutative BCK-algebras. In ([18, 19]) Jun and Xin have studied fuzzy prime ideals and invertible fuzzy

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ideals in BCK-algebras. Abdullah ([3]) introduced the notion of intuitionistic fuzzy prime ideals of commutative BCK-algebras. The concept of fuzzy point introduced by Ming and Ming in [25] and also they introduced the idea of relation "belongs to" and "quasi coincident with" between fuzzy point and a fuzzy set. Bhakat and Das ([7, 8])used the relation of "belongs to" and "quasi-coincident with" between fuzzy point and fuzzy set to introduced the concept of$(\in, \in \vee q)$-fuzzy subgroup, $(\in, \in \vee q)$-fuzzy subring and $(\in \vee q)$-level subset. Basnet and Singh ([10])introduced $(\in, \in \vee q)$-fuzzy ideals of BG-algebra in 2011. Dhanani and Pawar([12]) introduced the concept of $(\in, \in \vee q)$-fuzzy ideals(prime) of lattice.

As a generalization of fuzzy set interval-valued fuzzy set were proposed by Zadeh ([29]) as a natural extension of fuzzy sets. Interval-valued fuzzy subsets have many applications in several areas. Biswas ([9]) defined interval valued fuzzy subgroups i.e., interval valued fuzzy subgroups of Rosenfelds nature, and investigated some elementary properties. The concept of interval-valued fuzzy sets have been studied in various algebraic structures, see ([13, 21, 17, 22]). Lee et al[21] introduced interval-valued $(\in, \in \vee q_k)$-fuzzy ideals of rings and Ma et al. [22] studied interval valued fuzzy $(p-, q-, a-)$ideals of BCI-algebras and $(\in, \in \vee q)$-interval-valued fuzzy (p-,q-,a-) ideals of BCI-algebras with some related properties. Dutta et al. [13] investigated interval-valued fuzzy prime and semiprime ideals of a hyper semiring. In this paper, we introduced the notion of $(\in, \in \vee q)$-interval-valued fuzzy prime ideals of commutative BCK-algebras and got some interesting result.

2 Preliminaries

Definition 2.1 *[18, 19] An algebra $(X, *, 0)$ of type (2, 0) is called a BCK-algebra if it satisfies the following axioms:*
(i) $((x*y)*(x*z))*(z*y) = 0$
(ii) $(x*(x*y))*y = 0$
(iii) $x*x = 0$
(iv) $0*x = 0$
(v) $x*y = 0$ *and* $y*x = 0 \Rightarrow x = y$ *for all* $x, y, z \in X$
We can define a partial ordering $'' \leq ''$ *on X by* $x \leq y$ *iff* $x*y = 0$
A BCK-algebra X is said to be commutative if it satisfies the identity $x \wedge y = y \wedge x$ *where* $x \wedge y = y*(y*x) \forall x, y \in X$*. In a commutative BCK-algebra, it is known that* $x \wedge y$ *is the greatest lower bound of x and y.*
In a BCK-algebra X, the following hold:
(i) $x*0 = x$
(ii) $(x*y)*z = (x*z)*y$
(iii) $x*y \leq x$
(iv) $(x*y)*z \leq (x*z)*(y*z)$
(v) $x \leq y$ *implies* $x*z \leq y*z$ *and* $z*y \leq z*x$.
A nonempty subset I of a BCK-algebra X is called an ideal of X if (I_1) $0 \in I$ (I_2) $x*y \in I$ *and* $y \in I \Rightarrow x \in I$ *for all* $x, y \in X$*. A fuzzy set* μ *in BCK-algebra X is called a fuzzy ideal of X if it satisfies* (FI_1) $\mu(0) \geq \mu(x)$ (FI_2) $\mu(x) \geq min\{\mu(x*y), \mu(y)\}$ *for all* $x, y \in X$.
An ideal I of a commutative BCK-algebra X is said to be prime if $x \wedge y \in I \Rightarrow x \in I$ *or* $x \in I$*. A non constant fuzzy ideal*

μ of a commutative BCK-algebra X is said to be fuzzy prime ([19]) if $\mu(x \wedge y) \leq max\{\mu(x), \mu(y)\}$ for all $x, y \in X$ Since $x \wedge y \leq x, y$ and μ is order reversing, it follows that $\mu(x) \leq \mu(x * y)$ and $\mu(y) \leq \mu(x * y)$ therefore a non constant fuzzy ideal μ of a commutative BCK-algebra X is fuzzy prime iff $\mu(x \wedge y) = max\{\mu(x), \mu(y)\}$ for all $x, y \in X$ or equivalently $\mu(x \wedge y) = \mu(x)$ or $\mu(y)$ for all $x, y \in X$.

Definition 2.2 *([7]) A fuzzy set μ of the form*

$$\mu(y) = \begin{cases} t & if \quad y = x, \quad t \in (0,1] \\ 0 & if \quad y \neq x \end{cases}$$

is called a fuzzy point with support x and value t and it is denoted by x_t.

Let μ be a fuzzy set in X and x_t be a fuzzy point then ([7])
(i) If $\mu(x) \geq t$ then we say x_t belongs to μ and write $x_t \in \mu$
(ii)If $\mu(x) + t > 1$ then we say x_t quasi coincident with μ and write $x_t q \mu$
(iii)If $x_t \in \vee q\mu \Leftrightarrow x_t \in \mu$ or $x_t q \mu$
(iv) If $x_t \in \wedge q\mu \Leftrightarrow x_t \in \mu$ and $x_t q \mu$
The symbol $x_t \overline{\alpha} \mu$ means $x_t \alpha \mu$ does not hold and $\overline{\in \wedge q}$ means $\overline{\in} \vee \overline{q}$
For a fuzzy point x_t. and a fuzzy set μ in set X, Pu and Liu ([25])gave meaning to the symbol $x_t \alpha \mu$ where $\alpha \in \{\in, q, \in \vee q, \in \wedge q\}$.

Definition 2.3 *([20]) A fuzzy set μ of a BCK-algebra Xis said to be (α, β)-fuzzy ideal of X, Where $\alpha \neq \in \wedge q$ if*
(i) $x_t \alpha \mu \Rightarrow 0_t \beta \mu$
(ii) $(x * y)_t, y_s \alpha \mu \Rightarrow x_{m(t,s)} \beta \mu$ for all $x, y \in X$ where $t, s \in (0, 1]$.

Example 2.4 *Consider BCK-algebra $X = \{0, x, y, z\}$ with the following cayley table.*

*	0	x	y	z	w
0	0	0	0	0	0
x	x	0	x	0	x
y	y	y	0	y	0
z	z	x	z	0	z
w	w	w	y	w	0

Define a map $\mu : X \to [0, 1]$ by $\mu(0) = 0.7, \mu(x) = \mu(z) = 0.3, \mu(y) = \mu(w) = 0.2$, then μ is an $(\in, \in \vee q)$-fuzzy ideal X.

Definition 2.5 *An (α, β)-fuzzy ideal μ of a BCK-algebra X is said to be (α, β)-fuzzy prime ideal of X, where $\alpha \neq \in \wedge q$ if $(x \wedge y)_t \alpha \mu \Rightarrow x_t \beta \mu$ or $y_t \beta \mu$ for all $x, y \in X$ where $t \in (0\,1]$.*

Definition 2.6 *An $(\in, \in \vee q)$-fuzzy ideal μ of a BCK-algebra X is said to be $(\in, \in \vee q)$-fuzzy prime ideals of X if $(x \wedge y)_t \in \mu \Rightarrow x_t \in \vee q\mu$ or $y_t \in \vee q\mu$ for all $x, y \in X$, where $t \in (0, 1]$.*

The notion of interval-valued fuzzy set was introduced by Zadeh([29]). To consider the notion of interval-valued fuzzy sets, we need the following notations. By an interval number $\hat{a}$, we mean an interval $[\underline{a}, \overline{a}]$, where $0 \leq \underline{a} \leq \overline{a} \leq 1$. Let D[0, 1] denote the set of all such interval numbers of [0, 1].
Define on D[0, 1] the relations $\preceq, =, \prec$ by

1. $\hat{a_1} \preceq \hat{a_2} \Leftrightarrow \underline{a_1} \leq \underline{a_2}$ and $\overline{a_1} \leq \overline{a_2}$
2. $\hat{a_1} = \hat{a_2} \Leftrightarrow \underline{a_1} = \underline{a_2}$ and $\overline{a_1} = \overline{a_2}$
3. $\hat{a_1} \prec \hat{a_2} \Leftrightarrow \underline{a_1} < \underline{a_2}$ and $\overline{a_1} < \overline{a_2}$
4. $\hat{a_1} + \hat{a_2} \Leftrightarrow [\underline{a_1} + \underline{a_2},\ \overline{a_1} + \overline{a_2}]$
5. $\hat{a_1}.\hat{a_2} \Leftrightarrow [min(\underline{a_1 a_2}, \underline{a_1}\overline{a_2}, \overline{a_1}\underline{a_2}, \overline{a_1 a_2}), max(\underline{a_1 a_2}, \underline{a_1}\overline{a_2}, \overline{a_1}\underline{a_2}, \overline{a_1 a_2})] = [\underline{a_1 a_2}, \overline{a_1 a_2}]$
6. $k\hat{a} = [k\underline{a}, k\overline{a}]$ where $0 \leq k \leq 1$

Now consider two intervals $\hat{a_1} = [\underline{a_1}, \overline{a_1}], \hat{a_2} = [\underline{a_2}, \overline{a_2}] \in D[0,\ 1]$ then we define refine minimum rmin as $rmin(\hat{a_1}, \hat{a_2}) = [min(\underline{a_1}, \underline{a_2}), min(\overline{a_1}, \overline{a_2})]$ and refine maximum as rmax $rmax(\hat{a_1}, \hat{a_2}) = [max(\underline{a_1}, \underline{a_2}), max(\overline{a_1}, \overline{a_2})]$ generally if $\hat{a_i} = [\underline{a_i}, \overline{a_i}], \hat{b_i} = [\underline{b_i}, \overline{b_i}] \in D[0,\ 1]$ for i= 1,2,3,...then we define $rmax(\hat{a_i}, \hat{b_i}) = [max(\underline{a_i}, \underline{b_i}), max(\overline{a_i}, \overline{b_i})]$ and $rmin(\hat{a_i}, \hat{b_i}) = [min(\underline{a_i}, \underline{b_i}), min(\overline{a_i}, \overline{b_i})]$ and $rinf_i(\hat{a_i}) = [\wedge_i \underline{a_i}, \wedge_i \overline{a_i}]$ and $rsup_i(\hat{a_i}) = [\vee_i \underline{a_i}, \vee_i \overline{a_i}]$
$(D[0,\ 1], \preceq)$ is a complete lattice with $\wedge = rmin,\ \vee = rmax, \hat{0} = [0\,0]$ and $\hat{1} = [1\,1]$ being the least and the greatest element respectively.

Definition 2.7 *An interval-valued fuzzy set defined on a non empty set X as an objects having the form* $\hat{\mu} = \{x, [\underline{\mu}(x), \overline{\mu}(x)]\}, \forall x \in X$ *where* $\underline{\mu}$ *and* $\overline{\mu}$ *are two fuzzy sets in X such that* $\underline{\mu}(x) \leq \overline{\mu}(x)$ *for all* $x \in X$*. Let* $\hat{\mu}(x) = [\underline{\mu}(x), \overline{\mu}(x)], \forall x \in X$*, Then* $\hat{\mu}(x) \in D[0,1], \forall x \in X$
If $\hat{\mu}$ *and* $\hat{\nu}$ *be two interval-valued fuzzy sets in X, then we define*

- $\hat{\mu} \subseteq \hat{\nu} \Leftrightarrow$ *for all* $x \in X, \quad \underline{\mu}(x) \leq \underline{\nu}(x)$ *and* $\overline{\mu}(x) \leq \overline{\nu}(x)$.
- $\hat{\mu} = \hat{\nu} \Leftrightarrow$ *for all* $x \in X, \quad \underline{\mu}(x) = \underline{\nu}(x)$ *and* $\overline{\mu}(x) = \overline{\nu}(x)$.
- $(\hat{\mu} \hat{\cup} \hat{\nu})(x) = \hat{\mu}(x) \vee \hat{\nu}(x) = rmax\{\hat{\mu}(x), \hat{\nu}(x)\} = [max\{\underline{\mu}(x), \underline{\nu}(x)\}, max\{\overline{\mu}(x), \overline{\nu}(x)\}]$.
- $(\hat{\mu} \hat{\cap} \hat{\nu})(x) = \hat{\mu}(x) \wedge \hat{\nu}(x) = rmax\{\hat{\mu}(x), \hat{\nu}(x)\} = [min\{\underline{\mu}(x), \underline{\nu}(x)\}, min\{\overline{\mu}(x), \overline{\nu}(x)\}]$.
- $(\hat{\mu} \hat{\times} \hat{\nu})(x, y) = \hat{\mu}(x) \wedge \hat{\nu}(y) = [min\{\underline{\mu}(x), \underline{\nu}(y)\}, min\{\overline{\mu}(x), \overline{\nu}(y)\}]$.
- $\hat{\mu}^c(x) = [1 - \overline{\mu}(x), 1 - \underline{\mu}(x)]$.

Definition 2.8 *Let* $\hat{\mu}$ *be an interval-valued fuzzy set in X. Then, for every* $[0,\ 0] \prec \hat{t} \preceq [1,\ 1]$*, the crisp set* $\hat{\mu}_t = \{x \in X | \hat{\mu}(x) \succeq \hat{t}\}$ *is called the level subset of* $\hat{\mu}$.

Definition 2.9 *([22]) An interval-valued fuzzy set* $\hat{\mu}$ *in BCK-algebra X is called an interval-valued fuzzy ideal of X if it satisfies*
(i) $\hat{\mu}(0) \succeq \hat{\mu}(x)$
(ii) $\hat{\mu}(x) \succeq rmin\{\hat{\mu}(x * y), \hat{\mu}(y)\}$ *for all* $x, y \in X$.

Definition 2.10 *A non constant interval-valued fuzzy ideal* $\hat{\mu}$ *of a commutative BCK-algebra X is said to be an interval-valued fuzzy prime ideal of X if* $\hat{\mu}(x \wedge y) \preceq rmax\{\hat{\mu}(x), \hat{\mu}(y)\}$ *for all* $x, y \in X$.

Definition 2.11 *Ma et al [22] extended the notion of belongingness and quasi-coincidence of a fuzzy point with a fuzzy set and defined the notions of belongingness and quasi-coincidence of an interval-valued fuzzy point with an interval-valued fuzzy set. For any interval-valued fuzzy set* $\hat{\mu} = \{x, [\underline{\mu}(x), \overline{\mu}(x)]\}$ *and* $\hat{t} = [\underline{t}, \overline{t}]$*, we define* $\hat{\mu} + \hat{t} = [\underline{\mu}(x) + \underline{t}, \overline{\mu}(x) + t]$ *for all* $x \in X$*. In particular if* $\underline{\mu}(x) + \underline{t} \geq 1$*, we write as* $\hat{\mu} + \hat{t} \succ [1, 1] = \hat{1}$
Let $x \in X$ *and* $\hat{t} \in D[0, 1]$*, an interval-valued fuzzy set* $\hat{\mu}$ *of X of the form*

$$\hat{\mu}(y) = \begin{cases} \hat{t} \neq [0, 0], & if \quad y = x, \quad \hat{t} \in D(0, 1] \\ \hat{0} = [0, 0], & if \quad y \neq x \end{cases}$$

is said to be an interval-valued fuzzy point with support x and interval-valued value $\hat{t}$ *and is denoted by* $x_{\hat{t}}$*.*

Definition 2.12 *[22] Let* $\hat{\mu}$ *be an interval-value fuzzy set in X and* $x_{\hat{t}}$ *be an interval-value fuzzy point then*
(i) If $\hat{\mu}(x) \succeq \hat{t}$ *then we say* $x_{\hat{t}}$ *belongs to* $\hat{\mu}$ *and write* $x_{\hat{t}} \in \hat{\mu}$
(ii)If $\hat{\mu}(x) + \hat{t} \succ [1, 1]$ *then we say* $x_{\hat{t}}$ *quasi coincident with* $\hat{\mu}$ *and write* $x_{\hat{t}} q \hat{\mu}$
(iii)If $x_{\hat{t}} \in \vee q \hat{\mu} \Leftrightarrow x_t \in \hat{\mu}$ *or* $x_{\hat{t}} q \hat{\mu}$
(iv) If $x_{\hat{t}} \in \wedge q \hat{\mu} \Leftrightarrow x_{\hat{t}} \in \hat{\mu}$ *and* $x_{\hat{t}} q \hat{\mu}$
The symbol $x_{\hat{t}} \overline{\alpha} \hat{\mu}$ *means* $x_{\hat{t}} \alpha \hat{\mu}$ *does not hold and* $\overline{\in \wedge q}$ *means* $\overline{\in} \vee \overline{q}$
For an interval-valued fuzzy point $x_{\hat{t}}$ *and an interval-valued fuzzy set* $\hat{\mu}$ *in set X, Pu and Liu gave meaning to the symbol* $x_{\hat{t}} \alpha \hat{\mu}$ *where* $\alpha \in \{\in, q, \in \vee q, \in \wedge q\}$*.*

Definition 2.13 *A interval-valued fuzzy set* $\hat{\mu}$ *of a BCK-algebra X is said to be* (α, β)*-interval-valued fuzzy ideal of X, Where* $\alpha \neq \in \wedge q$ *if*
(i) $x_{\hat{t}} \alpha \hat{\mu} \Rightarrow 0_{\hat{t}} \beta \hat{\mu}$
(ii) $(x * y)_{\hat{t}}, y_{\hat{s}} \alpha \hat{\mu} \Rightarrow x_{rmin(\hat{t}, \hat{s})} \beta \hat{\mu}$ *for all* $x, y \in X$ *where* $[0, 0] \prec \hat{t}, \hat{s} \preceq [1, 1]$.

Example 2.14 *Consider BCK-algebra* $X = \{0, x, y, z\}$ *with the following cayley table.*

*	0	x	y	z	w
0	0	0	0	0	0
x	x	0	x	0	x
y	y	y	0	y	0
z	z	x	z	0	z
w	w	w	y	w	0

Define an interval-valued fuzzy set $\hat{\mu} : X \to D[0, 1]$ *by* $\hat{\mu}(0) = [0.6, 0.70], \hat{\mu}(x) = \hat{\mu}(z) = [0.3, 0.4], \hat{\mu}(y) = \hat{\mu}(w) = [0.1, 0.2]$ *then* $\hat{\mu}$ *is an* $(\in, \in \vee q)$*-interval-valued fuzzy ideal X.*

3 $(\in, \in \vee q)$-Interval-valued Fuzzy prime ideals of BCK-algebras

In what follows, let X denote a commutative BCK-algebra unless otherwise stated.

Definition 3.1 *An (α,β)-interval-value fuzzy ideal $\hat{\mu}$ of X is said to be (α,β)-interval-value fuzzy prime ideal of X, Where $\alpha \neq \in \wedge q$ if $(x \wedge y)_{\hat{t}} \alpha \hat{\mu} \Rightarrow x_{\hat{t}} \beta \hat{\mu}$ or $y_{\hat{t}} \beta \hat{\mu}$ for all $x, y \in X$ where$[0,\ 0] \prec \hat{t}, \hat{s} \preceq [1,\ 1]$.*

Definition 3.2 *An $(\in, \in \vee q)$-interval-value fuzzy ideal $\hat{\mu}$ of X is said to be $(\in, \in \vee q)$-interval-value fuzzy prime ideals of X if $(x \wedge y)_{\hat{t}} \in \hat{\mu} \Rightarrow x_{\hat{t}} \in \vee q \hat{\mu}$ or $y_{\hat{t}} \in \vee q \hat{\mu}$ for all $x, y \in X$ where $[0,\ 0] \prec \hat{t}, \hat{s} \preceq [1,\ 1]$.*

Theorem 3.3 *An interval-valued fuzzy subset $\hat{\mu}$ of X is an interval-valued fuzzy prime ideal if and only if $\hat{\mu}$ is an $(\in, \in)$-interval-valued fuzzy prime ideal.*

Let μ be an interval-valued fuzzy prime ideal
Therefore $\hat{\mu}(x \wedge y) \preceq rmax\{\hat{\mu}(x), \hat{\mu}(y)\}$
Let $(x \wedge y)_{\hat{t}} \in \hat{\mu} \Rightarrow \hat{\mu}(x \wedge y) \succeq \hat{t}$
$\Rightarrow rmax\{\hat{\mu}(x), \hat{\mu}(y)\} \succeq \hat{\mu}(x \wedge y) \succeq \hat{t}$
$\Rightarrow \hat{\mu}(x) \succeq \hat{t}$ or $\hat{\mu}(y) \succeq \hat{t}$
$\Rightarrow x_{\hat{t}} \in \hat{\mu}$ or $y_{\hat{t}} \in \hat{\mu}$
$\Rightarrow x_{\hat{t}} \in \hat{\mu}$ or $y_{\hat{t}} \in \hat{\mu}$
Therefore $\hat{\mu}$ is an $(\in, \in)$-interval-value fuzzy prime ideal.
Conversely, Let $\hat{\mu}$ be an $(\in, \in)$-interval-valued fuzzy prime ideal.
Let $x, y \in X$ and $\hat{\mu}(x \wedge y) = \hat{t}$ where $\hat{t} \in D[0, 1]$
then $\hat{\mu}(x \wedge y) \succeq \hat{t} \Rightarrow (x \wedge y)_{\hat{t}} \in \hat{\mu}$
$\Rightarrow x_{\hat{t}} \in \hat{\mu}$ or $y_{\hat{t}} \in \hat{\mu}$
$\Rightarrow \hat{\mu}(x) \succeq \hat{t}$ or $\hat{\mu}(y) \succeq \hat{t}$
$\Rightarrow rmax\{\hat{\mu}(x), \hat{\mu}(y)\} \succeq \hat{t} = \hat{\mu}(x \wedge y)$
Therefore $\hat{\mu}$ is an interval-valued fuzzy prime ideal.

Theorem 3.4 *$\hat{\mu}$ is (q, q)-interval-valued fuzzy prime ideal if and only if $\hat{\mu}$ is an $(\in, \in)$-interval-valued fuzzy prime ideal.*

Let $\hat{\mu}$ be a (q, q)-interval-valued fuzzy ideal of a BCK-algebra X.
Let x,y $\in X$ such that $(x \wedge y)_{\hat{t}} \in \hat{\mu}$
$\Rightarrow \hat{\mu}(x \wedge y) \succeq \hat{t}$
$\Rightarrow \hat{\mu}(x \wedge y) + \hat{\delta} \succ \hat{t}$ Since $\hat{\delta}$ is arbitrary small interval belongs to $D[0,\ 1]$
$\Rightarrow \hat{\mu}(x \wedge y) + \hat{\delta} - \hat{t} + \hat{1} \succ \hat{1}$
$\Rightarrow (x \wedge y)_{\hat{\delta} - \hat{t} + \hat{1}} q \hat{\mu}$
Since $\hat{\mu}$ is a (q, q)-interval-value fuzzy prime ideal X.
Therefore we have $x_{\hat{\delta} - \hat{t} + \hat{1}} q \hat{\mu}$ or $y_{\hat{\delta} - \hat{t} + \hat{1}} q \hat{\mu}$
$\Rightarrow \hat{\mu}(x) + \hat{\delta} - \hat{t} + \hat{1} \succ \hat{1}$ or $\hat{\mu}(y) + \hat{\delta} - \hat{t} + \hat{1} \succ \hat{1}$
$\Rightarrow \hat{\mu}(x) + \hat{\delta} \succ \hat{t}$ or $\hat{\mu}(y) + \hat{\delta} \succ \hat{t}$
$\Rightarrow \hat{\mu}(x) \succeq \hat{t}$ or $\hat{\mu}(y) \succeq \hat{t}$ Since $\hat{\delta}$ is arbitrary
$\Rightarrow x_{\hat{t}} \in \hat{\mu}$ or $y_{\hat{t}} \in \hat{\mu}$

therefore $(x \wedge y)_{\hat{t}} \in \hat{\mu} \Rightarrow x_{\hat{t}} \in \hat{\mu}$ or $y_{\hat{t}} \in \hat{\mu}$

Hence $\hat{\mu}$ is $(\in, \in)$-interval-valued fuzzy prime ideal of X.

Conversely,

Assume $\hat{\mu}$ be an $(\in, \in)$-interval-valued fuzzy prime ideal of X.

Let $(x \wedge y)_{\hat{t}} q \hat{\mu}$

$\Rightarrow \hat{\mu}(x \wedge y) + \hat{t} \succ \hat{1}$

$\Rightarrow \hat{\mu}(x \wedge y) \succ \hat{1} - \hat{t}$

$\Rightarrow \hat{\mu}(x \wedge y) \succeq \hat{\delta} - \hat{t} + \hat{1} \succ \hat{1} - \hat{t}$ where $\hat{\delta} \succ \hat{0}$ is arbitrary

$\Rightarrow (x \wedge y)_{\hat{\delta} - \hat{t} + \hat{1}} \in \hat{\mu}$

Since $\hat{\mu}$ is an $(\in, \in)$-interval-valued fuzzy prime ideal X.

Therefore we have $x_{\hat{\delta} - \hat{t} + \hat{1}} \in \hat{\mu}$ or $y_{\hat{\delta} - \hat{t} + \hat{1}} \in \hat{\mu}$

$\Rightarrow \hat{\mu}(x) \succeq \hat{\delta} - \hat{t} + \hat{1} \succ \hat{1} - \hat{t}$ or $\hat{\mu}(y) \succeq \hat{\delta} - \hat{t} + \hat{1} \succ \hat{1} - \hat{t}$

$\Rightarrow \hat{\mu}(x) \succ \hat{1} - \hat{t}$ or $\hat{\mu}(y) \succ \hat{1} - \hat{t}$

$\Rightarrow \hat{\mu}(x) + \hat{t} \succ \hat{1}$ or $\hat{\mu}(y) + \hat{t} \succ \hat{1}$

$\Rightarrow x_{\hat{t}} q \hat{\mu}$ or $y_{\hat{t}} q \hat{\mu}$

therefore $(x \wedge y)_{\hat{t}} q \hat{\mu} \Rightarrow x_{\hat{t}} q \hat{\mu}$ or $y_{\hat{t}} q \hat{\mu}$

Hence $\hat{\mu}$ is a (q, q)-interval-valued fuzzy prime ideal of X.

Remark 3.5 *The notion of interval-valued fuzzy prime ideal, $(\in, \in)$-interval-valued fuzzy prime ideal and (q, q)-interval-valued fuzzy prime ideal are equivalent.*

Theorem 3.6 *An interval-valued fuzzy set $\hat{\mu}$ of a BCK-algebra X is a $(\in, \in \vee q))$-interval-valued fuzzy prime ideal of X iff*

$$rmax\{\hat{\mu}(x), \hat{\mu}(y)\} \succeq rmin\{\hat{\mu}(x \wedge y), [0.5,\ 0.5]\} \quad \forall x, y \in X.$$

First let $\hat{\mu}$ be an $(\in, \in \vee q)$-interval-valued fuzzy prime ideal of X. To prove

$$rmax\{\hat{\mu}(x), \hat{\mu}(y)\} \succeq rmin\{\hat{\mu}(x \wedge y), [0.5,\ 0.5]\} \tag{3.1}$$

Assume that(3.1) is not valid, then there exists some $x, y \in X$such that

$$rmax\{\hat{\mu}(x), \hat{\mu}(y)\} \prec rmin\{\hat{\mu}(x \wedge y), [0.5,\ 0.5]\}$$

choose an interval $\hat{t} \in [[0\,0],\ [0.5,\ 0.5]]$ such that

$$rmax\{\hat{\mu}(x), \hat{\mu}(y)\} \prec \hat{t} \prec rmin\{\hat{\mu}(x \wedge y), [0.5,\ 0.5]\} \tag{3.2}$$

then $[0\,0] \prec \hat{t} \preceq [0.5,\ 0.5]$ and $\hat{\mu}(x \wedge y) \succ \hat{t} \Rightarrow (x \wedge y)_{\hat{t}} \in \hat{\mu}$

and also $\hat{\mu}(x) \prec \hat{t}$ or $\hat{\mu}(y) \prec \hat{t}$ i.e., $x_{\hat{t}} \overline{\in} \hat{\mu}, y_{\hat{t}} \overline{\in} \hat{\mu}$

also $\hat{\mu}(x) + \hat{t} \prec 2\hat{t} \prec 2[0.5,\ 0.5] = [1\,1] = \hat{1}$ i.e., $x_{\hat{t}} \overline{\in \vee q} \hat{\mu},\ y_{\hat{t}} \overline{\in \vee q} \hat{\mu}$ which is a contradiction, since $\hat{\mu}$ is an $(\in, \in \vee q)$-interval-valued fuzzy prime ideal

Hence we must have

$$rmax\{\hat{\mu}(x), \hat{\mu}(y)\} \succeq rmin\{\hat{\mu}(x \wedge y), [0.5,\ 0.5]\}$$

Conversely, Suppose

$$rmax\{\hat{\mu}(x), \hat{\mu}(y)\} \succeq rmin\{\hat{\mu}(x \wedge y), [0.5,\ 0.5]\} \tag{3.3}$$

Let $x, y \in X$ such that $(x \wedge y)_{\hat{t}} \in \hat{\mu}$ where $\hat{t} \in D(0,1]$

i.e., $\hat{\mu}(x \wedge y) \succeq \hat{t}$ Now (3.3) $\Rightarrow rmax\{\hat{\mu}(x), \hat{\mu}(y)\} \succeq rmin\{\hat{t}, [0.5,\ 0.5]\}$ Now we have

Case I: $\hat{t} \preceq [0.5\,0.5]$

then $rmax\{\mu(x), \mu(y)\} \succeq \hat{t} \ \Rightarrow \hat{\mu}(x) \succeq \hat{t}$ or $\hat{\mu}(y) \succeq \hat{t} \Rightarrow x_{\hat{t}} \in \hat{\mu}$ or $y_{\hat{t}} \in \hat{\mu}$

Case II: $\hat{t} \succ [0.5\,0.5]$ then $rmax\{\hat{\mu}(x), \hat{\mu}(y)\} \succeq [0.5,\ 0.5] \Rightarrow \hat{\mu}(x) \succeq [0.5,\ 0.5]$ or $\hat{\mu}(y) \succeq [0.5,\ 0.5]$

$\Rightarrow \hat{\mu}(x) + \hat{t} \succeq [0.5,\ 0.5] + \hat{t} \succ [0.5\,0.5] + [0.5, 0.5] = [1,\ 1]$ or $\hat{\mu}(y) + \hat{t} \succ [0.5,\ 0.5] + \hat{t} \succ [0.5,\ 0.5] + [0.5,\ 0.5] = [1,\ 1]$

$\Rightarrow x_{\hat{t}} q \hat{\mu}$ or $y_{\hat{t}} q \hat{\mu}$ combining case I and case II $x_{\hat{t}} \in \vee q \hat{\mu}$ or $y_{\hat{t}} \in \vee q \hat{\mu}$

Hence $(x \wedge y)_{\hat{t}} \in \hat{\mu} \Rightarrow x_{\hat{t}} \in \vee q \hat{\mu}$ or $y_{\hat{t}} \in \vee q \hat{\mu}$

that is $\hat{\mu}$ is an $(\in, \in \vee q)$-interval-valued fuzzy prime ideal of X.

Remark 3.7 *Every interval-value fuzzy prime ideal is an $(\in, \in \vee q)$-interval-value fuzzy prime ideal.*

Remark 3.8 *Converse of above is not true as seen from the following example.*

Example 3.9 *Consider BCK-algebra $X = \{0, x, y, z\}$ with the following cayley tables.*

Table.1: Illustration of converse of Remark 3.7

*	0	x	y	z
0	0	0	0	0
x	x	0	0	x
y	y	x	0	y
z	z	z	z	0

∧	0	x	y	z
0	0	0	0	0
x	0	x	x	0
y	0	x	y	0
z	0	0	0	z

Define an interval-valued fuzzy set $\hat{\mu} : X \to D[0,1]$ by $\hat{\mu}(0) = [0.65,\ 0.70], \hat{\mu}(x) = [0.5,\ 0.6], \hat{\mu}(y) = [0.62,\ 0.75], \hat{\mu}(z) = [0.53,\ 0.58]$ then $\hat{\mu}$ is an $(\in, \in \vee q)$-interval-value fuzzy prime ideal X by Theorem 3.6, but not an interval-value fuzzy prime ideal of X because $\hat{\mu}(x \wedge z) = \hat{\mu}(0) = [0.65,\ 0.70] \succ rmax\{\hat{\mu}(x), \hat{\mu}(z)\} = rmax\{[0.5,\ 0.6], [0.53,\ 0.58]\} = [0.53,\ 0.6]$.

Theorem 3.10 *If a fuzzy subset $\hat{\mu}$ of a BCK-algebra X is an $(\in, \in \vee q)$-interval-valued fuzzy prime ideal of X and $\hat{\mu}(x) \prec [0.5\,0.5]\ \forall x, y \in X$, then $\hat{\mu}$ is also an $(\in, \in)$-interval-valued fuzzy prime ideal of X.*

Let $\hat{\mu}$ be an $(\in, \in \vee q)$-interval-value fuzzy prime ideal of X and $\hat{\mu}(x) \succ [0.5,\ 0.5]\ \forall x \in X$. Let$(x \wedge y)_{\hat{t}} \in \hat{\mu} \Rightarrow \hat{\mu}(x \wedge y) \succeq \hat{t}$ therefore $\hat{t} \preceq \hat{\mu}(x \wedge y) \prec [0.5\,0.5]$ and also $\hat{\mu}(x) \preceq [0.5,\ 0.5]$, $\hat{\mu}(y) \prec [0.5,\ 0.5]$ therefore $\hat{t} \prec [0.5,\ 0.5]$ and also $\hat{\mu}(x) + \hat{t} \prec [0.5\,0.5] + [0.5,\ 0.5] = [1,\ 1]$ and $\hat{\mu}(y) + \hat{t} \prec [0.5\,0.5] + [0.5,\ 0.5] = [1,\ 1] \Rightarrow x_t \overline{q} \hat{\mu}$ and $y_t \overline{q} \hat{\mu}$. Since $\hat{\mu}$ is an $(\in, \in \vee q)$-interval-value fuzzy prime ideal, so we must have $x_{\hat{t}} \in \hat{\mu}$ or $y_{\hat{t}} \in \hat{\mu}$ i.e. $\hat{\mu}$ is an $(\in, \in)$-interval-value fuzzy prime ideal of X.

Theorem 3.11 *An interval-valued fuzzy set $\hat{\mu}$ in X is an $(\in, \in \vee q)$-interval-valued fuzzy prime ideal of X if and only if the set $\hat{\mu}_{\hat{t}} = \{x \in X | \hat{\mu}(x) \succeq \hat{t}\}$ is a prime ideal of X for all $[0,\ 0] \prec \hat{t} \preceq [0.5,\ 0.5]$ and $\hat{\mu}_{\hat{t}} \neq \phi$*

Assume that $\hat{\mu}$ be an $(\in, \in \vee q)$-interval-valued fuzzy prime ideal of X. Let $[0,\ 0] \prec \hat{t} \preceq [0.5,\ 0.5]$ and $x \wedge y \in \hat{\mu}_{\hat{t}}$. therefore $\hat{\mu}(x \wedge y) \succeq \hat{t}$. It follows that $rmax\{\hat{\mu}(x), \hat{\mu}(y)\} \succeq rmin\{\hat{\mu}(x \wedge y), [0.5\ 0.5]\} \succeq rmin\{\hat{t}, [0.5\ 0.5]\} = \hat{t}$ therefore $\hat{\mu}(x) \succeq \hat{t}$ or $\hat{\mu}(y) \succeq \hat{t}$ that is $x \in \hat{\mu}_{\hat{t}}$, or $y \in \hat{\mu}_{\hat{t}}$ therefore $\hat{\mu}_{\hat{t}}$ is a prime ideal of X.
Conversely
Suppose that $\hat{\mu}_{\hat{t}}$ is a prime ideal of X where $[0,\ 0] < \hat{t} \prec [0.5,\ 0.5]$ and let $rmax\{\hat{\mu}(x), \hat{\mu}(y)\} \succeq rmin\{\hat{\mu}(x \wedge y), [0.5,\ 0.5]\}$ is not valid, then their exists some $a, b \in X$ such that $rmax\{\hat{\mu}(a), \hat{\mu}(b)\} \prec rmin\{\hat{\mu}(a \wedge b), [0.5,\ 0.5]\}$ hence we can take $[0,\ 0] \succ \hat{t} \succeq [0.5,\ 0.5]$ such that

$$rmax\{\hat{\mu}(a), \hat{\mu}(b)\} \prec \hat{t} \prec rmin\{\hat{\mu}(a \wedge b), [0.5,\ 0.5]\} \tag{3.4}$$

(3.4)$\Rightarrow a \wedge b \in \hat{\mu}_{\hat{t}}$. Since $\hat{\mu}_{\hat{t}}$ is an ideal of X, it follows that $a \in \hat{\mu}_{\hat{t}}$ *or* $b \in \hat{\mu}_{\hat{t}}$, so that $\hat{\mu}(a) \succeq \hat{t}$ or $\hat{\mu}(b) \succeq \hat{t}$. which contradicts (3.4), therefore we must have $rmax\{\hat{\mu}(x), \hat{\mu}(y)\} \succeq rmin\{\hat{\mu}(x \wedge y), [0.5,\ 0.5]\}$ consequently $\hat{\mu}$ is an $(\in, \in \vee q)$-interval-valued fuzzy prime ideal of X.

Theorem 3.12 *Let A be a non empty subset of a BCK- algebra X. Consider the interval-valued fuzzy set $\hat{\mu}_A$ in X defined by*

$$\hat{\mu}_A(x) = \begin{cases} \hat{1} & \text{if} \quad x \in A \\ \hat{0} & \text{otherwise} \end{cases}$$

Then A is a prime ideal of X iff $\hat{\mu}_A$ is an $(\in, \in \vee q)$-interval-valued fuzzy prime ideal of X.

Let A be an ideal of X, then $(\hat{\mu}_A)_{\hat{t}} = \{x \in X | \hat{\mu}_A(x) \succ \hat{t}\} = A$ where $[0\,0] \prec \hat{t} \preceq [0.5,\ 0.5]$ which is a prime ideal. Hence by above theorem $\hat{\mu}_A$ is an $(\in, \in \vee q)$-interval-valued fuzzy prime ideal of X.
Conversely, assume that $\hat{\mu}_A$ is an $(\in, \in \vee q)$-interval-value fuzzy prime ideal of X. Let $x \wedge y \in A$, then $\hat{\mu}_A(x \wedge y) = \hat{1}$. Now $rmax\{\hat{\mu}_A(x), \hat{\mu}_A(y)\} \succeq rmin\{\hat{\mu}_A(x \wedge y), [0.5,\ 0.5]\} = rmin\{[1,\ 1], [0.5,\ 0.5]\} = [0.5,\ 0.5] \Rightarrow rmax\{\hat{\mu}_A(x), \hat{\mu}_A(y)\} \succeq [0.5,\ 0.5] \Rightarrow \hat{\mu}_A(x) \succeq [0.5,\ 0.5]$ or $\hat{\mu}_A(y)\} \succeq [0.5,\ 0.5] \Rightarrow \hat{\mu}_A(x) = [1,\ 1] = \hat{1}$ or $\hat{\mu}_A(y)\} = [1,\ 1] = \hat{1} \Rightarrow x \in A$ or $y \in A$. Therefore $x \wedge y \in A \Rightarrow x \in A$ or $y \in A$
Hence A is a prime ideal of X.

Theorem 3.13 *Let A be a prime ideal of X, then for every $[0,\ 0] \prec \hat{t} \preceq [0.5,\ 0.5]$ their exists an $(\in, \in \vee q)$-interval-valued fuzzy prime ideal $\hat{\mu}$ of X, such that $\hat{\mu}_{\hat{t}} = A$.*

Let $\hat{\mu}$ be an interval-valued fuzzy set in X defined by

$$\hat{\mu}(x) = \begin{cases} \hat{1} & \text{if} \quad x \in A \\ \hat{s} & \text{otherwise} \end{cases}$$

for all $x \in X$ where $[0\,0] \prec \hat{s} \prec \hat{t} \preceq [0.5,\ 0.5]$
$(\hat{\mu})_{\hat{t}} = \{x \in X | \hat{\mu}(x) \succeq \hat{t} \succ \hat{s}\} = A$. Hence $(\hat{\mu})_{\hat{t}}$ is a prime ideal. Now if $\hat{\mu}$ is not an $(\in, \in \vee q)$-interval-valued fuzzy prime ideal of X. Then there exists some $a, b \in X$ such that $rmax\{\hat{\mu}(a), \mu(b)\} \prec rmin\{\hat{\mu}(a \wedge b), [0.5,\ 0.5]\}$ choose $[0,\ 0] \prec \hat{t} \preceq [0.5,\ 0.5]$ such that

$$rmax\{\hat{\mu}(a), \hat{\mu}(b)\} \prec \hat{t} \prec rmin\{\hat{\mu}(a \wedge b), [0.5,\ 0.5]\} \tag{3.5}$$

therefore $\hat{\mu}(a \wedge b) \succeq \hat{t}$ for all $a, b \in X \Rightarrow a \wedge b \in (\hat{\mu})_{\hat{t}} = A$, a prime ideal therefore $a \in A$ or $b \in A \Rightarrow \hat{\mu}(a) = [1,\ 1] = \hat{1}$ or $\hat{\mu}(b) = [1,\ 1] = \hat{1}$ which contradicts(3.5). Hence $\hat{\mu}$ is an $(\in, \in \vee q)$-interval-valued fuzzy prime ideal of X.

Definition 3.14 *Let $\hat{\mu}$ be an interval-valued fuzzy set in BCK-algebra X and $[0\,0] \prec \hat{t} \preceq [0.5,\ 0.5]$, let*
$\hat{\mu}_{\hat{t}} = \{x \in X | x_{\hat{t}} \in \hat{\mu}\} = \{x \in X | \hat{\mu}(x) \succeq \hat{t}\}$
$< \hat{\mu} >_{\hat{t}} = \{x \in X | x_{\hat{t}} q \hat{\mu}\} = \{x \in X | \hat{\mu}(x) + \hat{t} \succ 1\}$
$[\hat{\mu}]_{\hat{t}} = \{x \in X | x_{\hat{t}} \in \vee q \hat{\mu}\} = \{x \in X | \hat{\mu}(x) \succeq \hat{t}\, or\, \hat{\mu}(x) + \hat{t} \succ 1\}$
Here $\hat{\mu}_{\hat{t}}$ is called $\hat{t}$ level set of $\hat{\mu}$, $< \hat{\mu} >_{\hat{t}}$ is called q level set of $\hat{\mu}$ and $[\hat{\mu}]_{\hat{t}}$ is called $(\in \vee q)$ level set of $\hat{\mu}$
Clearly $[\hat{\mu}]_{\hat{t}} = < \hat{\mu} >_{\hat{t}} \cup \hat{\mu}_{\hat{t}}$

Theorem 3.15 *Let $\hat{\mu}$ be a fuzzy set in BCK-algebra X. Then $\hat{\mu}$ is an $(\in, \in \vee q)$-interval-valued fuzzy prime ideal of X iff $[\hat{\mu}]_{\hat{t}}$ is a prime ideal of X for all $[0,\ 0] \prec \hat{t} \preceq [0.5,\ 0.5]$. We call $[\hat{\mu}]_{\hat{t}}$ as $(\in \vee q)$-level prime ideal of $\hat{\mu}$.*

Assume that $\hat{\mu}$ is an $(\in, \in \vee q)$-interval-valued fuzzy prime ideal of X, to prove $[\hat{\mu}]_{\hat{t}}$ is a prime ideal of X. Let $x \wedge y \in [\hat{\mu}]_{\hat{t}}$ for $\hat{t} \in D(0\,1]$ then $(x \wedge y)_{\hat{t}} \in \vee q \hat{\mu}$ then $\hat{\mu}(x \wedge y) \succeq \hat{t}$ or $\hat{\mu}(x \wedge y) + \hat{t} \succ [1,\ 1]$ since $\hat{\mu}$ is an $(\in, \in \vee q)$-interval-value fuzzy prime ideal of X. Therefore $rmax\{\hat{\mu}(x), \hat{\mu}(y)\} \succeq rmin\{\hat{\mu}(x \wedge y), [0.5,\ 0.5]\}$. Now we have the following cases:
Case I: $\hat{\mu}(x \wedge y) \succ \hat{t}$
$rmax\{\hat{\mu}(x), \hat{\mu}(y)\} \succeq rmin\{\hat{t}, [0.5,\ 0.5]\}$
Subcase I: $\hat{t} \succ [0.5\,0.5]$
$rmax\{\hat{\mu}(x), \hat{\mu}(y)\} \succeq [0.5\,0.5]$
$\Rightarrow \hat{\mu}(x) \succeq [0.5\,0.5]$ or $\hat{\mu}(y) \succeq [0.5\,0.5]$
$\Rightarrow \hat{\mu}(x) + \hat{t} \succ [0.5,\ 0.5] + [0.5,\ 0.5] = [1,\ 1]$ or $\hat{\mu}(y) + \hat{t} \succ [0.5,\ 0.5] + [0.5,\ 0.5] = [1,\ 1]$
$\Rightarrow x_{\hat{t}} q \hat{\mu}$ or $y_{\hat{t}} q \hat{\mu}$
Subcase II: $\hat{t} \preceq [0.5,\ 0.5]$
$rmax\{\hat{\mu}(x), \hat{\mu}(y)\} \succeq \hat{t}$
$\Rightarrow \hat{\mu}(x) \succeq \hat{t}$ or $\hat{\mu}(y) \succeq \hat{t}$
$\Rightarrow x_{\hat{t}} \in \hat{\mu}$ or $y_{\hat{t}} \in \hat{\mu}$
Hence $(x \wedge y)_{\hat{t}} \in \vee q \hat{\mu} \Rightarrow x_{\hat{t}} \in \vee q \hat{\mu}$ or $y_{\hat{t}} \in \vee q \hat{\mu}$
i.e $(x \wedge y)_{\hat{t}} \in [\hat{\mu}]_{\hat{t}} \Rightarrow x_{\hat{t}} \in [\hat{\mu}]_{\hat{t}}$ or $y_{\hat{t}} \in [\hat{\mu}]_{\hat{t}}$
Case II: $\hat{\mu}(x \wedge y) + \hat{t} \succ [1,\ 1]$
$rmax\{\hat{\mu}(x), \hat{\mu}(y)\} \succeq rmin\{[1,\ 1] - \hat{t}, [0.5,\ 0.5]\}$
Subcase I: $\hat{t} \preceq [0.5,\ 0.5]$
$rmax\{\hat{\mu}(x), \hat{\mu}(y)\} \succeq [0.5,\ 0.5] \succeq \hat{t}$
$\Rightarrow \hat{\mu}(x) \succeq \hat{t}$ or $\hat{\mu}(y) \succeq \hat{t}$
$\Rightarrow x_{\hat{t}} \in \hat{\mu}$ or $y_{\hat{t}} \in \hat{\mu}$
Subcase II: $\hat{t} \succ [0.5,\ 0.5]$
$rmax\{\hat{\mu}(x), \hat{\mu}(y)\} \succeq [1,\ 1] - \hat{t}$
$\Rightarrow \hat{\mu}(x) \succeq [1\,1] - \hat{t}$ or $\hat{\mu}(y) \succeq [1,\ 1] - \hat{t}$

$\Rightarrow \hat{\mu}(x) + \hat{t} \succeq [1,\ 1]$ or $\hat{\mu}(y) + \hat{t} \succeq [1,\ 1]$
$\Rightarrow x_{\hat{t}} q \hat{\mu}$ or $y_{\hat{t}} q \hat{\mu}$
Hence $(x \wedge y)_{\hat{t}} \in \vee q \hat{\mu} \Rightarrow x_{\hat{t}} \in \vee q \hat{\mu}$ or $y_{\hat{t}} \in \vee q \hat{\mu}$
i.e $(x \wedge y)_{\hat{t}} \in [\hat{\mu}]_t \Rightarrow x_{\hat{t}} \in [\hat{\mu}]_{\hat{t}}$ or $y_{\hat{t}} \in [\hat{\mu}]_{\hat{t}}$
Conversely, let $\hat{\mu}$ be an interval-value fuzzy set in X and $\hat{t} \in D(0,\ 1]$ such that $[\hat{\mu}]_t$ is an prime ideal of X. To prove $\hat{\mu}$ is an $(\in, \in \vee q)$-fuzzy prime ideal of X. If $\hat{\mu}$ is not an $(\in, \in \vee q)$-interval-valued fuzzy prime ideal of X, then there exists some $a, b \in X$ such that $rmax\{\hat{\mu}(a), \hat{\mu}(b)\} \prec rmin\{\hat{\mu}(a \wedge b), [0.5,\ 0.5]\}$ holds. Choose $\hat{t}$ such that $rmax\{\hat{\mu}(a), \hat{\mu}(b)\} \prec \hat{t} \prec rmin\{\hat{\mu}(a \wedge b), 0.5\}$ then $\hat{\mu}(a \wedge b) \succ \hat{t} \Rightarrow a \wedge b \in \hat{\mu}_{\hat{t}} \subset [\hat{\mu}]_{\hat{t}}$ which is a prime ideal
$\Rightarrow a \in [\hat{\mu}]_{\hat{t}}$ or $b \in [\hat{\mu}]_{\hat{t}}$
$\Rightarrow \hat{\mu}(a) \succeq \hat{t}$ or $\hat{\mu}(a) + \hat{t} \succ [1,\ 1]$ or $\hat{\mu}(b) \succeq \hat{t}$ or $\hat{\mu}(b) + \hat{t} \succ [1,\ 1]$ which contradicts hence we must have $rmax\{\hat{\mu}(x), \hat{\mu}(y)\} \succeq rmin\{\hat{\mu}(x \wedge y), [0.5,\ 0.5]\}$.

Theorem 3.16 *Let $\hat{\lambda}$ and $\hat{\mu}$ be two $(\in, \in \vee q)$-interval-valued fuzzy prime ideals of X, then $\hat{\lambda} \hat{\cup} \hat{\mu}$ is an $(\in, \in \vee q)$-interval-valued fuzzy prime ideal of X.*

Here $\hat{\lambda}$ and $\hat{\mu}$ both are $(\in, \in \vee q)$-interval-valued fuzzy prime ideals of X. Therefore $\forall x, y \in X$.

$$rmax\{\hat{\mu}(x), \hat{\mu}(y)\} \succeq rmin\{\hat{\mu}(x \wedge y), [0.5,\ 0.5]\},$$
$$rmax\{\hat{\lambda}(x), \hat{\lambda}(y)\} \succeq rmin\{\hat{\lambda}(x \wedge y), [0.5,\ 0.5]\}, \tag{3.6}$$

To prove $\hat{\lambda} \hat{\cup} \hat{\mu}$ is an $(\in, \in \vee q)$-fuzzy prime ideal of X. It is enough to show that

$$rmax\{(\hat{\lambda} \hat{\cup} \hat{\mu})(x), (\hat{\lambda} \hat{\cup} \hat{\mu})(y)\} \succeq rmin\{(\hat{\lambda} \hat{\cup} \hat{\mu})(x \wedge y), [0.5,\ 0.5]\} \quad \forall x, y \in X. \tag{3.7}$$

Now $rmax\{\hat{\lambda} \hat{\cup} \hat{\mu})(x), \hat{\lambda} \hat{\cup} \hat{\mu})(y)\}$
$= rmax\{rmax\{\hat{\lambda}(x), \mu(x)\}, rmax\{\hat{\lambda}(y), \hat{\mu}(y)\}\}$
$= rmax\{rmax\{\hat{\lambda}(x), \hat{\lambda}(y)\}, rmax\{\hat{\mu}(x), \hat{\mu}(y)\}\}$
$\succeq rmax\{rmin\{\hat{\lambda}(x \wedge y), [0.5,\ 0.5]\}, rmin\{\hat{\mu}(x \wedge y), [0.5,\ 0.5]\}\}$ by (3.6)

$$\Rightarrow rmax\{(\hat{\lambda} \hat{\cup} \hat{\mu})(x), \hat{\lambda} \hat{\cup} \hat{\mu})(y)\} \succeq rmax\{rmin\{(\hat{\lambda}(x \wedge y), [0.5,\ 0.5]\}, rmin\{\hat{\mu}(x \wedge y), [0.5,\ 0.5]\}\} \tag{3.8}$$

Now we have the following cases:
Case I: $\hat{\lambda}(x \wedge y) \preceq [0.5,\ 0.5]$ and $\hat{\mu}(x \wedge y) \preceq [0.5,\ 0.5]$, then

$$\begin{aligned} (3.8) \Rightarrow rmax\{(\hat{\lambda} \hat{\cup} \hat{\mu})(x), (\hat{\lambda} \hat{\cup} \hat{\mu})(y)\} \quad & \succeq rmax\{\hat{\lambda}(x \wedge y), \hat{\mu}(x \wedge y)\} \\ & \succeq (\hat{\lambda} \hat{\cup} \hat{\mu})(x \wedge y) \\ & = rmin\{(\hat{\lambda} \hat{\cup} \hat{\mu})(x \wedge y), [0.5,\ 0.5]\} \end{aligned}$$

Case II: $\hat{\lambda}(x \wedge y) \preceq [0.5,\ 0.5]$ and $\hat{\mu}(x \wedge y) \succ [0.5,\ 0.5]$, then

$$\begin{aligned} (3.8) \Rightarrow rmax\{(\hat{\lambda} \hat{\cup} \hat{\mu})(x), (\hat{\lambda} \hat{\cup} \hat{\mu})(y)\} \quad & \succeq rmax\{\hat{\lambda}(x \wedge y), [0.5,\ 0.5]\} = [0.5,\ 0.5] \\ & = rmin\{rmax\{\hat{\lambda}(x \wedge y), \hat{\mu}(x \wedge y)\}, [0.5,\ 0.5]\} \\ & = rmin\{(\hat{\lambda} \hat{\cup} \hat{\mu})(x \wedge y), [0.5,\ 0.5]\} \end{aligned}$$

Case III: $\hat{\lambda}(x \wedge y) \succ [0.5, 0.5]$ and $\hat{\mu}(x \wedge y) \preceq [0.5, 0.5]$

$$(3.8) \Rightarrow rmax\{(\hat{\lambda}\hat{\cup}\hat{\mu})(x), (\hat{\lambda}\hat{\cup}\hat{\mu})(y)\} \succeq rmax\{[0.5, 0.5], \hat{\mu}(x \wedge y)\} = [0.5, 0.5]$$
$$= rmin\{rmax\{\hat{\lambda}(x \wedge y), \hat{\mu}(x \wedge y)\}, [0.5, 0.5]\}$$
$$= rmin\{(\hat{\lambda}\hat{\cup}\hat{\mu})(x \wedge y), [0.5, 0.5]\}$$

Case IV: $\hat{\lambda}(x \wedge y) \succ [0.5, 0.5]$ and $\hat{\mu}(x \wedge y) \succ [0.5, 0.5]$, then

$$(3.8) \Rightarrow rmax\{(\hat{\lambda}\hat{\cup}\hat{\mu})(x), (\hat{\lambda}\hat{\cup}\hat{\mu})(y)\} \succeq rmax\{[0.5, 0.5], [0.5, 0.5]\} = [0.5, 0.5]$$
$$= rmin\{rmax\{\hat{\lambda}(x \wedge y), \hat{\mu}(x \wedge y)\}, [0.5, 0.5]\}$$
$$= rmin\{(\hat{\lambda}\hat{\cup}\hat{\mu})(x \wedge y), [0.5, 0.5]\}$$

Hence from above (3.7) holds $\forall\, x, y \in X$.
Hence $\hat{\lambda}\hat{\cup}\hat{\mu}$ is an $(\in, \in \vee q)$-interval-valued fuzzy prime ideal of X.

Theorem 3.17 *Let $\{\hat{\mu}_i : i \in \wedge\}$ be a family of $(\in, \in \vee q)$-interval-valued fuzzy prime ideals of X, then $\hat{\mu} = \hat{\cup}\{\hat{\mu}_i : i \in \wedge\}$ is an $(\in, \in \vee q)$-interval-valued fuzzy prime ideal of X.*

4 Conclusions

In this paper, we have studied $(\in, \in \vee q)$-interval-valued fuzzy prime ideals of commutative BCK-algebras and investigated several properties. In my opinion, these definitions and results can be extended to other algebraic systems also. In the notions of (α, β)-fuzzy prime ideals, (α, β)-intuitionistic fuzzy prime ideals and (α, β)-interval-valued fuzzy prime ideals, where $\alpha, \beta \in \{\in, q, \in \vee q, \in \wedge q\}$, we can define twelve different types of ideals by three choices of α and four choices of β. We can apply (α, β)-fuzzy prime ideals is in the field of fuzzy medical diagnosis, artificial intelligence, information science, agriculture etc.

References

[1] K. T. Atanassov and G. Gargov, *Interval-valued intuitionistic fuzzy sets,* Fuzzy sets and systems, **31(1)** (1989), 343-349.

[2] K. T. Atanassov, *Operations over interval-valued fuzzy sets,* Fuzzy sets and systems, **64** (1994), 159-174.

[3] S. Abdullah, *Intuitionistic fuzzy prime ideals of BCK-algebras,* Annals of Fuzzy Mathematics and Informatics, **7(4)** (April 2014) 661-668.

[4] J. Ahsan, E. Y. Deeba and A. B. Thaheem, *On prime ideal of BCK-algebras,* Mathematica Japonica,**36** (1991) 875-882.

[5] R. Ameri, H. Hedayati and M. Norouzi, $(\overline{\in}, \overline{\in \wedge q_k})$*-Fuzzy Subalgebras in BCK/BCI-Algebras,* The Journal of Mathematics and Computer Science. **2(1)**(2011)130-140.

[6] M. Attallah, *Completely Fuzzy Prime Ideals of distributed lattices,* The Journal of Fuzzy Mathematics, **8(1)** (2000), 153-156.

[7] S. K. Bhakat and P. Das, $(\in, \in \vee q)$*-fuzzy subgroup,* Fuzzy sets and systems, **80** (1996), 359-368.

[8] S. K. Bhakat and P. Das, $(\in \vee q)$*-level subset*, Fuzzy sets and systems, **103(3)** (1999), 529-533.

[9] R. Biswas, Rosenfelds fuzzy subgroups with interval valued membership functions, Fuzzy Sets and Systems, 63 (1994) 87-90.

[10] D. K. Basnet and L. B. Singh, $(\in, \in \vee q)$*-fuzzy ideal of BG-algebra*, International Journal of Algebra, **5(15)** (2011) 703-708.

[11] S. R. Barbhuiya and K. D. Choudhury, $(\in, \in \vee q)$*-Fuzzy ideal of d-algebra.* International Journal of Mathematics Trends and Technology **9(1)** (2014), 16-26.

[12] S. H. Dhanani and Y. S. Pawar, $(\in, \in \vee q)$*-Fuzzy Ideals of Lattice*, International Journal of Algebra, **4(26)** (2010), 1277-1288.

[13] T. K. Dutta, S. Kar, and S. Purkait, Interval-valued fuzzy prime and semiprime ideals of a hyper semiring, Annals of Fuzzy Mathematics and Informatics, 9(2), (2015), 261-278.

[14] Y. Imai and K. Iseki, *On Axiom systems of Propositional calculi XIV*, Proc, Japan Academy, **42** (1966)19-22.

[15] K. Iseki, *An algebra related with a propositional calculus, Proc. Japan Academy,* **42** (1966), 26-29.

[16] K. Iseki, *On some ideals in BCK-algebras, Math.Seminar Notes,* **3** (1975) 65-70.

[17] Y. B. Jun, Interval-valued fuzzy subalgebras/ideals in BCK-algebras, Scientiae Mathematicae, 3(3) (2000) 435-444.

[18] Y. B. Jun and X. L. Xin, *Involutory and invertible fuzzy BCK-algebras,* Fuzzy sets and systems, **117**(2001), 463-469.

[19] Y.B.Jun and X. L. Xin, *Fuzzy prime ideals and invertible fuzzy ideals in BCK-algebras,* Fuzzy sets and systems, **117**(2001), 471-476.

[20] Y. B. Jun, *On* (α, β)*-Fuzzy ideals of BCK/BCI-Algebras Scientiae Mathematicae Japonicae,* online, e-2004, 101-105.

[21] D. S. Lee and C. H. Park, *Interval-valued* $(\in, \in \vee q)$ *fuzzy ideals in Rings,* International Mathematical Forum, **4(13)** (2009), 623-630.

[22] X. Ma, J. Zhan, B. Davvaz and Y. B.Yun , *Some kinds of* $(\in, \in \vee q)$*-Interval-valued fuzzy ideals of BCI-algebras,* Information Sciences , **178** (2008), 3738-3754.

[23] B. B. N. Koguep, C. Nkuimi, and C. Lele, *On Fuzzy Ideals of Hyperlattice,* International Journal of Algebra, **2(15)** (2008),739-750.

[24] M. A. Larimi,*On* $(\in, \in \vee q_k)$*-Intuitionistic Fuzzy Ideals of Hemirings,* World Applied Sciences Journal **21** (special issue of Applied Math), 54-67,2013.

[25] P. P. Ming and L. Y. Ming, Fuzzy topology I, Neighbourhood structure of a fuzzy point and Moore-Smith convergence, J. Maths. Anal. Appl, 76(1980), 571-599.

[26] V. Murali, *Fuzzy points of equivalent fuzzy subsets,* inform Sci, **158** (2004) 277-288.

[27] A. Rosenfeld, *Fuzzy subgroups,* J Math Anal Appl. 35 (1971),512517.

[28] L. A. Zadeh, *Fuzzy sets,* Information and Control **8** (1965), 338-353.

[29] L. A. Zadeh, *The concept of a linguistic variable and its application to approximate reasoning-I,* Information Science **8** (1975), 199-249.