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Interactive Approach for Multi-level Quadratic Fractional Programming Problems

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ABSTRACT. This paper, present an Interactive decision making method for multi-level quadratic fractional programming **(MLQFP)** problems. The upper-level objective function is quadratic fractional problem in which both the numerator and denominator of the objective function can be factorized into linear function, the second-level objective function is quadratic fractional programming **(QPP)** problem in which the objective function can be factorized into linear functions and the denominator of the objective function in linear type and the third-level objective function is linear fractional with linear constraint. **(MLQFP)** problem is transformed into an equivalent bi-level quadratic fractional programming **(QFP)** problem by forcing the duality gap of the lower-level problem to zero. Then by using interactive approach for solving **(BLFQP)** problem, the first level decision maker **(FLDM)** give the preferred as satisfactory solution that are acceptable in rank order to second level decision maker. **(SLDM)** take the satisfactory solution one by one to seek the solutions, who will search for the preferred solution of **(FLDM)** until the preferred solution is reached. Finally, an illustrative numerical example is provided.

1 Introduction

Multi-level optimization plays an important role in engineering design, management, and decision making in general. Three-level optimization is a kind of multi-level optimization which is a technique developed to solve decentralized problems with multiple decision-makers in hierarchical organization. Three-level programming problem is concerned with minimizing or maximizing some quantity represented by an objective function [6,13,14,17].

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Quadratic programming (QP) is one of the most popular models used in decision-making and in optimization problems. The quadratic fractional programming (QFP) problems are the topic of great importance in non-linear programming. They are useful in many fields such as production planning, financial and corporative planning, health care and hospital planning. In various applications of nonlinear programming, one often encounters the problem in which the ratio of given two quadratic functions is to be maximized or minimized.

The linear fractional programming (LFP) problem arises when a ratio (i.e. numerator and denominator) objective that have linear function has to be maximized over a compact set. The field of LFP, largely developed by Hungarian Mathematician B. Martors and his associates in the 1960's, is concerned with problem of optimization. Several methods to solve this problem are proposed. In (1962), Charnes and Cooper have proposed their method that depends on transforming the LFP to equivalent linear programs (LP). Another method called up dated objective function method derived from Bitran and Novaes (1973) is used to solve the LFP by solving a sequence of linear programs only re-computing the local gradient of the objective function [3]. Also some aspects concerning duality and sensitivity analysis in LFP was discussed by Bitran and Magnant I (1976). Singh. C. (1981) in his paper made a useful study about the optimality condition in LFP.

In H.I. Calvetea et al, provided solving linear fractional bi-level programs [2]. S.F. Tantawy in presented a new procedure for solving linear fractional programming problems [20] and A.O. Odior presented an approach for solving linear fractional programming problems [4]. On the other hand M. Saraja et al. used a Taylor series approach for solving linear fractional decentralized bi-level multi-objective decision-making under fuzziness [16].

T. Antczak, present a modified objective function method for solving nonlinear multi-objective fractional programming problems [1] , and M.B. Hasan et al, present Solving LFP by converting it into a single LP,[7] . G Wang et al., present a global optimization algorithm for solving the bi-level linear fractional programming problem [21] also, N.A. Suleiman et al, provided solving quadratic fractional programming problem [18] . Suleiman et al., used a new modified simplex method to solve quadratic fractional programming problem and compared it to a traditional simplex method by using Pseudo affinity of quadratic fractional functions [19].

Factorized quadratic fractional programming QFP problem in which the ratio of the quadratic function in the objective can be factorized into two linear functions. Now, we proposed a new method namely, decomposition fractional separable method for finding an optimal solution to the given QFP problem with the help of LP simplex techniques.

An interactive approach for solving bi-level problem, the first level decision maker (FLDM) give the preferred as satisfactory solution that are acceptable in rank order to second level decision maker (SLDM). (SLDM) take the satisfactory solution one by one to seek the solutions, who will search for the preferred solution of (FLDM) until the preferred solution is reached, [4,5].

The purpose of the present paper is to find an optimal solution of the model of a three-level quadratic fractional programming problem with linear constrains. This paper is organized as follows: we start in section 2, Problem formulation and solution concept. In Section 3 an interactive model for problem (MLQFIP). Provides interactive algorithm of finding the optimal solution of the formulated model in Section 4. Section 5 a numerical example which illustrates the theory of the solution algorithm is suggested. Finally, the paper is concluded in Section 6 where some points of further research are reported.

2 Problem Formulation and Solution Concept

Multi-level quadratic fractional programming problem with linear constrains (MLQFP) have three-level can be stated as:

$[FLDM]$

$\underset{x_1}{Max}\ F_1(x_1, x_2, x_3) = \frac{C_1^T X + \frac{1}{2} X^T G_1 X}{D_1^T X + \frac{1}{2} X^T G_2 X}$

where x_2, x_3 solves

$[SLDM]$

$\underset{x_2}{Max}\ F_2(x_1, x_2, x_3) = \frac{C_2^T X + \frac{1}{2} X^T G_3 X}{D_2^T X + \gamma}$

where x_3 solves

$[TLDM]$

$\underset{x_3}{Max}\ F_3(x_1, x_2, x_3) = \frac{C_3^T X + \alpha}{D_3^T X + \beta}$

subject to $X = (x_1, x_2, x_3) \in M = \{X/AX \leq B, X \geq 0\}$.

Where G_1, G_2 and G_3 are $(n \times n)$ matrix of coefficient with G_1, G_2 and G_3 are symmetric matrices, X is an n-dimension column of decision variables and compact, C_1, D_1, C_2, D_2 and C_3, D_3 are n-dimensions column vector of coefficients constants, and α, β and γ are real numbers. M is the set of convex-polyhedron linear constraints and A is $(m \times n)$ rectangular matrix, and B is an m-dimensions column vector of constant. The quadratic functions $C_1^T X + \frac{1}{2} X^T G_1 X, D_1^T X + \frac{1}{2} X^T G_2 X$ and $C_2^T X + \frac{1}{2} X^T G_3 X$ are a concave non-linear functions. $D_2^T X + \gamma, C_3^T X + \alpha$ and $D_3^T X + \beta$ are linear functions.

3 An interactive Theoretical Results of (MLQFP) Problem

In this section, consider the quadratic fractional function of the **(FLDM)** and **(SLDM)** which can be factorized into two linear functions as:

$F_1(x_1, x_2, x_3) = \frac{(C_{11}^T X + \delta_1)(C_{12}^T X + \delta_2)}{(D_{11}^T X + \delta_3)(D_{12}^T X + \delta_4)}$ and $F_2(x_1, x_2, x_3) = \frac{(C_{21}^T X + \delta_5)(C_{22}^T X + \delta_6)}{(D_2^T X + \gamma)}$

So, three-level quadratic fractional programming $(TLQFP)$ problem can be written as:

(TLQFP):

$[FLDM]$

$\underset{x_1}{Max}\ F_1(x_1, x_2, x_3) = \frac{(C_{11}^T X + \delta_1)(C_{12}^T X + \delta_2)}{(D_{11}^T X + \delta_3)(D_{12}^T X + \delta_4)}$,

where x_2, x_3 solves

$[SLDM]$

$\underset{x_2}{Max}\ F_2(x_1, x_2, x_3) = \frac{(C_{21}^T X + \delta_5)(C_{22}^T X + \delta_6)}{(D_2^T X + \gamma)}$

where x_3 solves

$[TLDM]$

$\underset{x_3}{Max}\ F_3(x_1, x_2, x_3) = \frac{(C_3^T X + \alpha)}{(D_3^T X + \beta)}$; subject to $X \in M$.

3.1. Third-level Decision Maker (TLDM) Problem:

In this section, **(TLQFP)** problem is transformed into an equivalent bi-level quadratic fractional programming **(BLQFP)** problem with linear constraint by duality of the third-level problem [3].

The dual problem of the **(TLDM)** in **(TLQFP)** problem is as follows:

$(TLDM)_1$:Min$\psi(u) = u_0$,

Subject to,

$$\beta u_0 - Bu_i \geq \alpha,$$

$$D_3^T u_0 + Au_i \geq C_3^T,$$

$$u_i \geq 0, i = 1,2,3 \text{ and } u_0 - unrestricted.$$

(TLQFP) problem can be transformed into the following bi-level quadratic fractional programming **(BLQFP)** problem [12], as the following: $[FLDM]$

$\underset{x_1}{Max}\ F_1(x_1, x_2, x_3) = \frac{(C_{11}^T X+\delta_1)(C_{12}^T X+\delta_2)}{(D_{11}^T X+\delta_3)(D_{12}^T X+\delta_4)}$,

where x_2, x_3 solves

$[SLDM]$

$\underset{x_2}{Max}\ F_2(x_1, x_2, x_3) = \frac{(C_{21}^T X+\delta_5)(C_{22}^T X+\delta_6)}{(D_2^T X+\gamma)}$

Subject to,$X \in M$,

$$\beta u_0 - Bu_i \geq \alpha,$$

$$D_3^T u_0 + Au_i \geq C_3^T,$$

$$u_i \geq 0, i = 1,2,3 \text{ and } u_0 - unrestricted.$$

3.2. First-level Decision Maker (FLDM) Problem:

The (FLDM) solve the following problem:

$(P_F) : \underset{x_1}{Max} = F_1(x_1, x_2, x_3) = \frac{(C_{11}^T X+\delta_1)(C_{12}^T X+\delta_2)}{(D_{11}^T X+\delta_3)(D_{12}^T X+\delta_4)}$,

Subject to,$X \in M$,

$$\beta u_0 - Bu_i \geq \alpha,$$

$$D_3^T u_0 + Au_i \geq C_3^T,$$

$$u_i \geq 0, i = 1,2,3 \text{ and } u_0 - unrestricted.$$

By using the decomposition fractional separable method [8,9]. We solve the above problem as follows:

Construct two **(QP)** problems namely (P_1) and (P_2) from the above (P_F)as:

(P_1): numerator	(P_2): denominator
$\underset{x_1}{Max}\, f_1(X) = f_{11}(X).f_{12}(X)$, Subject to $X \in M$, $\beta u_0 - Bu_i \geq \alpha$, $D_3^T u_0 + Au_i \geq C_3^T$. $u_i \geq 0$, $i = 1,2,3$ and u_0-unrestricted.	$\underset{x_1}{Max}\, f_2(X) = f_{21}(X).f_{22}(X)$, Subject to $X \in M$, $\beta u_0 - Bu_i \geq \alpha$, $D_3^T u_0 + Au_i \geq C_3^T$. $u_i \geq 0$, $i = 1,2,3$ and u_0-unrestricted.

Definition1 [8,9]: Let $f_1(x)$ and $f_2(x)$ be two differentiable functions defined on $X \subset \mathbb{R}^n$, an n-dimensional Euclidean Space. The functions $f_1(x)$ and $f_2(x)$ are said to have the Gonzi property in $X \subset \mathbb{R}^n$ if $(f_1(x) - f_1(U))(f_2(x) - f_2(U)) \leq 0$, for all $x, U \in X$.

Theorem1[8,9]: The product $f_1(x).f_2(x)$ of two linear functions $f_1(x)$ and $f_2(x)$ are concave if and only if the functions $f_1(x)$ and $f_2(x)$ have the Gonzi property.

Now, we assume that the functions $C_{11}^T X + \delta_1, C_{12}^T X + \delta_2, D_{11}^T X + \delta_3$ and $D_{12}^T X + \delta_4$ satisfy the Gonzi property in the feasible set and the set of all feasible solutions to the problem (P_1) and (P_2) are non-empty and bounded. Thus, by Theorem1, it is concluded that the problem (P_1) and (P_2) is a concave non-linear programming problem with linear constraints. This implies that the optimal solution of the problem (P_1) and (P_2) exists and it occurs at an extreme point of the feasible region.

Now, in order to solve the above (QP) problem (P_1) and (P_2) by (LP) technique, we decompose it into two single objective by (LP) problems namely $(P_{11}), (P_{12}), (P_{21})$ and (P_{22}) as given below:

(P_1): numerator		(P_2): denominator	
(P_{11}): $Max_{x_1}\, f_{11}(X)$, Subject to $X \in M$, $\beta u_0 - Bu_i \geq \alpha$, $D_3^T u_0 + Au_i \geq C_3^T$. $u_i \geq 0$, $i = 1,2,3$ and u_0-unrestricted.	(P_{12}): $Max_{x_1}\, f_{12}(X)$, Subject to $X \in M$, $\beta u_0 - Bu_i \geq \alpha$, $D_3^T u_0 + Au_i \geq C_3^T$. $u_i \geq 0$, $i = 1,2,3$ and u_0-unrestricted.	(P_{21}):$Max_{x_1}\, f_{21}(X)$, Subject to $X \in M$, $\beta u_0 - Bu_i \geq \alpha$, $D_3^T u_0 + Au_i \geq C_3^T$. $u_i \geq 0$, $i = 1,2,3$ and u_0-unrestricted.	(P_{22}):$Max_{x_1}\, f_{22}(X)$, Subject to $X \in M$, $\beta u_0 - Bu_i \geq \alpha$, $D_3^T u_0 + Au_i \geq C_3^T$. $u_i \geq 0$, $i = 1,2,3$ and u_0-unrestricted

3.3. Second-Level Decision Maker (SLDM) Problem:

The **(SLDM)** solve the following problem:

(P_S) : $\underset{x_2}{Max} = F_2(x_1, x_2, x_3) = \frac{(C_{21}^T X + \delta_5)(C_{22}^T X + \delta_6)}{(D_2^T X + \gamma)}$,

Subject to,$x_1^*, x_2, x_3 \in M$,

$$\beta u_0 - Bu_i \geq \alpha,$$

$$D_3^T u_0 + Au_i \geq C_3^T,$$

$$u_i \geq 0, i = 1,2,3 \text{ and } u_0 - unrestricted.$$

By using the decomposition fractional separable method in [8,9]. In addition, the author put forward the satisfactoriness concept as the first-level decision-maker preference. We solve the above problem as follows: Now, from problem (P_S), we decompose it into two problem one of them is quadratic programming and the other is linear programming as (P_3) and (P_4) as constructed below:

(P_3): numerator	(P_4): denominator
$\underset{x_2}{Max}\, f_1(X) = f_{11}(X).f_{12}(X),$ Subject to $(x_1^*, x_2, x_3) \in M,$ $\beta u_0 - Bu_i \geq \alpha,$ $D_3^T u_0 + Au_i \geq C_3^T.$ $u_i \geq 0$, $i = 1,2,3$ and u_0-unrestricted.	$\underset{x_2}{Max}\, f_2(X),$ Subject to $(x_1^*, x_2, x_3) \in M,$ $\beta u_0 - Bu_i \geq \alpha,$ $D_3^T u_0 + Au_i \geq C_3^T.$ $u_i \geq 0$, $i = 1,2,3$ and u_0-unrestricted.

Now, (P_3) problem can be easily solved by objective separable method proposed by [8,9] for finding an optimal solution to a quadratic programming problem in which the objective function can be factorized into two linear functions.

Now, we assume that the two functions $(C_{21}^T X + \delta_5)$ and $(C_{22}^T X + \delta_6)$ have the Gonzi property in the feasible set and the set of all feasible solutions to the problem (P_3) are non-empty and bounded. Thus, by Theorem1. , it is concluded that the problem (P_3) is a concave non-linear programming problem with linear constraints. This implies that the optimal solution of the problem (P_3) exists and it occurs at an extreme point of the feasible region.

(P_3): numerator	
(P_{31}): $Max_{x_2}\, f_{11}(X),$ Subject to $(x_1^*, x_2, x_3) \in M,$ $\beta u_0 - Bu_i \geq \alpha,$ $D_3^T u_0 + Au_i \geq C_3^T.$ $u_i \geq 0$, $i = 1,2,3$ and u_0-unrestricted.	(P_{32}):$Max_{x_2}\, f_{12}(X)$, Subject to $(x_1^*, x_2, x_3) \in M,$ $\beta u_0 - Bu_i \geq \alpha,$ $D_3^T u_0 + Au_i \geq C_3^T.$ $u_i \geq 0$, $i = 1,2,3$ and u_0-unrestricted.

The optimal solution for the given $(MLQFP)$ problem is (x_1^*, x_2^*, x_3^*).

4 Interactive Algorithm for Solving (MLQFP) Problem

Step(1): Factorized the quadratic objective functions into linear function for (FLDM) and (SLDM).

Step(2): Construct the dual problem for $(TLDM)$ as $(TLDM)_1$ problem.

Step(3): Transform $(TLQFP)$ problem into $(BLQFP)$ problem.

Step(4): For $(FLDM)$, construct two (QP) problems namely (P_1) and (P_2) from the given (QFP) problem (P).

Step(5): Decompose the (QP) problem (P_1) and (P_2) into two single objective (LP) problems namely (P_{11}), (P_{12}), (P_{21}) and (P_{22}).

Step(6): Solve the problem (P_{11}) and (P_{21}) separately, with the help of (LP) technique. Let the optimal solution to the problem (P_{11}) be $X_{11}^* = (x_1^*, x_2^*, x_3^*)$ and $Max f_{11}(X) = f_{11}(X_{11}^*)$ and the optimal solution to the problem (P_{21}) be $X_{21}^* = (x_1^*, x_2^*, x_3^*)$ and $Max f_{21}(X) = f_{21}(X_{21}^*)$.

Step(7): Use the optimal table of the problem (P_{11}) and (P_{21}) as an initial table for the problem (P_{12}) and (P_{22}), continue to find a sequence of improved basic feasible solutions $\{X_n\}$ to the problem (P_{12}) and (P_{22}), also find the value of objective function of $f_1(X)$ and $f_2(X)$ at each of the improved basic feasible solution by (LP) technique.

Step(8): (a).If $f_r(X_k) \leq f_r(X_{k+1})$,$r = 1,2$ for all $k = 0,1,2,,n$ and $f_r(X_n) \leq f_r(X_{n+1}), r = 1,2$ for some n, stop the computation process and then, go to Step9.

(b).If $f_r(X_k) \leq f_r(X_{k+1})$, for all $k = 0,1,2,,n$ and X_{n+1} is an optimal solution to the (QP) problem for some n, stop the computation process and then, go to Step 10.

Step(9): X_n is an optimal solution to the (QP) problem.

Step(10): X_{n+1} is an optimal solution to the (QP) problem.

Step(11): Collect all the feasible solution from the iterations of the problem (P_{12}) and (P_{22}), also compute the value of the objective function of (QFP) problem (P), (i.e.) $f(X)$, then maximum value of $f(X)$, among all the feasible solution is the optimal solution for the (QFP) problem (P).

Step(12): For $(SLDM)$, Construct (P_3) and (P_4) problem from the given $(LFQFP)$. In addition, the author put forward the satisfactoriness concept as the first-level decision-maker preference (x_1^*, x_2, x_3).

Step(13): Solve (P_3) problem by separable method. Let the optimal solution of problem (P_3) be $X_{11}^* = (x_1^*, x_2^*, x_3^*)$ and $Max f_{11}(X) = f_{11}(X_{11}^*)$.

Step(14): Use the optimal table of the problem (P_3) as an initial simplex table for the problem (P_4), continue to find a sequence of improved basic feasible $\{X_n\}$ to the problem (P_4) and the value of $f(X)$,at each of the improved basic feasible solution by the simplex method.

Step(15): (a).If $f(X_k) \leq f(X_{k+1})$ for all $k = 0,1,2,,n-1$ and $f(X_n) \geq f(X_{n+1})$ for some n, stop the computation process and then, go to Step16.

(b).If $f_r(X_k) \leq f_r(X_{k+1})$, for all $k = 0,1,2,,n-1$ and X_{n+1} is an optimal solution to the (QP) problem for some n, stop the computation process and then, go to Step 17.

Step(16): X_n is an optimal solution to the $(TLQFP)$ problem and $Max f(X) = f(X_n)$.

Step(17): X_{n+1} is an optimal solution to the $(TLQFP)$ problem and $Max f(X) = f(X_{n+1})$.

The proposed method for solving the $(MLQFP)$ problem is illustrated by the following example.

5 An illustrative Example

To demonstrate the solution method for $(MLQFP)$ problem. Let us consider the following three-level quadratic fractional programming $(TLQFP)$ problem:

$[FLDM]$

$\underset{x}{Max} = F_1(x,y,z) = \frac{(y+z+1)(x+y+3)}{(x+4)(x+y+z+2)}$

where y,z solves $[SLDM]$

$\underset{y}{Max} = F_2(x,y,z) = \frac{(4x+6y-2)(x+3z+1)}{(6x+9y+3)}$

where y solves $[TLDM]$

$\underset{z}{Max} = F_3(x,y,z) = \frac{(-2x-y-z)}{(2x+3y+z)}$,

subject to

$$3x + 5y + z \leq 35,$$
$$2x - y + 12z \leq 0,$$
$$5x + 6y + z \leq 16,$$
$$x, y \text{ and } z \geq 0.$$

The dual problem of third-level can be written as:

$Min\psi(u) = u_0,$

Subject to

$$35u_1 + 20u_2 + 16u_3 \leq 0,$$
$$-2u_0 - 3u_1 - 2u_2 - 5u_3 \leq 2,$$
$$-3u_0 - 5u_1 + u_2 - 6u_3 \leq 1,$$
$$-u_0 - u_1 - 12u_2 - u_3 \leq 1,$$
$$u_i \geq 0, i = 0, 1, 2, 3.$$

$(TLQFP)$ problem (P) can be transformed into the following $(BLQFP)$ problem.

$[FLDM]$

$\underset{x}{Max} = F_1(X) = \frac{(y+z+1)(x+y+3)}{(x+4)(x+y+z+2)}$

where y, z solves $[SLDM]$

$\underset{y}{Max} = F_2(X) = \frac{(4x+6y-2)(x+3z+1)}{(6x+9y+3)}$

subject to

$$3x + 5y + z \leq 35,$$
$$2x - y + 12z \leq 20,$$
$$5x + 6y + z \leq 16,$$
$$x, y \text{ and } z \geq 0, 35u_1 + 20u_2 + 16u_3 \leq 0,$$
$$-2u_0 - 3u_1 - 2u_2 - 5u_3 \leq 2,$$
$$-3u_0 - 5u_1 + u_2 - 6u_3 \leq 1,$$
$$-u_0 - u_1 - 12u_2 - u_3 \leq 1,$$
$$x, y, z \text{ and } u_i \geq 0, i = 0, 1, 2, 3.$$

Now, solve the bi-level quadratic fractional programming $(BLQFP)$ problem by interactive approach. For $FLDM$, construct two QP problems namely (P_1) and (P_2) as follows:

(P_1): numerator	(P_2): denominator
$\underset{x}{Max}\, f_1(X) = f_{11}(X).f_{12}(X)$, Subject to $3x + 5y + z \leq 35$, $2x - y + 12z \leq 20$, $5x + 6y + z \leq 16$, $35u_1 + 20u_2 + 16u_3 \leq 0$, $-2u_0 - 3u_1 - 2u_2 - 5u_3 \leq 2$, $-3u_0 - 5u_1 + u_2 - 6u_3 \leq 1$, $-u_0 - u_1 - 12u_2 - u_3 \leq 1$, $x, y, z, u_i \geq 0, i = 1,2,3$. and u_0-unrestricted. where; $f_{11}(X) = (y + z + 1)$. $f_{12}(X) = (x + y + 3)$.	$\underset{x}{Max}\, f_2(X) = f_{21}(X).f_{22}(X)$, Subject to $3x + 5y + z \leq 35$, $2x - y + 12z \leq 20$, $5x + 6y + z \leq 16$, $35u_1 + 20u_2 + 16u_3 \leq 0$, $-2u_0 - 3u_1 - 2u_2 - 5u_3 \leq 2$, $-3u_0 - 5u_1 + u_2 - 6u_3 \leq 1$, $-u_0 - u_1 - 12u_2 - u_3 \leq 1$, $x, y, z, u_i \geq 0, i = 1,2,3$. and u_0-unrestricted. where; $f_{21}(X) = (x + 4)$. $f_{22}(X) = (x + y + z + 2)$.

Decomposes the above (QP) problem (P_1) and (P_2) into single objective (LP) problems as given below:

(P_1): numerator	
(P_{11}): $Max_x\, f_{11}(X) = y + z + 1$, Subject to $3x + 5y + z \leq 35$, $2x - y + 12z \leq 20$, $5x + 6y + z \leq 16$, $35u_1 + 20u_2 + 16u_3 \leq 0$, $-2u_0 - 3u_1 - 2u_2 - 5u_3 \leq 2$, $-3u_0 - 5u_1 + u_2 - 6u_3 \leq 1$, $-u_0 - u_1 - 12u_2 - u_3 \leq 1$, $x, y, z, u_i \geq 0, i = 1,2,3$. and u_0-unrestricted.	(P_{12}): $Max_x\, f_{12}(X) = x + y + 3$, Subject to $3x + 5y + z \leq 35$, $2x - y + 12z \leq 20$, $5x + 6y + z \leq 16$, $35u_1 + 20u_2 + 16u_3 \leq 0$, $-2u_0 - 3u_1 - 2u_2 - 5u_3 \leq 2$, $-3u_0 - 5u_1 + u_2 - 6u_3 \leq 1$, $-u_0 - u_1 - 12u_2 - u_3 \leq 1$, $x, y, z, u_i \geq 0, i = 1,2,3$. and u_0-unrestricted.
(P_2): denominator	
(P_{21}): $Max_x\, f_{21}(X) = x + 4$, Subject to $3x + 5y + z \leq 35$, $2x - y + 12z \leq 20$, $5x + 6y + z \leq 16$, $35u_1 + 20u_2 + 16u_3 \leq 0$, $-2u_0 - 3u_1 - 2u_2 - 5u_3 \leq 2$, $-3u_0 - 5u_1 + u_2 - 6u_3 \leq 1$, $-u_0 - u_1 - 12u_2 - u_3 \leq 1$, $x, y, z, u_i \geq 0, i = 1,2,3$. and u_0-unrestricted.	(P_{22}): $Max_x\, f_{22}(X) = x + y + z + 2$, Subject to $3x + 5y + z \leq 35$, $2x - y + 12z \leq 20$, $5x + 6y + z \leq 16$, $35u_1 + 20u_2 + 16u_3 \leq 0$, $-2u_0 - 3u_1 - 2u_2 - 5u_3 \leq 2$, $-3u_0 - 5u_1 + u_2 - 6u_3 \leq 1$, $-u_0 - u_1 - 12u_2 - u_3 \leq 1$, $x, y, z, u_i \geq 0, i = 1,2,3$. and u_0-unrestricted.

The optimal solution for problem (P_{11}) by simplex method is $X_{11}^* = (0, 2.36, 1.86)$ and the optimal solution for problem (P_{12}) is given $X_{12}^* = (3.17, 0, 0)$.

Since $f_1(X_{11}) > f_1(X_12)$, the optimal solution to the (QP) problem (P_1) is $(0, 2.36, 1.86)$ and $Max f_1(X) = 27.9792$.

The optimal solution for problem (P_{21}) by simplex method is $X_{21}^* = (3.20, 0, 0)$ and the optimal solution for problem (P_{22}) is given $X_{22}^* = (0, 2.36, 1.86)$.

Since $f_2(X_{21}^*) > f_2(X_{22}^*)$ the optimal solution to the (QP) problem (P_2) is $(3.20, 0, 0)$ and $Max f_2(X_{21}^*) = 37.44$.

Solution	Feasible Solutions (x, y, z)	Objective Value of $F_1(x, y, z)$
1	$X^* = (0,2.36,1.86)$.	1.124565916
2	$X^* = (3.20,0,0)$.	0.1655982906

To find the optimal solution for the given QFP problem, we collect all the feasible solution for the iterations of the problem (P_1) and (P_2) which are listed below:

Now, from the above table, the optimal solution for the given QFP problem of $FLDM$ is $(0, 2.36, 1.86)$ and $MaxF_1(X) = 1.124565916$.

Solve the $SLDM$ problem construct two (QP) problems namely (P_3) and (P_4), In addition, we put forward the satisfactoriness concept as the first-level decision-maker preference.

(P_3): numerator	(P_4): denominator
$\underset{y}{Max}\, f_1(X) = f_{11}(X).f_{12}(X)$, Subject to $3x + 5y + z \leq 35,$ $2x - y + 12z \leq 20,$ $5x + 6y + z \leq 16,$ $35u_1 + 20u_2 + 16u_3 \leq 0,$ $-2u_0 - 3u_1 - 2u_2 - 5u_3 \leq 2,$ $-3u_0 - 5u_1 + u_2 - 6u_3 \leq 1,$ $-u_0 - u_1 - 12u_2 - u_3 \leq 1,$ $x^* = 0,$ $y, z, u_i \geq 0, i = 1,2,3.$ and u_0-unrestricted. where; $f_{11}(X) = (4x + 6y - 2),$ $f_{12}(X) = (x + 3z + 1),$	$\underset{y}{Max}\, f_2(X)$, Subject to $3x + 5y + z \leq 35,$ $2x - y + 12z \leq 20,$ $5x + 6y + z \leq 16,$ $35u_1 + 20u_2 + 16u_3 \leq 0,$ $-2u_0 - 3u_1 - 2u_2 - 5u_3 \leq 2,$ $-3u_0 - 5u_1 + u_2 - 6u_3 \leq 1,$ $-u_0 - u_1 - 12u_2 - u_3 \leq 1,$ $x^* = 0,$ $y, z, u_i \geq 0, i = 1,2,3.$ and u_0-unrestricted. where; $f_2(X) = (6x + 9y + 3),$

Decomposes the above numerator (P_3) into single objective LP problems as given below:

(P_3)	
(P_{31}):	(P_{32}):
$\underset{y}{Max}\, f_{11}(X) = 4x + 6y - 2$, Subject to $3x + 5y + z \leq 35,$ $2x - y + 12z \leq 20,$ $5x + 6y + z \leq 16,$ $35u_1 + 20u_2 + 16u_3 \leq 0,$ $-2u_0 - 3u_1 - 2u_2 - 5u_3 \leq 2,$ $-3u_0 - 5u_1 + u_2 - 6u_3 \leq 1,$ $-u_0 - u_1 - 12u_2 - u_3 \leq 1,$ $x^* = 0,$ $y, z, u_i \geq 0, i = 1,2,3.$ and u_0-unrestricted.	$\underset{y}{Max}\, f_{12}(X) = x + 3z + 1$, Subject to $3x + 5y + z \leq 35,$ $2x - y + 12z \leq 20,$ $5x + 6y + z \leq 16,$ $35u_1 + 20u_2 + 16u_3 \leq 0,$ $-2u_0 - 3u_1 - 2u_2 - 5u_3 \leq 2,$ $-3u_0 - 5u_1 + u_2 - 6u_3 \leq 1,$ $-u_0 - u_1 - 12u_2 - u_3 \leq 1,$ $x^* = 0,$ $y, z, u_i \geq 0, i = 1,2,3.$ and u_0-unrestricted.

The optimal solution for problem (P_{31}) by simplex method is $X^*_{31} = (0, 2.67, 0)$ and the optimal solution for problem (P_{32}) is given $X^*_{32} = (0, 0, 1.86)$.
Since $f_1(X_{31}) > f_1(X_{32})$, the optimal solution to the (QP) problem (P_1) is $(0, 2.67, 0)$ and $Max f_1(X) = 14.02$.
The optimal solution for problem (P_4) by simplex method is $X^*_4 = (0, 2.67, 0)$.
To find the optimal solution for the given QFP problem, we collect all the feasible solution for the iterations of the problem (P_5) and (P_6) which are listed below:

Solution	**Feasible Solutions (x, y, z)**	**Objective Value of $F_2(x, y, z)$**
1	$X^* = (0,2.67,0)$.	0.5186829449

Now, from the above table, the optimal solution for the given QFP problem of $SLDM$ is $x = 0, y = 2.67, z = 0$ and $Max F_2(X) = 0.5186829449$.

Objective Value of $F_i(x^*, y^*, z^*)$, i=1,2,3
$F_1(x^*, y^*, z^*)$ =0.3035331906
$F_2(x^*, y^*, z^*)$ = 0.5186829449
$F_3(x^*, y^*, z^*)$=-0.3333333333

The optimal solution for the given $(MLQFP)$ problem is $(x^*, y^*, z^*) = (0, 2.36, 1.86)$.

6 Summary and Concluding Remarks

Our aim was to develop an easy technique for solving $(MLQFP)$ problems. So in this paper, we have The upper-level objective function is quadratic fractional problem in which both the numerator and denominator of the objective function can be factorized into linear function, the second-level objective function is quadratic fractional programming (QPP) problem in which the objective function can be factorized into linear functions and the denominator of the objective function in linear type and the third-level objective function is linear fractional with linear constraint. $(MLQFP)$ problem is transformed into an equivalent bi-level quadratic fractional programming (QFP) problem by forcing the duality gap of the lower-level problem to zero. Then by using interactive approach for solving $(BLFQP)$ problem, the first level decision maker $(FLDM)$ give the preferred as satisfactory solution that are acceptable in rank order to second level decision maker. $(SLDM)$ take the satisfactory solution one by one to seek the solutions, who will search for the preferred solution of $(FLDM)$ until the preferred solution is reached.
However, there are many open points for discussion in future, which should be explored and studied in the area of multilevel fractional optimization such as:

- Interactive approach for multi-level multi-objective quadratic fractional programming problems.

- Interactive approach for multi-level multi-objective quadratic fractional integer programming problems.
- Interactive algorithm is required for treating multi-level quadratic fractional multi-objective decision-making problems.
- Interactive algorithm is required for treating multi-level integer fractional multi-objective decision-making problems with rough parameters in the objective functions; in the constraints and in both.
- Interactive approach for multi-level multi-objective quadratic fractional programming problems with fuzzy parameters in the objective functions; in the constraints and in both.

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