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More On Fuzzy Regular Preopen Sets

Anjana Bhattacharyya
Victoria Institution (College)
78 B, A.P.C. Road, Kolkata - 700009, India.
Mail:anjanabhattacharyya@hotmail.com

Abstract. In this paper we have first introduced fuzzy p-closed and fuzzy regular p-closed spaces and then characterized these two concepts in several ways. Afterwards, we have introduced two new concepts of fuzzy cover, viz., fuzzy p-cover and fuzzy regular p-cover respectively and characterized fuzzy p-closed and fuzzy regular p-closed spaces via these two new concepts of fuzzy cover. Also fuzzy p-closed space is characterized by fuzzy net. Again we have introduced fuzzy p-closed and fuzzy regular p-closed sets and characterized them in different ways, especially via prefilterbases. In the last section, three different types of fuzzy continuous-like functions, viz. fuzzy regular precontinuous, fuzzy strongly θ-precontinuous and fuzzy p-continuous functions are introduced and some applications of these three functions are established.

1. Introduction

S. Nanda introduced fuzzy preopen set [7] in 1991. Using this concept we have introduced fuzzy regular preopen and fuzzy regular preclosed sets in [2]. In [1], we have introduced fuzzy p^*-closed and fuzzy p^*-open sets. Here we see that fuzzy regular preopen set is fuzzy p^*-open but not conversely. But in a fuzzy extremally disconnected space [2], the concepts of fuzzy regular preopen and fuzzy p^*-clopen sets coincide. Also in a fuzzy extremally p-disconnected space. the concepts of fuzzy p-closed and fuzzy regular p-closed spaces coincide.

Throughout the paper, (X, τ) or simply by X we shall mean a fuzzy topological space (fts, for short) in the sense of Chang [4]. In 1965, L.A. Zadeh introduced first fuzzy set as follows : A fuzzy set A is a mapping from a non-empty set X into the closed interval $I = [0, 1]$ and is denoted by $A \in I^X$. The support of a fuzzy set A,

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denoted by $suppA$, is defined by $suppA = \{x \in X : A(x) \neq 0\}$. A fuzzy point x_α with the singleton support $\{x\}$ and the value $\alpha(0 < \alpha \leq 1)$ is the fuzzy set taking value α at x and 0 otherwise [8]. 0_X and 1_X are the constant fuzzy sets taking values 0 and 1 respectively on X. For any two fuzzy sets A and B, $A \leq B$ means $A(x) \leq B(x)$ for all $x \in X$ while we write AqB, i.e., A is quasi-coincident (q-coincident, for short) with B [8] if there is $x \in X$ such that $A(x) + B(x) > 1$. The negation of these two statements are written as $A \not\leq B$ and $A \not q B$ respectively. For a fuzzy set A, clA and $intA$ stand for fuzzy closure and fuzzy interior of A in the sense of Chang [4]. A fuzzy set A in an fts (X, τ) is called fuzzy preopen (resp., fuzzy α-open) if $A \leq intclA$ [7] (resp., $A \leq intclintA$ [3]). The complement of a fuzzy preopen (resp., fuzzy α-open) set is called fuzzy preclosed [7] (resp., fuzzy α-closed [3]). A fuzzy set A is said to be fuzzy preclopen [7] if it is fuzzy preopen as well as fuzzy preclosed. A fuzzy set A in an fts (X, τ) is called a fuzzy neighbourhood (nbd, for short) [8] of a fuzzy point x_α, if there is a fuzzy open set U in X such that $x_\alpha \leq U \leq A$. If, in addition, A is fuzzy open (resp., fuzzy preopen), then A is called fuzzy open (resp., fuzzy preopen [7]) nbd [8] of x_α. A fuzzy set A is called fuzzy quasi neighbourhood (q-nbd, for short) [8] of a fuzzy point x_α in an fts X if there exists a fuzzy open set U in X such that $x_\alpha qU \leq A$. If, in addition, A is fuzzy open (resp., fuzzy preopen), then A is called fuzzy open q-nbd (resp., fuzzy pre-q-nbd [1]) [8] of x_α. The union of all fuzzy preopen sets contained in a fuzzy set A is called fuzzy preinterior of A, denoted by $pintA$ [7] and the intersection of all fuzzy preclosed sets containing a fuzzy set A is called fuzzy preclosure of A, denoted by $pclA$ [7]. The collection of all fuzzy preopen (resp., fuzzy preclosed), fuzzy α-open sets are denoted by $FPO(X)$ (resp., $FPC(X)$), $F\alpha O(X)$.

2. Some Well Known Definitions

In this section we first recall some definitions from [1, 2].

Definition 2.1.[2]. A fuzzy preopen set A in an fts (X, τ) is called fuzzy regular preopen if $A = pint(pclA)$. The complement of a fuzzy regular preopen set is called fuzzy regular preclosed.
The family of all fuzzy regular preopen (resp., fuzzy regular preclosed) sets is denoted by $FRPO(X)$ (resp., $FRPC(X)$).

Definition 2.2. [1]. A fuzzy point x_α in an fts (X, τ) is called fuzzy p^*-cluster point of a fuzzy set A in X if $pclUqA$ for every fuzzy preopen set U in X with $x_\alpha qU$. The union of all fuzzy p^*-cluster points of a fuzzy set A in X is called fuzzy p^*-closure of A, denoted by $[A]_p$. $A(\in I^X)$ is called fuzzy p^*-closed if $A = [A]_p$. The complement of a fuzzy p^*-closed set is called fuzzy p^*-open set. A fuzzy set A is called fuzzy p^*-clopen if A is both fuzzy p^*-open and fuzzy p^*-closed.

Definition 2.3. [2]. An fts (X, τ) is called fuzzy extremally p-disconnected if the fuzzy preclosure of every fuzzy preopen set in X is fuzzy preopen.

Definition 2.4. [1]. An fts (X, τ) is said to be fuzzy p^*-regular if for each fuzzy point x_α and each fuzzy pre-q-nbd U of x_α, there exists $V \in FPO(X)$ such that $x_\alpha qV \leq pclV \leq U$.

Definition 2.5. [4]. Let A be a fuzzy set in an fts (X, τ). A collection $\mathcal{U}$ of fuzzy sets in X is called a fuzzy cover of A if $sup\{U(x) : U \in \mathcal{U}\} = 1$ for each $x \in suppA$. If, in addition, the members of $\mathcal{U}$ are fuzzy open, then $\mathcal{U}$ is called a fuzzy open cover of A. In particular, if $A = 1_X$, we get the definition of fuzzy cover of the fts X.

Definition 2.6. [5, 4]. A fuzzy cover $\mathcal{U}$ of a fuzzy set A in an fts (X, τ) is said to have a finite subcover $\mathcal{U}_0$ if $\mathcal{U}_0$ is a finite subcollection of $\mathcal{U}$ such that $\bigcup \mathcal{U}_0 \geq A$. If, in particular, if $A = 1_X$, then the requirement is $\bigcup \mathcal{U}_0 = 1_X$.

Definition 2.7. [4]. An fts (X, τ) is said to be fuzzy compact if every fuzzy open cover of X has a finite subcover.

3. Some Results on Fuzzy Extremally p-Disconnected Space

In this section some results on fuzzy extremally p-disconnected space are established.

Result 3.1. *For a fuzzy set A in a fuzzy extremally p-disconnected space (X, τ), the following statements are equivalent :*
(i) A is fuzzy preclopen,
(ii) $A = pcl(pintA)$,
(iii) $1_X \setminus A \in FRPO(X)$,
(iv) $A \in FRPO(X)$.

Proof. $(i) \Rightarrow (ii)$. Obvious.
$(ii) \Rightarrow (iii)$. $1_X \setminus A = 1_X \setminus pcl(pintA) = pint(pcl(1_X \setminus A)) \Rightarrow 1_X \setminus A \in FRPO(X)$.
$(iii) \Rightarrow (iv)$. $A = 1_X \setminus (1_X \setminus A) \in FRPC(X)$ (by (iii)) $\Rightarrow A = pcl(pintA)$. Now X is fuzzy extremally p-disconnected $\Rightarrow A = pcl(pintA) \in FPO(X) \Rightarrow A = pintA$. Again $A \in FRPC(X) \Rightarrow A \in FPC(X) \Rightarrow A = pclA$ and so $A = pint(pclA) \Rightarrow A \in FRPO(X)$.
$(iv) \Rightarrow (i)$. $A \in FRPO(X) \Rightarrow A \in FPO(X)$ and $1_X \setminus A \in FRPC(X) \Rightarrow 1_X \setminus A = pcl(pint(1_X \setminus A))$. As X is fuzzy extremally p-disconnected and $pint(1_X \setminus A) \in FPO(X)$, $pcl(pint(1_X \setminus A)) = 1_X \setminus A \in FPO(X) \Rightarrow A \in FPC(X)$. Hence A is fuzzy preclopen. □

Result 3.2. *For a fuzzy set A in an fts (X, τ), let clA (resp., $pclA$) be fuzzy regular preopen set in X. Then $A \in FPO(X)$. Moreover, if X is fuzzy extremally p-disconnected and $A \in FPO(X)$, then $pclA \in FRPO(X)$.*

Proof. Let $clA \in FRPO(X)$. Then $clA = pint(pcl(clA))$. Then $clA \in FPO(X)$. Now $A \leq clA \leq int(cl(clA)) = int(clA) \Rightarrow A \in FPO(X)$.
Again let $pclA \in FRPO(X)$. Then $pclA \in FPO(X)$. Then $A \leq pclA \leq int(cl(pclA)) \leq int(cl(clA)) = int(clA) \Rightarrow A \in FPO(X)$.
Let us assume that X is fuzzy extremally p-disconnected and $A \in FPO(X)$. Then $pclA = pcl(pintA) \in FPO(X) \Rightarrow pclA = pint(pclA) = pint(pcl(pclA)) \Rightarrow pclA \in FRPO(X)$. □

Result 3.3. *If a fuzzy set A in an fts (X, τ) is fuzzy α-open and fuzzy α-closed, then $A \in FRPO(X)$ and $A \in FRPC(X)$.*

Proof. A is fuzzy α-open as well as fuzzy α-closed. Then $A \in FPO(X)$ as well as $A \in FPC(X) \Rightarrow A = pint(pclA) \Rightarrow A \in FRPO(X)$ also $A = pcl(pintA) \Rightarrow A \in FRPC(X)$. □

Result 3.4. *For a fuzzy set A in an fts (X, τ), $A \in F\alpha O(X), A \in FRPO(X) \Rightarrow A = int(cl(intA))$.*

Proof. $pclA \in FPC(X) \Rightarrow cl(int(pclA)) \leq pclA \Rightarrow pint(cl(int(pclA))) \leq pint(pclA)$. Now $A \in F\alpha O(X) \Rightarrow A \leq int(cl(intA))$...(1).
Again, $A \in FRPO(X) \Rightarrow A = pint(pclA) \geq pint(cl(int(pclA))) \geq pint(cl(intA)) \geq int(cl(intA))$...(2).
Combining (1) and (2), we get the result. □

Result 3.5. *For a fuzzy set A in a fuzzy extremally p-disconnected space (X, τ),*

$[A]_p = \bigcap\{V : A \leq V \text{ and } V \in FRPO(X)\}$.

Proof. Let $x_\alpha \notin [A]_p$. Then there exists $V \in FPO(X)$ with $x_\alpha qV$, but $pclV \not{q} A \Rightarrow A \leq 1_X \setminus pclV$. By Result 3.1 and Result 3.2, $1_X \setminus pclV \in FRPO(X)$. Now $V(x) + \alpha > 1 \Rightarrow x_\alpha \notin 1_X \setminus V \in FPC(X)$. Then $1_X \setminus pclV \leq 1_X \setminus V \Rightarrow x_\alpha \notin 1_X \setminus pclV \Rightarrow x_\alpha \notin \bigcap\{V : A \leq V$ and $V \in FRPO(X)\} \Rightarrow$ R.H.S. $\leq$ L.H.S.

Conversely, let $x_\alpha \in$ L.H.S. Let $V \in FRPO(X)$ with $A \leq V$...(1). If possible, let $x_\alpha \notin V$. Then $V(x) < \alpha \Rightarrow 1 - V(x) > 1 - \alpha \Rightarrow x_\alpha q(1_X \setminus V)$. By Result 3.1, $1_X \setminus V \in FRPO(X) \Rightarrow 1_X \setminus V \in FPO(X)$. By hypothesis, $pcl(1_X \setminus V) = 1_X \setminus V$ (as $V \in FRPO(X) \Rightarrow 1_X \setminus V \in FRPC(X) \Rightarrow 1_X \setminus V \in FPC(X))qA \Rightarrow A \leq V$, contradicts (1). So $x_\alpha \in$ R.H.S. Therefore, L.H.S. $\leq$ R.H.S. □

Result 3.6. *Let A be a fuzzy set in a fuzzy extremally p-disconnected space (X, τ). Then*

$(i)x_\alpha \in [A]_p$ iff every fuzzy regular preopen set V in X with $x_\alpha qV$, VqA,

$(ii)A$ is fuzzy p^*-open iff for any fuzzy point x_α with $x_\alpha qA$, there exists $V \in FRPO(X)$ with $x_\alpha qV$ such that $V \leq A$,

$(iii)A \in FRPO(X)$ iff A is fuzzy p^*-clopen.

Proof. (i). Let $x_\alpha \in [A]_p$ and $V \in FRPO(X)$ with $x_\alpha qV$. Then $V \in FPO(X)$ with $x_\alpha qV$. By hypothesis, $pclVqA$. As X is fuzzy extremally p-disconnected, $pclV \in FPO(X)$. By Result 3.1, V is fuzzy preclopen and so $pclV = V$. Thus VqA.

Conversely, let VqA for every fuzzy regular preopen set V in X with $x_\alpha qV$. Let $U \in FPO(X)$ with $x_\alpha qU$. by Result 3.2, $pclU \in FRPO(X)$. As $x_\alpha qU, x_\alpha qpclU$. By assumption, $pclUqA$. Consequently, $x_\alpha \in [A]_p$.

(ii) Suppose A is fuzzy p^*-open. Then $[1_X \setminus A]_p = 1_X \setminus A$. Let x_α be any fuzzy point in X such that $x_\alpha qA$. Then $x_\alpha \notin 1_X \setminus A = [1_X \setminus A]_p$. Then by (1), there exists $V \in FRPO(X)$ with $x_\alpha qV, V \not{q}(1_X \setminus A) \Rightarrow V \leq A$.

Conversely, suppose that $x_\alpha \notin 1_X \setminus A$, but $x_\alpha \in [1_X \setminus A]_p$. Then $1 - A(x) < \alpha \Rightarrow x_\alpha qA$. By hypothesis, there exists $V \in FRPO(X)$ with $x_\alpha qV$ such that $V \leq A$. By assumption, $pclVq(1_X \setminus A) \Rightarrow pclV \not\leq A$. Now as X is fuzzy extremally p-disconnected, $pclV \in FPO(X)$. Then $pint(pclV) = pclV \not\leq A$...(1). But $V \leq A \Rightarrow pint(pclV) = V \leq A$, contradicts (1). So, $1_X \setminus A = [1_X \setminus A]_p \Rightarrow 1_X \setminus A$ is fuzzy p^*-closed in $X \Rightarrow A$ is fuzzy p^*-open in X.

(iii) Let $A \in FRPO(X)$. Then $A \in FPO(X)$ and $A \in FPC(X)$ by Result 3.1. Then $A = pclA = [A]_p$ by Result 3.5 $\Rightarrow A$ is fuzzy p^*-closed. Again by Result 3.1, $1_X \setminus A \in FRPO(X)$ and so by above, $1_X \setminus A$ is fuzzy p^*-closed $\Rightarrow A$ is fuzzy p^*-open. Consequently, A is fuzzy p^*-clopen.

Converse is obvious by Result 3.5. □

It is obvious that fuzzy regular preopen set is fuzzy p^*-open but not conversely, follows from the next example recalling from [1].

Example 3.7. Fuzzy p^*-closed $\not\Rightarrow$ fuzzy regular preclosed.

Let $X = \{a, b\}$, $\tau = \{0_X, 1_X, B\}$ where $B(a) = 0.7, B(b) = 0.5$. Then (X, τ) is an fts. The collection of fuzzy preopen sets in (X, τ) is $\{0_X, 1_X, B, U\}$ where $U \not\leq 1_X \setminus B$ and that of fuzzy preclosed sets is $\{0_X, 1_X, 1_X \setminus B, 1_X \setminus U\}$ where $1_X \setminus U \not\geq B$. Consider the fuzzy set V defined by $V(a) = V(b) = 0.1$. Then V is fuzzy p^*-closed, but V is not fuzzy regular preclosed as $pcl(pintV) = 0_X \neq V$.

4. Fuzzy p-Closed and Fuzzy Regular p-Closed Spaces

In this section two new types of fuzzy spaces are introduced and characterized them in several ways and established some of their properties.

Definition 4.1. An fts (X, τ) is said to be fuzzy p-closed if every fuzzy cover $\mathcal{U}$ of X by fuzzy preopen sets has a finite p-proximate subcover, i.e., there exists a finite subcollection $\mathcal{U}_0$ of $\mathcal{U}$ such that $\{pclU : U \in \mathcal{U}_0\}$ is again a fuzzy cover of X.

Definition 4.2. An fts (X, τ) is said to be fuzzy regular p-closed if every fuzzy cover of X by fuzzy regular preopen sets has a finite subcover.

Remark 4.3. From Definition 4.1, Definition 4.2 and Theorem 3.3, it is clear that a fuzzy extremally p-disconnected space X is fuzzy p-closed iff it is fuzzy regular p-closed.

Theorem 4.4. *In an fts (X, τ), the following statements are equivalent:*

$(i) X$ is fuzzy p-closed,

(ii) for each family $\mathcal{F} = \{F_\alpha : \alpha \in \Lambda\}$ of fuzzy preclosed sets of X such that $\bigcap\limits_{\alpha\in\Lambda} F_\alpha = 0_X$, there exists a finite subset Λ_0 of Λ such that $\bigcap\limits_{\alpha\in\Lambda_0} pintF_\alpha = 0_X$,

(iii) for each family $\mathcal{F} = \{F_\alpha : \alpha \in \Lambda\}$ of fuzzy preclosed subsets of X, if $\bigcap\limits_{\alpha\in\Lambda_0} pintF_\alpha \neq 0_X$, for every finite subset Λ_0 of Λ, then $\bigcap\limits_{\alpha\in\Lambda} F_\alpha \neq 0_X$,

(iv) for any collection $\{F_\alpha : \alpha \in \Lambda\}$ of fuzzy preopen sets in X having finite intersection property, $\bigcap\{pclF_\alpha : \alpha \in \Lambda\} \neq 0_X$.

Proof. $(i) \Rightarrow (ii)$. Let $\mathcal{G} = \{F_\alpha : \alpha \in \Lambda\}$ be a family of fuzzy preclosed sets in X such that $\bigcap\limits_{\alpha\in\Lambda} F_\alpha = 0_X$. Then $\bigcup\limits_{\alpha\in\Lambda} (1_X \setminus F_\alpha) = 1_X \Rightarrow \mathcal{F} = \{1_X \setminus F_\alpha : \alpha \in \Lambda\}$ is a fuzzy cover of X by fuzzy preopen sets of X. By (i), there exists a finite subset Λ_0 of Λ such that $\bigcup\limits_{\alpha\in\Lambda_0} pcl(1_X \setminus F_\alpha) = 1_X \Rightarrow 1_X \setminus \bigcap\limits_{\alpha\in\Lambda_0} pintF_\alpha = 1_X \Rightarrow \bigcap\limits_{\alpha\in\Lambda_0} pintF_\alpha = 0_X$.

$(ii) \Rightarrow (i)$. Let $\mathcal{A} = \{A_\alpha : \alpha \in \Lambda\}$ be a family of fuzzy preopen sets in X such that $\bigcup\limits_{\alpha\in\Lambda} A_\alpha = 1_X$. Then $\{1_X \setminus A_\alpha : \alpha \in \Lambda\}$ is a family of fuzzy preclosed sets in X satisfying the hypothesis of (ii). Then by (ii), there exists a finite subset Λ_0 of Λ such that $\bigcap\limits_{\alpha\in\Lambda_0} pint(1_X \setminus A_\alpha) = 0_X \Rightarrow 1_X \setminus \bigcup\limits_{\alpha\in\Lambda_0} pclA_\alpha = 0_X \Rightarrow \bigcup\limits_{\alpha\in\Lambda_0} pclA_\alpha = 1_X \Rightarrow X$ is fuzzy p-closed.

$(ii) \Leftrightarrow (iii)$. Obvious.

$(i) \Rightarrow (iv)$. Let X be fuzzy p-closed and $\mathcal{F} = \{F_\alpha : \alpha \in \Lambda\}$ be a collection of fuzzy preopen sets in X with finite intersection property. If possible, let $\bigcap\{pclF_\alpha : \alpha \in \Lambda\} = 0_X$. Then $\bigcup\limits_{\alpha\in\Lambda} (1_X \setminus pclF_\alpha) = 1_X \Rightarrow \mathcal{U} = \{1_X \setminus pclF_\alpha : \alpha \in \Lambda\}$ is a fuzzy cover of X by fuzzy preopen sets of X. By (i), there is a finite subcollection Λ_0 of Λ such that $1_X = \bigcup\limits_{\alpha\in\Lambda_0} pcl(1_X \setminus pclF_\alpha) = \bigcup\limits_{\alpha\in\Lambda_0} (1_X \setminus pint(pclF_\alpha)) \leq \bigcap\limits_{\alpha\in\Lambda_0} (1_X \setminus F_\alpha) = 1_X \setminus \bigcap\limits_{\alpha\in\Lambda_0} F_\alpha \Rightarrow \bigcap\limits_{\alpha\in\Lambda_0} F_\alpha = 0_X$, a contradiction.

$(iv) \Rightarrow (i)$. Let X be not fuzzy p-closed. Then there exists a fuzzy cover $\mathcal{F} = \{F_\alpha : \alpha \in \Lambda\}$ by fuzzy preopen sets of X such that for every finite subset Λ_0 of Λ, $\bigcup\limits_{\alpha\in\Lambda_0} pclF_\alpha \neq 1_X$. Then $\bigcap\limits_{\alpha\in\Lambda_0} (1_X \setminus pclF_\alpha) \neq 0_X$ for every finite subset Λ_0 of Λ. Thus $\{1_X \setminus pclF_\alpha : \alpha \in \Lambda\}$ is a collection of fuzzy preopen sets having finite intersection property. By (iv), $\bigcap\limits_{\alpha\in\Lambda} pcl(1_X \setminus pclF_\alpha) \neq 0_X \Rightarrow 0_X \neq 1_X \setminus \bigcup\limits_{\alpha\in\Lambda} pint(pclF_\alpha) \Rightarrow \bigcup\limits_{\alpha\in\Lambda} pint(pclF_\alpha) \neq 1_X \Rightarrow \bigcup\limits_{\alpha\in\Lambda} F_\alpha \neq 1_X$, a contradiction.
□

Theorem 4.5. In an fts (X, τ), the following statements are true:

$(i) X$ is fuzzy regular p-closed,

$\Leftrightarrow$ (ii) for each family $\{A_\alpha : \alpha \in \Lambda\}$ of fuzzy regular preclosed sets in X with $\bigcap_{\alpha \in \Lambda} A_\alpha = 0_X$, there exists a finite subset Λ_0 of Λ such that $\bigcap_{\alpha \in \Lambda_0} A_\alpha = 0_X$,

$\Leftrightarrow$ (iii) for each family $\{A_\alpha : \alpha \in \Lambda\}$ of fuzzy regular preclosed sets in X with $\bigcap_{\alpha \in \Lambda_0} A_\alpha \neq 0_X$, for every finite subset Λ_0 of Λ, then $\bigcap_{\alpha \in \Lambda} A_\alpha \neq 0_X$,

$\Rightarrow$ (iv) every fuzzy cover of X by fuzzy preclopen sets has a finite subcover,

$\Rightarrow$ (v) every family of fuzzy preclopen sets having finite intersection property has a non-null intersection.

Proof. $(i) \Rightarrow (ii)$. Let $\mathcal{U} = \{U_\alpha : \alpha \in \Lambda\}$ be a family of fuzzy regular preclosed sets in X with $\bigcap_{\alpha \in \Lambda} U_\alpha = 0_X$. Then $\mathcal{F} = \{1_X \setminus U_\alpha : \alpha \in \Lambda\}$ is a fuzzy regular preopen cover of X. By (i), there exists a finite subset Λ_0 of Λ such that $\bigcup_{\alpha \in \Lambda_0} (1_X \setminus U_\alpha) = 1_X \Rightarrow \bigcap_{\alpha \in \Lambda_0} U_\alpha = 0_X$.

$(ii) \Rightarrow (i)$. Let $\mathcal{A} = \{A_\alpha : \alpha \in \Lambda\}$ be a family of fuzzy regular preopen sets in X with $\bigcup_{\alpha \in \Lambda} A_\alpha = 1_X$. Then $\bigcap_{\alpha \in \Lambda} (1_X \setminus A_\alpha) = 0_X$ where each $1_X \setminus A_\alpha$ is fuzzy regular preclosed in X. By (ii), there exists a finite subset Λ_0 of Λ such that $\bigcap_{\alpha \in \Lambda_0} (1_X \setminus A_\alpha) = 0_X \Rightarrow \bigcup_{\alpha \in \Lambda_0} A_\alpha = 1_X$.

$(ii) \Leftrightarrow (iii)$. Obvious.

$(i) \Rightarrow (iv)$. Obvious.

$(iv) \Rightarrow (v)$. Let $\mathcal{U} = \{U_\alpha : \alpha \in \Lambda\}$ be a family of fuzzy preclopen sets in X having finite intersection property. If possible, let $\bigcap_{\alpha \in \Lambda} U_\alpha = 0_X$. Then $\bigcup_{\alpha \in \Lambda} (1_X \setminus U_\alpha) = 1_X$. By (iv), there exists a finite subset Λ_0 of Λ such that $\bigcup_{\alpha \in \Lambda_0} (1_X \setminus U_\alpha) = 1_X \Rightarrow \bigcap_{\alpha \in \Lambda_0} U_\alpha = 0_X$, a contradiction. □

Definitio 4.6. Let $\{S_n : n \in (D, \geq)\}$ be a fuzzy net of fuzzy preopen sets in X. A fuzzy point x_α in X is said to be a fuzzy p-cluster point of the fuzzy net if for every $n \in D$ and every fuzzy pre-q-nbd V of x_α, there exists $m \in D$ with $m \geq n$ such that $S_m q V$.

Theorem 4.7. *An fts X is fuzzy p-closed iff every fuzzy net of fuzzy preopen sets in X has a fuzzy p-cluster point in X.*

Proof. Let $\mathcal{U} = \{S_n : n \in (D, \geq)\}$ be a fuzzy net of fuzzy preopen sets in X. For each $n \in D$, let us put $F_n = pcl[\bigcup\{S_m : m \in D \text{ and } m \geq n\}]$. Then $\mathcal{F} = \{F_n : n \in D\}$ is a family of fuzzy preclosed sets in X with the condition that for every finite subcollection $\mathcal{F}_0$ of $\mathcal{F}$, $\bigcap\{pintF : F \in \mathcal{F}_0\} \neq 0_X$. By Theorem 4.4 $(i) \Rightarrow (iii)$, $\bigcap_{n \in D} F_n \neq 0_X$. Let $x_\alpha \in \bigcap_{n \in D} F_n$. Then $x_\alpha \in F_n$, for all $n \in D$. Thus for any fuzzy pre-q-nbd A of x_α and any $n \in D$, $Aq[\bigcup\{S_m : m \geq n\}]$ and so there exists some $m \in D$ with $m \geq n$ and $AqS_m \Rightarrow x_\alpha$ is a fuzzy p-cluster point of $\mathcal{U}$.

Conversely, let $\mathcal{F}$ be a collection of fuzzy preclosed sets in X with the condition that for every finite subcollection $\mathcal{F}_0$ of $\mathcal{F}$, $\bigcap\{pintF : F \in \mathcal{F}_0\} \neq 0_X$. Let $\mathcal{F}^*$ denote the family of all finite intersections of members of $\mathcal{F}$ directed by the relation '$\gg$' such that for $F_1, F_2 \in \mathcal{F}^*$, $F_1 \gg F_2$ iff $F_1 \leq F_2$. Let $F^* = pintF$, for each $F \in \mathcal{F}^*$. Then $F^* \neq 0_X$. Consider the fuzzy net $\mathcal{U} = \{F^* : F \in (\mathcal{F}^*, \gg)\}$ of non-null fuzzy preopen sets of X. By hypothesis, $\mathcal{U}$ has a fuzzy p-cluster point, say x_α. We claim that $x_\alpha \in \bigcap \mathcal{F}$. In fact, let $F \in \mathcal{F}$ be arbitrary and A be any fuzzy pre-q-nbd of x_α. Since $F \in \mathcal{F}^*$ and x_α is a fuzzy p-cluster point of $\mathcal{U}$, there exists $G \in \mathcal{F}^*$ such that $G \gg F$ (i.e., $G \leq F$) and $G^*qA \Rightarrow GqA \Rightarrow FqA \Rightarrow x_\alpha \in pclF = F$, for each $F \in \mathcal{F} \Rightarrow x_\alpha \in \bigcap \mathcal{F} \Rightarrow \bigcap \mathcal{F} \neq 0_X$. By Theorem 4.4 $(iii) \Rightarrow (i)$, X

is fuzzy p-closed. □

Definition 4.8. A fuzzy set A is said to be fuzzy regular prenbd of a fuzzy point x_α in an fts (X,τ) if there exists a fuzzy regular preopen set U in X such that $x_\alpha \leq U \leq A$.

Definition 4.9. A fuzzy cover $\mathcal{U}$ by fuzzy preclosed (resp., fuzzy regular preclosed) sets of an fts (X,τ) will be called a fuzzy p-cover (resp., fuzzy regular p-cover) of X if for each fuzzy point x_α $(0<\alpha<1)$ in X, there exits $U \in \mathcal{U}$ such that U is a fuzzy preopen (resp., fuzzy regular pre) nbd of x_α.

Theorem 4.10. *An fts (X,τ) is fuzzy p-closed (resp., fuzzy regular p-closed) iff every fuzzy p-cover (resp., fuzzy regular p-cover) of X has a finite subcover.*

Proof. Let X be fuzzy p-closed (resp., fuzzy regular p-closed) and $\mathcal{U}$ be any fuzzy p-cover (resp., fuzzy regular p-cover) of X. Then for each $n \in N$ with $n>1$, there exists $U_x^n \in \mathcal{U}$ and a fuzzy preopen (resp., fuzzy regular preopen) set V_x^n in X such that $x_{1-1/n} \leq V_x^n \leq U_x^n$. Then $V_x^n(x) \geq 1-1/n \Rightarrow sup\{V_x^n(x) : n \in N\} = 1 \Rightarrow \mathcal{V} = \{V_x^n : x \in X, n \in N, n>1\}$ is a fuzzy cover of X by fuzzy preopen (resp., fuzzy regular preopen) sets of X. As X is fuzzy p-closed (resp., fuzzy regular p-closed), there exist finitely many points $x_1, x_2, ..., x_m \in X$ and $n_1, n_2, ..., n_m \in N \setminus \{1\}$ such that $1_X = \bigcup_{k=1}^{m} pclV_{x_k}^{n_k} \leq \bigcup_{k=1}^{m} U_{x_k}^{n_k}$ (resp., $1_X = \bigcup_{k=1}^{m} V_{x_k}^{n_k} \leq \bigcup_{k=1}^{m} U_{x_k}^{n_k}$).

Conversely, let $\mathcal{U}$ be fuzzy cover of X by fuzzy preopen (resp., fuzzy regular preopen) sets of X. For any fuzzy point x_α $(0<\alpha<1)$ in X, there exists $U_{x_\alpha} \in \mathcal{U}$ such that $U_{x_\alpha}(x) \geq \alpha$ $(0<\alpha<1)$. Then $\mathcal{V} = \{pclU : U \in \mathcal{U}\}$ is a fuzzy p-cover (resp., fuzzy regular p-cover) of X and the rest is clear. □

Definition 4.11. A prefilterbase $\mathcal{B}$ in an fts (X,τ) is said to have a fuzzy p^*-cluster point x_α if $x_\alpha \in \bigcap\{[B]_p : B \in \mathcal{B}\}$. In otherwords, we can write that $\mathcal{B}$, p-adheres at x_α, denoted by $x_\alpha \leq p$-ad $\mathcal{B}$.

Theorem 4.12. *If an fts (X,τ) is fuzzy p-closed then every fuzzy prefilterbase $\mathcal{B}$ in X has a fuzzy p^*-cluster point.*

Proof. Let X be fuzzy p-closed and $\mathcal{B} = \{B_\alpha : \alpha \in \Lambda\}$ be a prefilterbase in X having no fuzzy p^*-cluster point. Let $x \in X$. For each $n \in N$, consider the fuzzy point $x_{1/n}$. Then $x_{1/n} \not\in \bigcap\{[B]_p : B \in \mathcal{B}\} \Rightarrow$ there exists $B_x^n \in \mathcal{B}$ such that $x_{1/n} \not\in B_x^n \Rightarrow$ there exists $U_x^n \in FPO(X)$ with $x_{1/n}qU_x^n$ such that $pclU_x^n \not{q} B_x^n$. Now $U_x^n(x) + 1/n > 1 \Rightarrow U_x^n(x) > 1-1/n \Rightarrow sup_{n\in N} U_x^n(x) = 1 \Rightarrow \mathcal{U} = \{U_x^n : x \in X, n \in N\}$ is a fuzzy cover of X by fuzzy preopen sets in X. As X is fuzzy p-closed, there exist finitely many points $x_1, x_2, ..., x_k \in X$ and $n_1, n_2, ..., n_k \in N$ such that $\bigcup_{i=1}^{k} pclU_{x_i}^{n_i} = 1_X$. Then $\bigcap_{i=1}^{k} B_{x_i}^{n_i} \not{q} 1_X \Rightarrow \bigcap_{i=1}^{k} B_{x_i}^{n_i} = 0_X$, a contradiction. □

Definition 4.13. A fuzzy point x_α is said to be a fuzzy regular p^*-cluster point of a fuzzy set A in an fts (X,τ) if for every fuzzy regular preopen set U with $x_\alpha qU$, $pclUqA$. The union of all fuzzy regular p^*-cluster points of a fuzzy set A is called fuzzy regular p^*-closure of A, denoted by $[A]_{p^*}^r$.

Definition 4.14. A prefilterbase $\mathcal{B}$ in an fts (X,τ) is said to have a fuzzy regular p^*-cluster point x_α if $x_\alpha \in \bigcap\{[B]_{p^*}^r : B \in \mathcal{B}\}$.

Theorem 4.15. *If an fts (X,τ) is fuzzy regular p-closed then every prefilterbase $\mathcal{B}$ in X has a fuzzy regular p^*-cluster point.*

Proof. Let X be fuzzy regular p-closed and $\mathcal{B} = \{B_\alpha : \alpha \in \Lambda\}$ be a prefilterbase in X having no fuzzy regular p^*-cluster point. Consider any $x \in X$. Then for each $n \in N$, $x_{1/n} \not\in \bigcap\{[B]_{p^*}^r : B \in \mathcal{B}\} \Rightarrow$ there exists fuzzy regular preopen set U_x^n with $x_{1/n}qU_x^n$, but $pclU_x^n \not{q} B_x^n$ for some $B_x^n \in \mathcal{B}$. Now $U_x^n(x) > 1-1/n \Rightarrow \{supU_x^n(x) : n \in N\} = 1 \Rightarrow \mathcal{U} = \{U_x^n : x \in X, n \in N\}$ is a fuzzy cover of X by fuzzy regular preopen sets in X. As

X is fuzzy regular p-closed, there exist finitely many points $x_1, x_2, ..., x_k \in X$ and $n_1, n_2, ..., n_k \in N$ such that $\bigcup_{i=1}^{k} U_{x_i}^{n_i} = 1_X \Rightarrow \bigcup_{i=1}^{k} pclU_{x_i}^{n_i} = 1_X \Rightarrow \bigcap_{i=1}^{k} B_{x_i}^{n_i} \not{q} 1_X \Rightarrow \bigcap_{i=1}^{k} B_{x_i}^{n_i} = 0_X$, a contradiction. □

Theorem 4.16. *If an fts (X, τ) is fuzzy p^*-regular and fuzzy p-closed, then it is fuzzy compact.*

Proof. Let $\mathcal{V} = \{V_\alpha : \alpha \in \Lambda\}$ be a fuzzy cover of X by fuzzy open sets in X. For each $n \in N$, $x_{1/n}$ is a fuzzy point in X and so there exists $\alpha_{n(x)} \in \Lambda$ such that $x_{1/n} q V_{\alpha_{n(x)}}$. Since fuzzy open set is fuzzy preopen and X is fuzzy p^*-regular, there exists $U_x^n \in FPO(X)$ such that $x_{1/n} q U_x^n \leq pclU_x^n \leq V_{\alpha_{n(x)}}$. Since $U_x^n(x) > 1 - 1/n$, $\{supU_x^n(x) : n \in N\} = 1 \Rightarrow \mathcal{U} = \{U_x^n : x \in X, n \in N\}$ is a fuzzy cover of X by fuzzy preopen sets of X. Since X is fuzzy p-closed, there exist finitely many points $x_1, x_2, ..., x_k \in X$ and $n_1, n_2, ..., n_k \in N$ such that $\bigcup_{i=1}^{k} pclU_{x_i}^{n_i} = 1_X \Rightarrow \bigcup_{i=1}^{k} V_{\alpha_{n_i}(x_i)} = 1_X \Rightarrow X$ is fuzzy compact. □

Theorem 4.17. *If an fts (X, τ) is fuzzy extremally p-disconnected, fuzzy p^*-regular and fuzzy regular p-closed, then X is fuzzy compact.*

Proof. The proof follows from Theorem 4.16 and Result 3.2. □

5. Fuzzy p-Closed and Fuzzy Regular p-Closed Sets

In this section, fuzzy p-closed and fuzzy regular p-closedness of a fuzzy set are introduced and characterized them via fuzzy net and prefilterbases. Also some properties of fuzzy p-closed sets are discussed here.

Definition 5.1. A fuzzy set A in an fts (X, τ) is said to be fuzzy p-closed if every fuzzy cover $\mathcal{U}$ of A by fuzzy preopen sets of X has a finite p-proximate subcover, i.e., there exists a finite subcollection $\mathcal{U}_0$ of $\mathcal{U}$ such that $\{pclU : U \in \mathcal{U}_0\}$ covers A.

Definition 5.2. A fuzzy set A in an fts (X, τ) is said to be fuzzy regular p-closed if every fuzzy cover of A by fuzzy regular preopen sets of X has a finite subcover, i.e., there exists a finite subcollection $\mathcal{U}_0$ of $\mathcal{U}$ such that $\bigcup \mathcal{U}_0 \geq A$.

Definition 5.3. A fuzzy set A in an fts (X, τ) is said to be fuzzy regular pre-q-nbd of a fuzzy point x_α in X if there exists a fuzzy regular preopen set U in X such that $x_\alpha q U \leq A$.

Definition 5.4. Let x_α be a fuzzy point in an fts (X, τ). A prefilterbase $\mathcal{F}$ on X is said to rp-adhere at x_α, written as $x_\alpha \leq rp\text{-}ad\mathcal{F}$, if for each fuzzy regular pre-q-nbd U of x_α and each $F \in \mathcal{F}$, FqU.

Theorem 5.5. *For a fuzzy set A in an fts (X, τ), the following statements are equivalent:*

(i) *A is fuzzy p-closed set,*

(ii) *for every prefilterbase $\mathcal{B}$ in X, $[\bigcap\{pclB : B \in \mathcal{B}\}] \bigcap A = 0_X \Rightarrow$ there exists a finite subcollection $\mathcal{B}_0$ of $\mathcal{B}$ such that $\bigcap\{pintB : B \in \mathcal{B}_0\} \not{q} A$,*

(iii) *for any family $\mathcal{F}$ of fuzzy preclosed sets in X with $\bigcap\{F : F \in \mathcal{F}\} \bigcap A = 0_X$, there exists a finite subcollection $\mathcal{F}_0$ of $\mathcal{F}$ such that $\bigcap\{pintF : F \in \mathcal{F}_0\} \not{q} A$,*

(iv) *every prefilterbase on X, each member of which is q-coincident with A, p-adheres at some fuzzy point in A.*

Proof. $(i) \Rightarrow (ii)$. Let $\mathcal{B}$ be a prefilterbase in X such that $[\bigcap\{pclB : B \in \mathcal{B}\}] \bigcap A = 0_X$. Then for every $x \in suppA$, $\bigcap\{pclB : B \in \mathcal{B}\}(x) = 0 \Rightarrow \bigcup_{B \in \mathcal{B}} (1 - pclB)(x) = 1 \Rightarrow \{sup(1 - pclB)(x) : B \in \mathcal{B}\} = 1 \Rightarrow \{1_X \setminus pclB : B \in \mathcal{B}\}$ is a fuzzy cover of A by fuzzy preopen sets of X. By (i), there exists a finite subcollection $\{1_X \setminus pclB_i : i = 1, 2, ..., n\}$

(say) of it such that $A \leq \bigcup_{i=1}^{n} pcl(1_X \setminus pclB_i) = \bigcup_{i=1}^{n}(1_X \setminus pint(pclB_i)) \Rightarrow 1_X \setminus A \geq 1_X \setminus \bigcup_{i=1}^{n}(1_X \setminus pint(pclB_i)) = \bigcap_{i=1}^{n} pint(pclB_i) \Rightarrow A \not{q} \bigcap_{i=1}^{n} pintB_i$.

$(ii) \Rightarrow (i)$. Suppose A is not fuzzy p-closed. Then there exists a fuzzy cover $\mathcal{U}$ of A by fuzzy preopen sets of X having no finite p-proximate subcover. Then for every subcollection $\mathcal{U}_0$ of $\mathcal{U}$, there exists $x \in suppA$ such that $\{sup[(pclU)(x)] : U \in \mathcal{U}_0\} < A(x) \Rightarrow \{inf(1 - pclU)(x) : U \in \mathcal{U}_0\} > 1 - A(x) \geq 0$. Then $\mathcal{B} = \{\bigcap_{U \in \mathcal{U}_0} (1_X \setminus pclU) : \mathcal{U}_0$ is a finite subcollection of $\mathcal{U}\}$ is a prefilterbase in X. We claim that there does not exist a finite subcollection $\{U_1, U_2, ..., U_n\}$ (say) of $\mathcal{U}$ such that $\bigcap_{i=1}^{n} pint(1_X \setminus pclU_i) \not{q} A$. If not, there exists a finite subcollection $\{U_1, U_2, ..., U_n\}$ (say) of $\mathcal{U}$ such that $\bigcap_{i=1}^{n} pint(1_X \setminus pclU_i) \not{q} A$. Then $A \leq 1_X \setminus \bigcap_{i=1}^{n} pint(1_X \setminus pclU_i) = \bigcup_{i=1}^{n} pclU_i$, contradicts the property of $\mathcal{U}$. Hence for every finite subcollection $\{\bigcap_{U \in \mathcal{U}_1} (1_X \setminus pclU), ..., \bigcap_{U \in \mathcal{U}_k} (1_X \setminus pclU)\}$ of $\mathcal{B}$ where $\mathcal{U}_1, ..., \mathcal{U}_k$ are finite subsets of $\mathcal{U}$, we have $[\bigcap_{U \in \mathcal{U}_1 \bigcup ... \bigcup \mathcal{U}_k} pint(1_X \setminus pclU)] q A$. By (ii), $[\bigcap_{U \in \mathcal{U}} \{pcl(1_X \setminus pclU)\}] \bigcap A \neq 0_X \Rightarrow$ there exists $x \in suppA$ such that $[\bigcap_{U \in \mathcal{U}} pcl(1_X \setminus pclU)](x) > 0 \Rightarrow 1 - [\bigcap_{U \in \mathcal{U}} pcl(1_X \setminus pclU)](x) < 1 \Rightarrow \bigcup_{U \in \mathcal{U}} [1 - pcl(1 - pclU)](x) < 1 \Rightarrow [\bigcup_{U \in \mathcal{U}} pint(pclU)](x) < 1 \Rightarrow \{supU(x) : U \in \mathcal{U}\} \leq \{sup[pint(pclU)(x)] : U \in \mathcal{U}\} < 1$ (as U is fuzzy preopen) $\Rightarrow \mathcal{U}$ is not a fuzzy cover of A by fuzzy preopen set of X, a contradiction.

$(i) \Rightarrow (iii)$. Let $\mathcal{F}$ be a family of fuzzy preclosed sets in X with $\bigcap\{F : F \in \mathcal{F}\} \bigcap A = 0_X$. Then for each $x \in suppA$ and for each $n \in N$, there exists some $F_n \in \mathcal{F}$ such that $F_n(x) < 1/n \Rightarrow 1 - F_n(x) > 1 - 1/n \Rightarrow \{sup(1_X \setminus F)(x) : F \in \mathcal{F}\} = 1 \Rightarrow \{1_X \setminus F : F \in \mathcal{F}\}$ is a fuzzy cover of A by fuzzy preopen sets on X. By (i), there exists a finite subcollection $\mathcal{F}_0$ of $\mathcal{F}$ such that $A \leq \{sup[pcl(1_X \setminus F)] : F \in \mathcal{F}_0\} \Rightarrow 1_X \setminus A \geq 1 - \{sup[pcl(1_X \setminus F)] : F \in \mathcal{F}_0\} = \{inf[1_X \setminus pcl(1_X \setminus F)] : F \in \mathcal{F}_0\} = \bigcap_{F \in \mathcal{F}_0} pintF \Rightarrow A \not{q} \bigcap\{pintF : F \in \mathcal{F}_0\}$.

$(iii) \Rightarrow (ii)$. Let $\mathcal{B}$ be a prefilterbase in X with $[\bigcap\{pclB : B \in \mathcal{B}\}] \bigcap A = 0_X$. Then $\mathcal{F} = \{pclB : B \in \mathcal{B}\}$ is a family of fuzzy preclosed sets in X with $(\bigcap \mathcal{F}) \bigcap A = 0_X$. By (iii), there exists a finite subcollection $\mathcal{B}_0$ of $\mathcal{B}$ such that $\bigcap\{pint(pclB) : B \in \mathcal{B}_0\} \not{q} A \Rightarrow (\bigcap_{B \in \mathcal{B}_0} pintB) \not{q} A$.

$(i) \Rightarrow (iv)$. let $\mathcal{F}$ be a prefilterbase on X each member of which is q-coincident with A. If possible, let $\mathcal{F}$ have no p-adhere point in A. Then for each $x \in suppA$, there exists $n_x \in N$ such that $x_{1/n_x} \leq A$. Then there exists a fuzzy preopen set $U^x_{n_x}$ and a member $F^x_{n_x}$ of $\mathcal{F}$ such that $x_{1/n_x} q U^x_{n_x}$, but $pclU^x_{n_x} \not{q} F^x_{n_x}$. Then $U^x_{n_x} > 1 - 1/n_x$ so that $sup\{U^x_n(x) : n \in N$ and $n \geq n_x\} = 1 \Rightarrow \{U^x_n : n \geq n_x, n \in N, x \in suppA\}$ forms a fuzzy cover of A by fuzzy preopen sets on X. By (i), there exist finitely many points $x_1, x_2, ..., x_k \in suppA$ and $n_1, n_2, ..., n_k \in N$ such that $A \leq \bigcup_{i=1}^{K} pclU^{x_i}_{n_i}$. Choose $F \in \mathcal{F}$ such that $F \leq \bigcap_{i=1}^{k} F^{x_i}_{n_i}$. Then $F \not{q} (\bigcup_{i=1}^{k} pclU^{x_i}_{n_i}) \Rightarrow F \not{q} A$, a contradiction.

$(iv) \Rightarrow (i)$. If possible, let there exist a fuzzy cover of A by fuzzy preopen sets in X having no finite p-proximate subcover, i.e., for every finite subcollection $\mathcal{U}_0$ of $\mathcal{U}$, $\bigcup\{pclU : U \in \mathcal{U}_0\} \not\geq A$. Then $\mathcal{F} = \{1_X \setminus \bigcup_{U \in \mathcal{U}_0} pclU : \mathcal{U}_0$ is a finite subcollection of $\mathcal{U}\}$ is a prefilterbase on X such that FqA, for all $F \in \mathcal{F}$. By (iv), $\mathcal{F}$ p-adheres at some fuzzy point $x_\alpha \leq A$. As $\mathcal{U}$ covers A, $\{supU(x) : U \in \mathcal{U}\} = 1 \Rightarrow$ there exists $U_k \in \mathcal{U}$ such that $U_k(x) > 1 - \alpha$. Then $x_\alpha q U_k$. As $x_\alpha \leq p\text{-}ad\mathcal{F}$ and $1_X \setminus pclU_k \in \mathcal{F}$, we have $pclU_k q(1_X \setminus pclU_k)$, a contradiction. □

Theorem 5.6. *For a fuzzy set A in an fts (X, τ), the following statements are equivalent:*

$(i) A$ is fuzzy regular p-closed set,

(ii) for every prefilterbase $\mathcal{B}$ in X where each member of $\mathcal{B}$ is fuzzy regular preclosed in X, $[\bigcap\{B : B \in \mathcal{B}\}] \bigcap A = 0_X \Rightarrow$ there exists a finite subcollection $\mathcal{B}_0$ of $\mathcal{B}$ such that $\bigcap\{B : B \in \mathcal{B}_0\} \not{q} A$,

(iii) for any family $\mathcal{F}$ of fuzzy regular preclosed sets in X with $\bigcap\{F : F \in \mathcal{F}\} \bigcap A = 0_X$, there exists a finite subcollection $\mathcal{F}_0$ of $\mathcal{F}$ such that $\bigcap\{F : F \in \mathcal{F}_0\} \not{q} A$,

(iv) every prefilterbase on X, each member of which is q-coincident with A, rp-adheres at some fuzzy point in A.

Proof. $(i) \Rightarrow (ii)$. Let $\mathcal{B}$ be a prefilterbase in X each member of $\mathcal{B}$ is fuzzy regular preclosed such that $[\bigcap\{B : B \in \mathcal{B}\}] \bigcap A = 0_X$. Then for every $x \in suppA$, $\bigcap\{B : B \in \mathcal{B}\}(x) = 0 \Rightarrow \bigcup\limits_{B \in \mathcal{B}} (1 - B)(x) = 1 \Rightarrow sup_{B \in \mathcal{B}}(1 - pclB)(x) = 1 \Rightarrow \{1_X \setminus B : B \in \mathcal{B}\}$ is a fuzzy cover of A by fuzzy regular preopen sets of X. By (i), there exists a finite subcollection $\{1_X \setminus B_i : i = 1, 2, ..., k\}$ (say) of it such that $A \leq \bigcup\limits_{i=1}^{k}(1_X \setminus B_i) = 1_X \setminus \bigcap\limits_{i=1}^{k} B_i \Rightarrow A \not{q} \bigcap\limits_{i=1}^{k} B_i$.

$(ii) \Rightarrow (i)$. Suppose that there exists a fuzzy cover $\mathcal{U}$ of A by fuzzy regular preopen sets in X having no finite subcover for A. Then for every finite subcollection $\mathcal{U}_0$ of $\mathcal{U}$, there exists $x \in suppA$ such that $\{supU(x) : U \in \mathcal{U}_0\} < A(x) \Rightarrow 1 - \{supU(x) : U \in \mathcal{U}_0\} > 1 - A(x) \geq \{inf[1 - U(x)] : U \in \mathcal{U}_0\} > 0$. Thus $\mathcal{B} = \{\bigcap\limits_{U \in \mathcal{U}_0} (1_X \setminus U) : \mathcal{U}_0$ is a finite subcollection of $\mathcal{U}\}$ is a prefilterbase in X. We claim that there does not exist a finite subcollection $\{U_1, U_2, ..., U_n\}$ (say) of $\mathcal{U}$ such that $\bigcap\limits_{i=1}^{n}(1_X \setminus U_i) \not{q} A$. If not, there exists a finite subcollection $\{U_1, U_2, ..., U_n\}$ (say) of $\mathcal{U}$ such that $\bigcap\limits_{i=1}^{n}(1_X \setminus U_i) \not{q} A$. Then $A \leq 1_X \setminus \bigcap\limits_{i=1}^{n}(1_X \setminus U_i) = \bigcup\limits_{i=1}^{n} U_i \Rightarrow \mathcal{U}$ has a finite subcover for A, a contradiction. Hence for every finite subcollection $\{\bigcap\limits_{U \in \mathcal{U}_1} (1_X \setminus U), ..., \bigcap\limits_{U \in \mathcal{U}_k} (1_X \setminus U)\}$ of $\mathcal{B}$ where $\mathcal{U}_1, ..., \mathcal{U}_k$ are finite subsets of $\mathcal{U}$, we have $[\bigcap\limits_{U \in \mathcal{U}_1 \bigcup ... \bigcup \mathcal{U}_k} pint(1_X \setminus U)] q A$. Then by (ii), $[[\bigcap\limits_{U \in \mathcal{U}} \{(1_X \setminus U)\}]] \bigcap A \neq 0_X$. Then there exists $x \in suppA$ such that $\bigcap\limits_{U \in \mathcal{U}} (1_X \setminus U)(x) > 0 \Rightarrow 1 - \bigcup\limits_{U \in \mathcal{U}} U(x) > 0 \Rightarrow \bigcup\limits_{U \in \mathcal{U}} U(x) < 1$ which contradicts the fact that $\mathcal{U}$ is a fuzzy cover of A.

$(i) \Rightarrow (iii)$. Let $\mathcal{F}$ be a family of fuzzy regular preclosed sets in X with $\bigcap\{F : F \in \mathcal{F}\} \bigcap A = 0_X$. Then for each $x \in suppA$ and for each $n \in N$, there exists some $F_n \in \mathcal{F}$ such that $F_n(x) < 1/n \Rightarrow 1 - F_n(x) > 1 - 1/n \Rightarrow \{sup(1_X \setminus F)(x) : F \in \mathcal{F}\} = 1 \Rightarrow \{1_X \setminus F : F \in \mathcal{F}\}$ is a fuzzy cover of A by fuzzy regular preopen sets on X. By (i), there exists a finite subcollection $\mathcal{F}_0$ of $\mathcal{F}$ such that $A \leq \bigcup\limits_{F \in \mathcal{F}_0} (1_X \setminus F)$ for some finite subcollection $\mathcal{F}_0$ of $\mathcal{F} \Rightarrow A \not{q} \bigcap\limits_{F \in \mathcal{F}_0} F$.

$(iii) \Rightarrow (ii)$. Let $\mathcal{B}$ be a prefilterbase in X where each member of $\mathcal{B}$ is fuzzy regular preclosed such that $[\bigcap\{B : B \in \mathcal{B}\}] \bigcap A = 0_X$. Then the family $\mathcal{F} = \{B : B \in \mathcal{B}\}$ is a family of fuzzy regular preclosed sets in X with $(\bigcap \mathcal{F}) \bigcap A = 0_X$. By (iii), there exists a finite subcollection $\mathcal{B}_0$ of $\mathcal{B}$ such that $[\bigcap_{B \in \mathcal{B}_0} B] \not{q} A$.

$(i) \Rightarrow (iv)$. Let $\mathcal{F}$ be a prefilterbase on X each member of which is q-coincident with A. If possible, let $\mathcal{F}$ have no rp-adhere point in A. Then for each $x \in suppA$, there exists $n_x \in N$ such that $x_{1/n_x} \leq A$. Then there exists a fuzzy regular preopen set $U^x_{n_x}$ and a member $F^x_{n_x}$ of $\mathcal{F}$ such that $x_{1/n_x} q U^x_{n_x}$ and $U^x_{n_x} \not{q} F^x_{n_x}$. Then $U^x_{n_x}(x) > 1 - 1/n_x$ so that $sup\{U^x_n(x) : n \in N$ and $n \geq n_x\} = 1 \Rightarrow \{U^x_n : n \geq n_x, n \in N, x \in suppA\}$ forms a fuzzy cover of A by fuzzy regular preopen sets on X. By (i), there exist finitely many points $x_1, x_2, ..., x_k \in suppA$ and $n_1, n_2, ..., n_k \in N$ such that $A \leq \bigcup\limits_{i=1}^{k} pclU^{x_i}_{n_i}$. Choose $F \in \mathcal{F}$ such that $F \leq \bigcap\limits_{i=1}^{k} F^{x_i}_{n_i}$. Then $F \not{q} (\bigcup\limits_{i=1}^{k} pclU^{x_i}_{n_i}) \Rightarrow F \not{q} A$, a contradiction.

$(iv) \Rightarrow (i)$. If possible, let there exist a fuzzy cover $\mathcal{U}$ of A by fuzzy regular preopen sets in X such that for every finite subcollection $\mathcal{U}_0$ of $\mathcal{U}$, $\bigcup\{U : U \in \mathcal{U}_0\} \not\geq A$. Then $\mathcal{F} = \{1_X \setminus \bigcup\limits_{U \in \mathcal{U}_0} U : \mathcal{U}_0$ is a finite subcollection of $\mathcal{U}\}$ is a prefilterbase on X such that FqA, for all $F \in \mathcal{F}$. By (iv), $\mathcal{F}$ rp-adheres at some fuzzy point $x_\alpha \leq A$. As $\mathcal{U}$ covers A, $\{supU(x) : U \in \mathcal{U}\} = 1 \Rightarrow$ there exists $U_k \in \mathcal{U}$ such that $U_k(x) > 1 - \alpha$. Then $x_\alpha q U_k$. As $x_\alpha \leq rp\text{-}ad\mathcal{F}$ and $1_X \setminus U_k \in \mathcal{F}$, we have $U_k q (1_X \setminus U_k)$, a contradiction. □

Definition 5.7. Let x_α be a fuzzy point in an fts (X, τ). A fuzzy net $\{S_n : n \in (D, \geq)\}$ is said to

(i) p-adhere at x_α, denoted by $x_\alpha \leq p\text{-}ad(S_n)$ if for each fuzzy pre-q-nbd U of x_α and each $n \in D$, there exists $m \in D$ with $m \geq n$ such that $S_m q pclU$;

(ii) p-converge to x_α, denoted by $S_n \overrightarrow{p} x_\alpha$ if for each fuzzy pre-q-nbd W of x_α, there exists $m \in D$ such that $S_n q pclW$ for all $n \geq m (n \in D)$.

Theorem 5.8. *For a fuzzy set A in an fts (X, τ), the following implications hold:*

(a) every fuzzy net in A p-adheres at some fuzzy point in A,

$\Leftrightarrow$(b) every fuzzy net in A has a p-convergent fuzzy subnet,

$\Leftrightarrow$ (c) every prefilterbase in A p-adheres at some fuzzy point in A,

$\Rightarrow$ (d) for every family $\{B_\alpha : \alpha \in \Lambda\}$ of non-null fuzzy sets with $[\bigcap\limits_{\alpha \in \Lambda} [B_\alpha]_p] \bigcap A = 0_X$, there is a finite subset Λ_0 of Λ such that $(\bigcap\limits_{\alpha \in \Lambda_0} B_\alpha) \bigcap A = 0_X$,

$\Rightarrow$ (e) A is fuzzy p-closed.

Proof. *(a) $\Rightarrow$ (b). Let a fuzzy net $\{S_n : n \in (D, \geq)\}$ in A where $(D, \geq)$ is a directed set, p-adhere at a fuzzy point $x_\alpha \leq A$. Let Q_{x_α} denote the set of the fuzzy preclosures of all fuzzy pre-q-nbds of x_α. For any $B \in Q_{x_\alpha}$, we can choose some $n \in D$ such that $S_n q B$. Let E denote the set of all ordered pairs (n, B) with the property that $n \in D$, $B \in Q_{x_\alpha}$ and $S_n q B$. Then $(E, \gg)$ is a directed set where $(m, C) \gg (n, B)$ iff $m \geq n$ in D and $C \leq B$. Then $T : (E, \gg) \rightarrow (X, \tau)$ given by $T(n, B) = S_n$, is a fuzzy subnet of $\{S_n : n \in (D, \geq)\}$. Let V be any fuzzy pre-q-nbd of x_α. Then there is $n \in D$ such that that $(n, pclV) \in E$ and hence $S_n q pclV$. Now, for any $(m, U) \gg (n, pclV)$, $T(m, U) = S_m q U \leq pclV \Rightarrow T(m, U) q pclV$. Hence $T \overrightarrow{p} x_\alpha$.*

(b) $\Rightarrow$ (a). If a fuzzy net $\{S_n : n \in (D, \geq)\}$ does not p-adhere at a fuzzy point x_α, then there is a fuzzy pre-q-nbd U of x_α and an $n \in D$ such that $S_m \not{q} pclU$, for all $m \geq n$. Then obviously no fuzzy subnet of the fuzzy net can p-converge to x_α.

(a) $\Rightarrow$ (c). Let $\mathcal{F} = \{F_\alpha : \alpha \in \Lambda\}$ be a prefilterbase in A. For each $\alpha \in \Lambda$, choose a fuzzy point $x_{F_\alpha} \leq F_\alpha$ and construct the fuzzy net $S = \{x_{F_\alpha} : F_\alpha \in \mathcal{F}\}$ in A with $(\mathcal{F}, \gg)$ as domain, where for two members $F_\alpha, F_\beta \in \mathcal{F}$, $F_\alpha \gg F_\beta$ iff $F_\alpha \leq F_\beta$. By (a), the fuzzy net S p-adheres at some fuzzy point x_t $(0 < t \leq 1) \leq A$. Then for any fuzzy pre-q-nbd U of x_t and any $F_\alpha \in \mathcal{F}$, there exists $F_\beta \in \mathcal{F}$ such that $F_\beta \gg F_\alpha$ and $x_{F_\beta} q pclU$. Then $F_\beta q pclU$ and hence $F_\alpha q pclU$. Thus $\mathcal{F}$ p-adheres at x_t.

(c) $\Rightarrow$ (a). Let $\{S_n : n \in (D, \geq)\}$ be a fuzzy net in A. Consider the prefilterbase $\mathcal{F} = \{T_n : n \in D\}$ generated by the net, where $T_n = \{S_m : m \in D, m \geq n\}$. By (c), there exists a fuzzy point $a_\alpha \leq A$ such that $\mathcal{F}$ p-adhere at a_α. Then for each fuzzy pre-q-nbd U of a_α and each $F \in \mathcal{F}$, $FqpclU$, i.e., $pclUqT_n$, for all $n \in D$. Hence the given fuzzy net p-adheres at a_α.

(c) $\Rightarrow$ (d). Let $\mathcal{B} = \{B_\alpha : \alpha \in \Lambda\}$ be a family of fuzzy sets in X such that for every finite subset Λ_0 of Λ, $(\bigcap\limits_{\alpha \in \Lambda_0} B_\alpha) \bigcap A \neq 0_X$. Then $\mathcal{F} = \{(\bigcap\limits_{\alpha \in \Lambda_0} B_\alpha) \bigcap A : \Lambda_0$ is a finite subset of $\Lambda\}$ is a prefilterbase in A. By (c), $\mathcal{F}$ p-adheres at some fuzzy point $a_t \leq A$ $(0 < t \leq 1)$. Then for each $\alpha \in \Lambda$ and each fuzzy pre-q-nbd U of a_t, $B_\alpha q pclU$, for each $\alpha \in \Lambda$. Consequently,

$[\bigcap_{\alpha\in\Lambda}[B_\alpha]_p]\bigcap A \neq 0_X$.

(d) $\Rightarrow$ (e). Let $\mathcal{U} = \{U_\alpha : \alpha \in \Lambda\}$ be a fuzzy cover of A by fuzzy preopen sets of X. Then $A\bigcap[\bigcap_{\alpha\in\Lambda}(1_X \setminus U_\alpha)] = A\bigcap[1_X \setminus \bigcup_{\alpha\in\Lambda} U_\alpha] = 0_X$. If for some $\alpha \in \Lambda$, $1_X \setminus pclU_\alpha = 0_X$, then we are done. If $1_X \setminus pclU_\alpha$ (=B_α, say) $\neq 0_X$, for each $\alpha \in \Lambda$, then $\mathcal{B} = \{B_\alpha : \alpha \in \Lambda\}$ is a family of non-null fuzzy sets. We show that $\bigcap_{\alpha\in\Lambda}[B_\alpha]_p \leq \bigcap_{\alpha\in\Lambda}(1_X \setminus U_\alpha)$. In fact, let x_t ($0 < t \leq 1$) be a fuzzy point such that $x_t \leq [B_\alpha]_p = [1_X \setminus pclU_\alpha]_p$. If $x_t qU_\alpha$, then $pclU_\alpha q(1_X \setminus pclU_\alpha)$, a contradiction. Hence $x_t \not{q} U_\alpha \Rightarrow x_t \leq 1_X \setminus U_\alpha$. Then $[\bigcap_{\alpha\in\Lambda}[B_\alpha]_p]\bigcap A \leq A\bigcap[\bigcap_{\alpha\in\Lambda}(1_X \setminus U_\alpha)] = 0_X$. By (d), there exists a finite subset Λ_0 of Λ such that $[\bigcap_{\alpha\in\Lambda_0}[B_\alpha]_p]\bigcap A = 0_X$, i.e., $A \leq 1_X \setminus \bigcap_{\alpha\in\Lambda_0}[B_\alpha]_p = 1_X \setminus \bigcap_{\alpha\in\Lambda_0}[1_X \setminus pclU_\alpha]_p$ as $1_X \setminus pclU_\alpha \in FPO(X) = 1_X \setminus \bigcap_{\alpha\in\Lambda_0} pcl(1_X \setminus pclU_\alpha) = \bigcup_{\alpha\in\Lambda_0} pint(pclU_\alpha) \leq \bigcup_{\alpha\in\Lambda_0} pclU_\alpha \Rightarrow A$ is fuzzy p-closed. □

Theorem 5.9. *A fuzzy set A in an fts $(X.\tau)$ is fuzzy p-closed iff for each prefilterbase $\mathcal{F}$ in X which is such that for each set of finitely many members $F_1, F_2 < ..., F_n$ from $\mathcal{F}$ and for any fuzzy regular preclosed set C containing A, one has $(F_1 \bigcap ... \bigcap F_n)qC$, $\mathcal{F}$ p-adheres at a fuzzy point in A.*

Proof. Let A be fuzzy p-closed set and suppose $\mathcal{F}$ be a prefilterbase in X such that $[\bigcap\{pclF : F \in \mathcal{F}\}]\bigcap A = 0_X$...(1). Let $x \in suppA$. Consider any $n \in N$ such that $1/n < A(x)$, i.e., $x_{1/n} \leq A$. By (1), there exists a fuzzy pre-q-nbd U^n_x of $x_{1/n}$ and an $F^n_x \in \mathcal{F}$ such that $pclU^n_x \not{q} F^n_x$. Now $U^n_x(x) > 1 - 1/n \Rightarrow sup\{U^n_x(x) : 1/n < A(x), n \in N\} = 1 \Rightarrow \mathcal{U} = \{U^n_x : x \in suppA, n \in N\}$ forms a fuzzy cover of A by fuzzy preopen sets of X. By hypothesis, there exist finitely many members $U^{n_1}_{x_1}, , , , , U^{n_k}_{x_k}$ of $\mathcal{U}$ such that $A \leq \bigcup_{i=1}^{k} pclU^{n_k}_{x_k} = pcl(\bigcup_{i=1}^{k} U^{n_k}_{x_k})$ (=U, say). Now $F^{n_1}_{x_1}, ..., F^{n_k}_{x_k} \in \mathcal{F}$ such that $U^{n_i}_{x_i} \not{q} F^{n_i}_{x_i}$ for $i = 1, 2, ..., k$. Then U is a fuzzy regular preclosed set containing A such that $pclU \not{q}(F^{n_1}_{x_1} \bigcap .. \bigcap F^{n_k}_{x_k}) \Rightarrow U \not{q}(F^{n_1}_{x_1} \bigcap .. \bigcap F^{n_k}_{x_k})$.

Conversely, let $\mathcal{B}$ be a prefilterbase in X which do not p-adhere at any point in A. Then by hypothesis, there is a fuzzy regular preclosed set C containing A such that for some finite subcollection $\mathcal{B}_0$ of $\mathcal{B}$, $(\bigcap \mathcal{B}_0) \not{q}C$. Then $(\bigcap \mathcal{B}_0) \not{q}A$. By Theorem 5.5 (ii) $\Rightarrow$ (i), A is fuzzy p-closed. □

Theorem 5.10. *Let (X, τ) be fuzzy extremally p-disconnected and fuzzy p-closed space. Let $A \in I^X$ be fuzzy preclosed in X. Then A is fuzzy p-closed in X.*

Proof. Let $\mathcal{U} = \{U_\alpha : \alpha \in \Lambda\}$ be a fuzzy cover of A by fuzzy preopen sets in X. Then $\mathcal{V} = \mathcal{U}\bigcup(1_X \setminus A)$ is a fuzzy cover of X by fuzzy preopen sets of X. Since X is fuzzy p-closed, $\mathcal{V}$ has a finite subcovering $\mathcal{V}_0$ such that $1_X = (\bigcup_{i=1}^{k} pclU_{\alpha_i})\bigcup pcl(1_X \setminus A)$. Since X is fuzzy extremally p-disconnected, $pcl(1_X \setminus A)$ is fuzzy preopen in X and so $1_X = (\bigcup_{i=1}^{k} pclU_{\alpha_i})\bigcup(1_X \setminus A)$ and so $A \leq \bigcup_{i=1}^{k} pclU_{\alpha_i}$. If $\mathcal{V}_0$ contains $1_X \setminus A$, we omit it and get the result. □

Theorem 5.11. *Let (X, τ) be an fts and $A \in I^X$. Then*

(a) if A is fuzzy p-closed, then so is pclA,

(b) union of two fuzzy p-closed sets is also so,

(c) if X is fuzzy p-closed, then every fuzzy regular preclosed set A in X is fuzzy p-closed.

Proof. (a). Let $\mathcal{U}$ be a fuzzy cover of pclA by fuzzy preopen sets in X. Then $\mathcal{U}$ is a fuzzy cover of A by fuzzy preopen sets in X. As A is fuzzy p-closed, there exists a finite subcollection $\mathcal{U}_0$ of $\mathcal{U}$ such that $A \leq \bigcup\{pclU : U \in \mathcal{U}_0\} = pcl\{\bigcup U : U \in \mathcal{U}_0\} \Rightarrow pclA \leq pcl\{\bigcup U : U \in \mathcal{U}_0\}$. Hence the result.

(b). Obvious.

(c). Let $\mathcal{U} = \{U_\alpha : \alpha \in \Lambda\}$ be a fuzzy cover of a fuzzy regular preclosed set A in X by fuzzy preopen sets of X. Then for each $x \notin suppA$, $A(x) = 0 \Rightarrow (1_X \setminus A)(x) = 1 \Rightarrow \mathcal{U} \bigcup (1_X \setminus A)$ is a fuzzy cover of X by fuzzy preopen sets of X. Since X is fuzzy p-closed, there are finitely many members $U_1, U_2, ..., U_n$ in $\mathcal{U}$ such that $1_X = (pclU_1 \bigcup ... \bigcup pclU_n) \bigcup pcl(1_X \setminus A)$. We claim that $pintA \leq pclU_1 \bigcup ... \bigcup pclU_n$. If not, there exists a fuzzy point $x_\beta \leq pintA$, but $x_\beta \not\leq (pclU_1 \bigcup ... \bigcup pclU_n)$, i.e., $\beta > max\{(pclU_1)(x), ..., (pclU_n)(x)\}$. As $1_X = (pclU_1 \bigcup ... \bigcup pclU_n) \bigcup pcl(1_X \setminus A)$, $[pcl(1_X \setminus A)](x) = 1 \Rightarrow 1 - pintA(x) = 1 \Rightarrow pintA(x) = 0 \Rightarrow x_\beta \notin pintA$, a contradiction. Hence $A = pcl(pintA) \leq pcl(pclU_1 \bigcup .. \bigcup pclU_n) = pclU_1 \bigcup ... \bigcup pclU_n \Rightarrow A$ is fuzzy p-closed. □

6. Fuzzy Regular Precontinuous, Fuzzy Strongly θ-Precontinuous and Fuzzy p-Continuous Functions

In this section three different types of fuzzy continuous functions are introduced and shown that under these types of functions fuzzy p-closed and fuzzy regular p-closed spaces become fuzzy compact space and a fuzzy compact space becomes fuzzy p-closed space.

Definition 6.1. *A function $f : X \to Y$ is said to be fuzzy regular precontinuous (resp., fuzzy strongly θ-precontinuous) if for each fuzzy point x_α in X and each fuzzy open q-nbd V of $f(x_\alpha)$, there exists a fuzzy regular preopen (resp., fuzzy preopen) set U with $x_\alpha qU$, $f(U) \leq V$ (resp., $f(pclU) \leq V$).*

Definition 6.2. *A function $f : X \to Y$ is said to be fuzzy p-continuous if for each fuzzy point x_α in X and each fuzzy pre-q-nbd V of $f(x_\alpha)$, there exists a fuzzy open q-nbd U of x_α such that $f(U) \leq pclV$.*

Theorem 6.3. *Let X be fuzzy extremally p-disconnected space. Then $f : X \to Y$ is fuzzy regular precontinuous iff f is fuzzy strongly θ-precontinuous.*

Proof. The proof follows from Result 3.1. □

Theorem 6.4. *If $f : X \to Y$ is fuzzy strongly θ-precontinuous surjection and X is fuzzy p-closed, then Y is fuzzy compact.*

Proof. Let $\mathcal{V} = \{V_\alpha : \alpha \in \Lambda\}$ be a fuzzy open cover of Y. For each $n \in N$, $x_{1/n}$ is a fuzzy point in X and so there exists $\alpha_{n(x)} \in \Lambda$ such that $f(x_{1/n})qV_{\alpha_{n(x)}}$. As f is fuzzy strongly θ-precontinuous surjection, there exists $U^n_x \in FPO(X)$ with $x_{1/n}qU^n_x$ such that $f(pclU^n_x) \leq V_{\alpha_{n(x)}}$. Then $U^n_x(x) > 1 - 1/n \Rightarrow \{supU^n_x : n \in N\} = 1 \Rightarrow \mathcal{U} = \{U^n_x : x \in X, n \in N\}$ is a fuzzy cover of X by fuzzy preopen sets of X. Since X is fuzzy p-closed, there exist finitely many points $x_1, x_2, ..., x_k$ of X and $n_1, n_2, ..., n_k \in N$ such that $\bigcup\limits_{i=1}^{k} pclU^{n_i}_{x_i} = 1_X \Rightarrow 1_Y = f(\bigcup\limits_{i=1}^{k} pclU^{n_i}_{x_i}) = \bigcup\limits_{i=1}^{k} f(pclU^{n_i}_{x_i}) \leq \bigcup\limits_{i=1}^{k} V_{\alpha_{n_i(x_i)}} \Rightarrow Y$ is fuzzy compact. □

Now we can state the following two theorems the proofs of which are as same as Theorem 6.4.

Theorem 6.5. *If $f : X \to Y$ is a fuzzy regular p-continuous surjection and X is fuzzy regular p-closed, then Y is fuzzy compact.*

Theorem 6.6. *If $f : X \to Y$ is fuzzy p-continuous surjection and X is fuzzy compact, then Y is fuzzy p-closed.*

Now we recall the following lemma for ready reference.

Lemma 6.7. *[6] Let Z, X, Y be fts's and $f_1 : Z \to X$ and $f_2 : Z \to Y$ be two functions. Let $f : Z \to X \times Y$ be defined by $f(z) = (f_1(z), f_2(z))$ for $z \in Z$, where $X \times Y$ is provided with the product topology. Then if B, U_1, U_2 are fuzzy sets in Z, X, Y respectively such that $f(B) \leq U_1 \times U_2$, then $f_1(B) \leq U_1$ and $f_2(B) \leq U_2$.*

Theorem 6.8. *Let Z, X, Y be fts's. For any function $f_1 : Z \to X$, $f_2 : Z \to Y$, if $f : Z \to X \times Y$, defined by $f(x) = (f_1(x), f_2(x))$ for all $x \in Z$, is fuzzy regular precontinuous, so are $f_1$and f_2.*

Proof. Let U_1 be any fuzzy open q-nbd of $f_1(x_\alpha)$ in X for any fuzzy point $x_\alpha \in Z$. Then $U_1 \times 1_Y$ is a fuzzy open q-nbd *of $f(x_\alpha)$, i.e., of $(f(x))_\alpha$ in $X \times Y$. Since f is fuzzy regular precontinuous, there exists a fuzzy regular preopen q-nbd V of x_α in Z such that $f(V) \leq U_1 \times 1_Y$. By Lemma 6.7, $f_1(V) \leq U_1$ and $f_2(V) \leq 1_Y$. Consequently, f_1 is fuzzy regular precontinuous.*

Similarly, f_2 is also fuzzy regular precontinuous. □

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