International Journal of Advances in Mathematics
Volume 2017, Number 1, Pages 30-40, 2017

On b-dislocated quasi-metric space

P Sumati Kumari*, A.H.Ansari[1] and M Sarwar[2]
*Department of Mathematics, National Institute of Technology-Andhra Pradesh, Tadepalligudem, A.P, India.
[1]Department of Mathematics, Karaj Branch, Islamic Azad University,Karaj, Iran.
[2]Department of Mathematics, University of Malakand, Chakdara Dir(L), Pakistan.

ABSTRACT. The purpose of this work is to study topological properties of b-dislocated quasi-metric space and derive some fixed point theorems. Moreover, few examples are given to illustrate the usability of the obtained results.

---

1. Introduction :

In 1922, Stefan Banach[1] established a remarkable fixed point theorem known as the "Banach Contraction Principle". This contraction principle assures the existence and uniqueness of fixed points of certain self-maps of metric spaces, and gives a constructive method to find those fixed points. Since then, many authors proved the Banach contraction principle in various generalized metric spaces. This theorem has several mathematical and real world illustrations.

In 1989, Bakhtin[2] introduced a very interesting concept "b-metric space" as an analog of a metric space.He proved the contraction mapping principle in b-metric space that generalized the renowned Banach contraction principle in metric spaces. Since then many mathematicians done several works on involving fixed point for single-valued and multi-valued operators in b-metric spaces(see for example[5-7]). Sumati[14] has established the existence of a topology induced by a dislocated quasi metric and prove fixed point theorems for dislocated quasi metric spaces.

Recently, the authors of [20] has introduced b-dislocated quasi metric space and derived related fixed point theorems by using cyclic contractions.

Motivated by the above, we establish the existence of a topology induced by a b-dislocated quasi metric space (simply b_{dq}-metric space). Then we derive pertinent unique fixed point theorems for such a space. Our main theo-

---

* Corresponding Author.
Received November14, 2016; revised December16, 2016; accepted December30, 2016.
2010 Mathematics Subject Classification:47H10, 54H25.
Key words and phrases: b-dislocated quasi-metric space, Zamfirescu contraction and $s-z$ quasi-contraction.

rems extend and unify existing results in the recent literature. Moreover, we present several examples to illustrate the theorems.

2.Preliminaries:

Definition 2.1:[2]Let X be a non empty set, let $d : X \times X \longrightarrow [0, \infty)$ and let $s \in \mathbb{R}$. Then (X, d) is said to be b-metric space if the following conditions satisfied.

$(i).d(x, y) = 0$ if and only if $x = y$ for all $x, y \in X$

$(ii).d(x, y) = d(y, x)$ for all $x, y \in X$

(iii). There exist a real number $s \geq 1$ such that $d(x, y) \leq s[d(x, z) + d(z, y)]$ for all $x, y, z \in X$

Example 2.2: Let $X = \mathbb{R}$, define $d : X \times X \to \mathbb{R}$ by $d(x, y) = (x - y)^2$. Then (X, d) is a b-metric space with coefficient $s = 2$.

The concept of b-metric space is very important as the class of b-metric space is larger than that of metric space since a b-metric space is a metric space when $s = 1$.

In [16], it was proved that each b-metric on X generates a topology $\Im$ whose base is the family of open balls $\mathcal{B}_d(x, \epsilon) = \{y \in X / d(x, y) < \epsilon\}$.

Definition 2.3[9]: Let X be a non empty set and let $d : X \times X \to \mathbb{R}$ is called a dislocated metric if the following conditions hold for all x, y, z in X.

$(i).d(x, y) \geq 0$;

$(ii).d(x, y) = d(y, x)$;

$(iii).d(x, y) = 0 = d(y, x) \Rightarrow x = y$ and

$(iv).d(x, y) \leq d(x, z) + d(z, y)$.

If d satisfies (i),(iii) and (iv) then d is called dislocated quasi metric on X and the pair (X, d) is called dislocated quasi metric space.

Examples:

1)Let $X = \mathbb{R}$,define $d : X \times X \to \mathbb{R}$ by $d(x, y) = |x| + y^2$. Then (X, d) is a dislocated quasi metric space.

2)Let $X = \mathbb{R}$,define $d : X \times X \to \mathbb{R}$ by $d(x, y) = |x|$. Then (X, d) is a dislocated quasi metric space.

In [14,15,18], Sumati and Sarma investigated some fixed point theorems in dislocated and dislocated quasi metric spaces and established the existence of topology induced by a dislocated and dislocated quasi metric respectively.

3. Topological Structures of b_{dq}-metric space:

In this section, we discuss some topological and geometric structures of b-dislocated quasi metric space.

Definition 3.1:[13] A b-dislocated quasi(simply b_{dq}) metric on a nonempty set X is a function $d : X \times X \to \mathbb{R}$ such that for all x, y, z in X and $s \in \mathbb{R}$, the following conditions hold.

$(i).b_{dq}(x, y) \geq 0$;

$(ii).b_{dq}(x, y) = 0 = b_{dq}(y, x) \Rightarrow x = y$;

(iii). There exists a real number $s \geq 1$ such that $b_{dq}(x, y) \leq s[b_{dq}(x, z) + b_{dq}(z, y)]$.

Then the pair (X, b_{dq}) is called a b_{dq}-metric space.

In [10], we note that if b_{dq} satisfies extra condition *i.e.* $b_{dq}(x,y) = b_{dq}(y,x)$ then (X, b_{dq}) is called b-dislocated metric space(simply b_d-metric space).

Example 3.2: Let $X = \{0,1,2\}$, and let $b_{dq}(0,0) = \frac{1}{2}, b_{dq}(0,1) = 0, b_{dq}(0,2) = 1, b_{dq}(1,0) = \frac{1}{2}, b_{dq}(1,1) = 0, b_{dq}(1,2) = 2, b_{dq}(2,0) = \frac{1}{2}, b_{dq}(2,1) = 1$ and $b_{dq}(2,2) = 2$ then (X, b_{dq}) is a b_{dq}-metric space with coefficient $s = 2$, but since $b_{dq}(0,1) \neq b_{dq}(1,0)$, it is not a b_d-metric space.

Obviously (X, d) is not a dislocated quasi metric space as triangle inequality does not hold.Since $b_{dq}(2,1) \nleq b_{dq}(2,0) + b_{dq}(0,1)$.

Now we start by introducing the notions of b_{dq}-limit points,b_{dq}-open set & b_{dq}-closed set which plays a distinguished role in analysis and primary results involving them in b_{dq}-metric.

Definition 3.3: Let (X, b_{dq}) be a b_{dq}-metric space and $\{x_\alpha\}_{\alpha\in\delta}$ be a net in X. We say that $\{x_\alpha\}_{\alpha\in\triangle}$ b_{dq}-converges to x in (X, b_{dq}) if $\lim_\alpha b_{dq}(x, x_\alpha) = \lim_\alpha b_{dq}(x_\alpha, x) = 0$.

In this case we write $\lim_\alpha x_\alpha = x$.

Definition 3.4: Let (X, b_{dq}) be a b_{dq}-metric space and let $A \subseteq X, x \in X$. We say that x is a b_{dq}-limit point of A if there exists a net $\{x_\alpha\}_{\alpha\in\triangle}$ in $A - \{x\}$ such that $\lim_\alpha x_\alpha = x$. The set of all b_{dq}-limit points of $A \subseteq X$ is denoted by $L(A)$.

Definition 3.5: Let (X, b_{dq}) be a b_{dq}-metric space with $\epsilon > 0, x_0 \in X$. The set $\mathcal{S}(x_0, \epsilon) = \{x/x \in X$ and $max\{b_{dq}(x_0, x), b_{dq}(x, x_0)\} < \epsilon\}$ is called b_{dq}-open ball of radius ϵ,center x_0 and $\mathcal{B}_\epsilon(x_0) = \{x_0\} \cup \mathcal{S}(x_0, \epsilon)$.
The set $\overline{\mathcal{S}}(x_0, \epsilon) = \{x/x \in X$and $max\{b_{dq}(x_0, x), b_{dq}(x, x_0)\} \leq \epsilon\}$ is called b_{dq}-closed ball of radius ϵ,center x_0 and $\overline{\mathcal{B}}_\epsilon(x_0) = \{x_0\} \cup \overline{\mathcal{S}}(x_0, \epsilon)$.

Above similar argument can be found in [21,22].

Remark 3.6:

1. b_{dq}-open balls are not necessarily non empty. For example, in example 3.2, the set $\mathcal{S}(2, \frac{1}{2})$ does not contain 2.
2. $\lim_\alpha x_\alpha = x$ iff for every $r > 0$, there exists $r_0 \in \triangle$ such that $x_\alpha \in \mathcal{S}(x, r)$ for all $\alpha \geq r_0$.
3. b_{dq}-limit point of a net is unique.

Proposition 3.7: Let $A, B \subseteq X$ then
$(i).L(A) = \phi$ if $A = \phi$
$(ii).L(A) \subseteq L(B)$ if $A \subseteq B$
$(iii).L(A \cup B) = L(A) \cup L(B)$
$(iv).L(L(A)) \subseteq L(A)$

Proof:To prove $(i), (ii)$ and (iii) we can refer to [13]. To prove (iv), suppose $x \in L(L(A))$. Then there exists a net $\{x_\alpha : \alpha \in \triangle\}$ in $L(A)$ such that $x = \lim_\alpha x_\alpha$. Since x_α is in $L(A)$, there exists another net $\{x_{\alpha\beta} : \beta \in \triangle(\alpha)\}$ in A such that $x_\alpha = \lim_\beta x_{\alpha\beta}$. For each positive integer i, there exists $\alpha_i \in \triangle$ such that $b_{dq}(x_{\alpha_i}, x) = b_{dq}(x, x_{\alpha_i}) < \frac{1}{2is}$ and $\beta_i \in \triangle(\alpha_i)$ such that $b_{dq}(x_{\alpha_{i\beta_i}}, x_{\alpha_i}) = b_{dq}(x_{\alpha_i}, x_{\alpha_{i\beta_i}}) < \frac{1}{2is}$. If we write $\alpha_{i\beta_i} = \gamma_i$ for all i, then $\{\gamma_1, \gamma_2, ...\}$ is

directed set with $\gamma_i < \gamma_j$ if $i < j$ and $b_{dq}(x_{\gamma_i}, x) \leq s[b_{dq}(x_{\gamma_i}, x_{\alpha_i}) + b_{dq}(x_{\alpha_i}, x)] < \frac{1}{i}$
This implies $x \in L(A)$.

Corollary 3.8: If we write $\overline{A} = A \cup L(A)$ for $A \subset X$ the operation $A \to \overline{A}$ satisfies Kuratowski's closure axioms[12] so that the set $\Im = \{A/A \subset X$ and $\overline{A^c} = A^c\}$ is a topology on X.

Corollary 3.9: We call $(X, b_{dq}, \Im)$ the topological space induced by b_{dq}. We call $A \subset X$ to be closed if $\overline{A} = A$ and open if $A \in \Im$.

Now we state some properties and corollaries in $(X, b_{dq}, \Im)$ which can be proved following similar arguments to those given in [14].

Proposition 3.10: Let (X, b_{dq}) be a b_{dq}-metric space.$x \in X$ is a b_{dq}-limit point of $A \subset X$ iff for every $r > 0, A \cap \mathcal{S}(x, r) \neq \phi$.

Corollary 3.11: $x \in \overline{A}$ if and only if $x \in A$ or $\mathcal{S}(x, r) \cap A \neq \phi$, for all $r > 0$.

Corollary 3.12: $A \subseteq X$ is open in $(X, b_{dq}, \Im)$ iff for every $x \in A$, there exists $\delta > 0$ such that $\mathcal{B}_\delta(x) \subseteq A$.

Proposition 3.13: If $x \in X$ and $\delta > 0$, then $\mathcal{B}_\delta(x)$ is an open set in $(X, b_{dq}, \Im)$.

Proof: Let $y \in \mathcal{B}_\delta(x)$ and $0 < r < \min\{\frac{\delta}{s} - b_{dq}(x, y), \frac{\delta}{s} - b_{dq}(y, x)\}$. Then $\mathcal{B}_r(y) \subset \mathcal{B}_\delta(x)$.
Hence $\mathcal{B}_\delta(x)$ is open.

Proposition 3.14:$(X, b_{dq}, \Im)$ is a Hausdorff space.

Proof: Let $x \neq y$, we can assume $b_{dq}(x, y) > 0$. Choose $\delta > 0$ such that $2\delta < b_{dq}(x, y)$. Then $\mathcal{B}_{\frac{\delta}{2s}}(x) \cap \mathcal{B}_{\frac{\delta}{2s}}(y) = \phi$.

Corollary 3.15: If $x \in X$, then the collection $\{\mathcal{B}_r(x)/x \in X\}$ is an open base at x in $(X, b_{dq}, \Im)$. Hence $(X, b_{dq}, \Im)$ is first countable.

The above corollary yields us to deal with sequences instead of nets.

Definition 3.16: A sequence $\{x_n\}$ in a b_{dq}-metric space is called a b_{dq}-Cauchy sequence if given $\epsilon > 0$, there exists $n_0 \in N$ such that for all $n, m \geq n_0$, we have $b_{dq}(x_n, x_m) < \epsilon$ and $b_{dq}(x_m, x_n) < \epsilon$ or $\lim_{n,m\to\infty} b_{dq}(x_n, x_m) = 0 = \lim_{n,m\to\infty} b_{dq}(x_m, x_n)$.

Proposition 3.17: Every b_{dq}-convergent sequence in a b_{dq}-metric space is b_{dq}-Cauchy.

Definition 3.18: A b_{dq}-metric space (X, b_{dq}) is called complete if every b_{dq}-Cauchy sequence in X is b_{dq}-convergent.

4.Main Results:

Theorem 4.1: Let (X, b_{dq}) be a complete b_{dq}-metric space with the coefficient $s \geq 1$, and let $T : X \to X$ be a mapping such that

$$b_{dq}(Tx, Ty) \leq \phi(b_{dq}(x, y)) \tag{1}$$

for all $x, y \in X$, where $\phi : [0, +\infty) \to [0, +\infty)$ is a continuous mapping such that $\phi(t) = 0$ iff $t = 0$ and $\phi(t) < t$ for all $t > 0$. If $\sum_{n=1}^{\infty} s^n \phi^n(t)$ b_{dq}-converges for all $t > 0$, where ϕ^n is the n^{th} iterate of ϕ, then T has a unique fixed point.

Proof: Let x_0 be an arbitrary point in X. Define the iterative sequence $\{x_n\}$ as follows:
$x_1 = T(x_0), x_2 = T(x_1), ..., x_{n+1} = T(x_n), ...$
If we assume that $x_{n+1} = x_n$ for some $n \in N$,then we have $x_n = x_{n+1} = T(x_n)$, so x_n is a fixed point of T and the proof is completed. From now on we will assume that for each $n \in N, x_{n+1} \neq x_n$.

$b_{dq}(x_n, x_{n+1}) = b_{dq}(Tx_{n-1}, Tx_n) \leq \phi b_{dq}(x_{n-1}, x_n)$
By continuing this process,

$$\begin{aligned} b_{dq}(x_n, x_{n+1}) &\leq \phi b_{dq}(x_{n-1}, x_n) \\ &\leq \phi^2 b_{dq}(x_{n-2}, x_{n-1}) \\ &\vdots \\ &\leq \phi^n b_{dq}(x_0, x_1) \text{ , for n} > 1. \end{aligned} \tag{2}$$

similarly, we can get

$$\begin{aligned} b_{dq}(x_{n+1}, x_n) &\leq \phi b_{dq}(x_n, x_{n-1}) \\ &\leq \phi^2 b_{dq}(x_{n-1}, x_{n-2}) \\ &\vdots \\ &\leq \phi^n b_{dq}(x_1, x_0) \text{ , for n} > 1. \end{aligned} \tag{3}$$

If $b_{dq}(x_0, Tx_0) = 0$ and $b_{dq}(Tx_0, x_0) = 0$ then $x_0 = Tx_0$. Which yields x_0 is a fixed point of T. Suppose that $b_{dq}(x_0, Tx_0) > 0$ and $b_{dq}(Tx_0, x_0) > 0$.

Now we prove that $\{x_n\}$ is a b_{dq}-Cauchy sequence. For all $m, n > 0$ and $p \in N$.
We have

$$\begin{aligned} b_{dq}(x_n, x_{n+p}) &\leq s[b_{dq}(x_n, x_{n+1}) + b_{dq}(x_{n+1}, x_{n+p})] \\ &\leq s b_{dq}(x_n, x_{n+1}) + s^2 b_{dq}(x_{n+1}, x_{n+2}) + s^2 b_{dq}(x_{n+2}, x_{n+p}) \\ &\leq s b_{dq}(x_n, x_{n+1}) + s^2 b_{dq}(x_{n+1}, x_{n+2}) + s^3 b_{dq}(x_{n+2}, x_{n+3}) \\ &\quad + ... + s^p b_{dq}(x_{n+p-1}, x_{n+p}) \end{aligned} \tag{4}$$

From (2)&(4), we get,

$$\begin{aligned} b_{dq}(x_n, x_{n+p}) &\leq s\phi^n b_{dq}(x_0, x_1) + s^2 \phi^{n+1} b_{dq}(x_0, x_1) + ... + s^p \phi^{n+p-1} b_{dq}(x_0, x_1) \\ &\leq s^n \phi^n b_{dq}(x_0, x_1) + s^{n+1} \phi^{n+1} b_{dq}(x_0, x_1) + ... + s^{n+p-1} \phi^{n+p-1} b_{dq}(x_0, x_1) \\ &= \sum_{k=n}^{n+p-1} s^k \phi^k b_{dq}(x_0, x_1) \end{aligned}$$

Since $\sum_{n=1}^{\infty} s^n \phi^n(t)$ b_{dq}-converges for all $t > 0$, then $\lim_{n\to\infty} b_{dq}(x_n, x_{n+p}) = 0$, which means that for $m > n$,

$$\lim_{n,m\to\infty} d(x_n, x_m) = 0 \tag{5}$$

Similarly, we can get $\lim_{n,m\to\infty} d(x_m, x_n) = 0$.
Thus $\{x_n\}$ is a b_{dq}-Cauchy sequence. Since (X, b_{dq}) is complete b_{dq}-metric space, we have the sequence $\{x_n\}$ b_{dq}-converges to some point $z \in X$.
Which yields, $\lim_{n\to\infty} b_{dq}(x_n, z) = \lim_{n\to\infty} b_{dq}(z, x_n) = 0$
Now we prove that z is a fixed point of T.

By using Definition 3.1, consider

$$\begin{aligned} b_{dq}(z,Tz) &\leq s[b_{dq}(z,x_{n+1}) + b_{dq}(x_{n+1},Tz)] \\ &= s[b_{dq}(z,x_{n+1}) + b_{dq}(Tx_n,Tz)] \\ &\leq sb_{dq}(z,x_{n+1}) + s\phi b_{dq}(x_n,z) \end{aligned}$$

By taking limits, $b_{dq}(z,Tz) = 0$

Similarly, $b_{dq}(Tz,z) = 0$

Hence $Tz = z$.

Uniqueness:Now we show that z is a unique fixed point of T.

Suppose there is another fixed point z^* of T such that $Tz^* = z^*$. To prove this,we claim $b_{dq}(z,z^*) = 0$.

Suppose not, then $b_{dq}(z,z^*) = b_{dq}(Tz,Tz^*) \leq \phi b_{dq}(z,z^*) < b_{dq}(z,z^*)$, which is a contradiction.

Similarly, $b_{dq}(z^*,z) = 0$.

Which implies $z = z^*$.

Hence z is a unique fixed point of T.

By taking $\phi(t) = \lambda t$ with $0 \leq \lambda < \frac{1}{s}$, we can get the following corollary which generalizes the famous Banach contraction principle in b_{dq}-metric space.

Corollary 4.2: Let (X,b_{dq}) be a complete b_{dq}-metric space with the coefficient $s \geq 1$, and let $T : X \to X$ be a continuous mapping such that $b_{dq}(Tx,Ty) \leq \lambda b_{dq}(x,y)$

for all $x,y \in X$, where $0 \leq \lambda < \frac{1}{s}$. Then T has a unique fixed point in X.

Corollary 4.3: Let (X,b_{dq}) be a complete b_{dq}-metric space. Let $T : X \to X$ be a continuous self mapping such that $b_{dq}(Tx,Ty) \leq \alpha \frac{b_{dq}(x,Tx).b_{dq}(y,Ty)}{b_{dq}(x,y)} + \beta b_{dq}(x,y)\ \forall x,y \in X, x \neq y$ where $0 \leq \alpha+\beta < \frac{1}{s}$. Then T has a unique fixed point.

Now, we will illustrate the validity of theorem 4.1 by giving below example.

Example 4.4: Let $X = \{0,1,2\}$.Define $b_{dq} : X \times X \to [0,\infty)$ as follows.

$d(0,0) = \frac{1}{2}, b_{dq}(0,1) = 0, b_{dq}(0,2) = 1, b_{dq}(1,0) = \frac{1}{2}, b_{dq}(1,1) = 0, b_{dq}(1,2) = 2, b_{dq}(2,0) = \frac{1}{2}, b_{dq}(2,1) = 1$ and $b_{dq}(2,2) = 2$. Then (X,b_{dq}) is a complete b_{dq}-metric space with the coefficient $s = 2$.

Define the mapping $T : X \to X$ by $T0 = 1, T1 = 1, T2 = 0$ and $\phi : [0,\infty) \to [0,\infty)$ by $\phi(t) = \frac{t}{t+1}$ then we have the following cases.

Case.1: For $x = y = 0$,

we get $b_{dq}(Tx,Ty) = b_{dq}(1,1) = 0 \leq \phi(b_{dq}(0,0)) = \frac{1}{3}$.

Case.2: If $x = 0$ and $y = 1$,

$b_{dq}(Tx,Ty) = b_{dq}(1,1) = 0 \leq \phi(b_{dq}(0,1)) = 0$.

Case.3: If $x = 0$ and $y = 2$,

$b_{dq}(Tx,Ty) = b_{dq}(1,0) = \frac{1}{2} \leq \phi(b_{dq}(0,2)) = \frac{1}{2}$.

Case.4: If $x = 1$ and $y = 0$,

$b_{dq}(Tx,Ty) = b_{dq}(1,1) = 0 \leq \phi(b_{dq}(1,0)) = \frac{1}{3}$.

Case.5: If $x = y = 1$

$b_{dq}(Tx,Ty) = b_{dq}(1,1) = 0 \leq \phi(b_{dq}(1,1)) = 0$.

Case.6: If $x = 1$ and $y = 2$,
$b_{dq}(Tx, Ty) = b_{dq}(1, 0) = \frac{1}{2} \leq \phi(b_{dq}(1, 2)) = \frac{2}{3}$.
Case.7: If $x = 2$ and $y = 0$,
$b_{dq}(Tx, Ty) = b_{dq}(0, 1) = 0 \leq \phi(b_{dq}(2, 0)) = \frac{1}{3}$.
Case.8: If $x = 2$ and $y = 1$,
$b_{dq}(Tx, Ty) = b_{dq}(0, 1) = 0 \leq \phi(b_{dq}(2, 1)) = \frac{1}{2}$.
Case.9: If $x = 2$ and $y = 2$,
$b_{dq}(Tx, Ty) = b_{dq}(0, 0) = \frac{1}{2} \leq \phi(b_{dq}(2, 2)) = \frac{2}{3}$.
Hence T, ϕ satisfies all the conditions of theorem 4.1.
Thus T has a unique fixed point.In fact '1' is the unique fixed point of T.

If we take $s = 1$ in theorem 4.1, we obtain the following corollary in the setting of dislocated quasi metric space.

Corollary 4.5: Let (X, d) be a complete dislocated quasi metric space and let $T : X \to X$ be a mapping such that $d(Tx, Ty) \leq \phi(d(x, y))$ for all $x, y \in X$, where $\phi : [0, +\infty) \to [0, +\infty)$ is a continuous mapping such that $\phi(t) = 0$ iff $t = 0$ and $\phi(t) < t$ for all $t > 0$. If $\sum_{n=1}^{\infty} \phi^n(t)$ b_{dq}-converges for all $t > 0$, where ϕ^n is the n^{th} iterate of ϕ, then T has a unique fixed point.

The following example shows that corollary 4.5 is a proper generalization.

Example 4.6: Let $X = [0, 1]$. Define $d : X \times X \to \mathbb{R}$ by $d(x, y) = x$. Then (X, d) is a dislocated quasi metric space.
Define $T : X \to X$ by $Tx = \frac{x}{2}$ then we have following cases.
Case(i) : If $x = y = 0$ then $d(Tx, Ty) = d(0, 0) = 0 \leq \phi(d(x, y)) = \phi(0) = 0$.
Case(ii) : If $x = 0$ and $0 < y \leq 1$ then
$d(Tx, Ty) = d(0, \frac{y}{2}) = 0 \leq \phi(d(0, y)) = \phi(0) = 0$.
Case(iii) : If $y = 0$ and $0 < x \leq 1$ then
$d(Tx, Ty) = d(\frac{x}{2}, 0) = \frac{x}{2} \leq \phi(d(x, 0)) = \phi(x) = \frac{x}{1+x}$.
Case(iv) : If $x > 0, y > 0$ and $x > y$ $(or\ x < y)$ then
$d(Tx, Ty) = d(\frac{x}{2}, \frac{y}{2}) = \frac{x}{2}$
$\phi(d(x, y)) = \phi(x) = \frac{x}{1+x}$.
Hence $d(Tx, Ty) \leq \phi(d(x, y))$.
Case(v) : If $x = 1, y = 1$ then $d(Tx, Ty) = d(\frac{1}{2}, \frac{1}{2}) = \frac{1}{2}$
$\phi(d(x, y)) = \phi(1) = \frac{1}{2}$.
Case(vi) : If $x = 1$ and $0 < y < 1$ then $d(Tx, Ty) = d(\frac{1}{2}, \frac{y}{2}) = \frac{1}{2}$
$\phi(d(x, y)) = \phi(1) = \frac{1}{2}$.
Case(vii) : If $y = 1$ and $0 < x < 1$ then $d(Tx, Ty) = d(\frac{x}{2}, \frac{1}{2}) = \frac{x}{2}$
$\phi(d(x, y)) = \phi(x) = \frac{x}{1+x}$.
Thus all conditions of corollary are satisfied and T has a unique fixed point in X. In fact '0' is the unique fixed point.

Historically, the below significant contractions, as originally defined by their respective authors.

Banach[1] :Let (X, d) be a metric space, let A be a subset of X, and $f : A \to X$, be a mapping. Then f is a contractive if there exists $\alpha \in [0,1)$ such that, for all $x, y \in A$,

$$d(f(x), f(y)) \leq \alpha d(x, y) \tag{6}$$

The famous Banach fixed point theorem contends that if $A = X$, f is contractive and (X, d) is complete, then f has a unique fixed point.

Above theorem has been extended by many famous authors to some classes of maps which do not satisfy the contractive condition (6).

For example, below mentioned two contractive conditions are proper generalizations of Banach's fixed point theorem.

(**Kannan**[11]) There exists $\alpha \in [0,1)$ such that, for all $x, y \in X$,

$$d(f(x), f(y)) \leq \frac{\alpha}{2}[d(x, f(x)) + d(y, f(y))] \tag{7}$$

(**Chatterjea**[8]) There exists $\alpha \in [0,1)$ such that, for all $x, y \in X$,

$$d(f(x), f(y)) \leq \frac{\alpha}{2}[d(x, f(y)) + d(y, f(x))] \tag{8}$$

After these three results, a huge number of papers have been written by several authors to generalize, extend and improve some of the conditions (6),(7) or (8), or even the three conditions simultaneously.

In 1972, Zamfirescu[19] consolidate the (6,7,8) conditions which is known as Zamfirescu contractive condition and proved a fixed point theorem. In [17],Rhoades state below conditions,

(**Rhoades 19$'$**[17])There exist non negative functions a, b, c satisfying

$\sup_{x,y\in X}\{a(x,y) + 2b(x,y) + 2c(x,y)\} \leq \lambda < 1$ such that, for each $x, y \in X$,

$$d(f(x), f(y)) \leq a(x,y)d(x,y) + b(x,y)[d(x, f(x)) + d(y, f(y))] + c(x,y)[d(x, f(y)) + d(y, f(x))] \tag{9}$$

(**Rhoades 19$''$**[17])There exist a constant h, $0 \leq h < 1$, such that for each $x, y \in X$,

$$d(f(x), f(y)) \leq h\ max\{d(x,y),\ \frac{[d(x, f(x)) + d(y, f(y))]}{2},\ \frac{[d(x, f(y)) + d(y, f(x))]}{2}\} \tag{10}$$

Moreover, he proved Zamfirescus condition is equivalent to Rhodes 19$'$&Rhodes 19$''$ conditions. Subsequently, many authors gave different generalizations of Zamfirescus theorem.(see for example[3,4]).

However, recently Berinde[4] showed that Banachs, Kannans, Chatterjeas and Zamfirescus mappings are weak contractions.

Based on the definition of *Zamfirescu Contraction*, we introduce the following definition in the setting of b_{dq}-metric space.

Definition 4.7: Let (X, b_{dq}) be a complete b_{dq} metric space with the coefficient $s > 1$. If $f : X \to X$ be a mapping such that for each $x, y \in X$ at least one of the following is true:

1. $s^2 b_{dq}(f(x), f(y)) \leq \alpha b_{dq}(x, y)$, $0 \leq \alpha < s$
2. $s^2 b_{dq}(f(x), f(y)) \leq \beta[b_{dq}(x, f(x)) + b_{dq}(y, f(y))]$, $0 \leq \beta < \frac{s^2}{s+1}$
3. $s^2 b_{dq}(f(x), f(y)) \leq \gamma[b_{dq}(x, f(y)) + b_{dq}(y, f(x))]$, $0 \leq \gamma < \frac{s^2}{s(1+\delta)+1}$
   where $\delta = max\{\frac{2\alpha}{s^2}, \frac{2\beta}{s^2}, \frac{2\gamma}{s^2}\}$.

Then f is called *"s-z quasi contraction"*.

Theorem 4.8: Let (X, b_{dq}) be a complete b_{dq}-metric space with the coefficient $s > 1$. If $f : X \to X$ is a continuous *s-z quasi contraction* then f has a unique fixed point in X.

Proof:By taking $y = x$ in the above and $\delta = max\{\frac{2\alpha}{s^2}, \frac{2\beta}{s^2}, \frac{2\gamma}{s^2}\}$ yields

$$b_{dq}(f(x), f(x)) \leq \delta b_{dq}(x, f(x)) \tag{11}$$

Again putting $y = f(x)$ in the above (1),(2) and (3) of Definition 4.7 gives,

$$b_{dq}(f(x), f^2(x)) \leq \frac{\alpha}{s^2} b_{dq}(x, f(x))$$

$$b_{dq}(f(x), f^2(x)) \leq \frac{\beta}{s^2-\beta} b_{dq}(x, f(x))$$

$$b_{dq}(f(x), f^2(x)) \leq \frac{\gamma(1+\delta)}{s^2-\gamma} b_{dq}(x, f(x))$$

Choose $h = max\{\frac{\alpha}{s^2}, \frac{\beta}{s^2-\beta}, \frac{\gamma(1+\delta)}{s^2-\gamma}\}$ and $0 \leq h < \frac{1}{s}$ then we get

$$b_{dq}(f(x), f^2(x)) \leq h b_{dq}(x, f(x)) \tag{12}$$

By repeating this procedure,we obtain

$$b_{dq}(f^n(x), f^{n+1}(x)) \leq h^n b_{dq}(x, f(x)) \tag{13}$$

Since $0 \leq h < 1$, $\lim_{n\to\infty} b_{dq}(f^n(x), f^{n+1}(x)) = 0$ Now we prove that $f^n(x)$ is a b_{dq}-Cauchy sequence.

To do this, let m, n are positive integers such that $m > n$ then by using the definition of b_{dq}-metric space, we get

$$\begin{aligned} b_{dq}(f^n(x), f^m(x)) &\leq s[b_{dq}(f^n(x), f^{n+1}(x)) + b_{dq}(f^{n+1}(x), f^m(x))] \\ &\leq s b_{dq}(f^n(x), f^{n+1}(x)) + s^2 b_{dq}(f^{n+1}(x), f^{n+2}(x)) + s^3 b_{dq}(f^{n+2}(x), f^{n+3}(x)) + ... \\ &\leq s h^n b_{dq}(x, f(x)) + s^2 h^{n+1} b_{dq}(x, f(x)) + ... \\ &\leq s h^n [1 + sh + (sh)^2 + ...] b_{dq}(x, f(x)) \\ &\leq \frac{sh^n}{1 - sh} b_{dq}(x, f(x)) \end{aligned} \tag{14}$$

Applying the limit as $n, m \to \infty$, we get, $b_{dq}(f^n(x), f^m(x)) \to 0$ as $hs < 1$.

Similarly, letting limit as $n, m \to \infty$, $b_{dq}(f^m(x), f^n(x)) \to 0$.

Thus $f^n(x)$ is a b_{dq}-Cauchy sequence in complete b_{dq}-metric space.

Which implies there exists some $u \in X$ such that

$\lim_{n\to\infty} b_{dq}(f^n(x), u) = \lim_{n\to\infty} b_{dq}(u, f^n(x)) = 0$.

Since f is continuous mapping, we get $f(u) = f(\lim_{n\to\infty}(f^n(x)) = \lim_{n\to\infty} f^{n+1}(x)$.

Thus $f(u) = \lim_{n\to\infty} f(f^n(x))$.

$b_{dq}(u, fu) \leq s[b_{dq}(u, f^{n+1}(x)) + b_{dq}(f^{n+1}(x), fu)]$

By taking limit as $n \to \infty$, we get $b_{dq}(u, fu) = 0$. Similarly $b_{dq}(fu, u) = 0$.

Hence $fu = u$.

Uniqueness: Suppose u and v are two fixed points of f.

If $u = v$ then there is a unique fixed point point of f.

Let us assume that $u \neq v$, thus,

$s^2 b_{dq}(u, v) = s^2 b_{dq}(f(u), f(v)) \leq \alpha d(u, v)$

$\Rightarrow (s^2 - \alpha)d(u, v) \leq 0.$

Since $s^2 - \alpha > 0$ implies $d(u, v) = 0$.

Similarly $d(v, u) = 0$. Thus $u = v$.

Hence f has a unique fixed point.

In theorem 4.8, by taking $s = 1$ we can derive the following corollary.

Corollary 4.9: Let (X, d) be a complete dislocated quasi metric space. If $f : X \to X$ be a continuous mapping such that for each $x, y \in X$ at least one of the following is true.

1. $d(f(x), f(y)) \leq \alpha d(x, y)$, $0 \leq \alpha < 1$
2. $d(f(x), f(y)) \leq \beta[d(x, f(x)) + d(y, f(y))]$, $0 \leq \beta < \frac{1}{2}$
3. $d(f(x), f(y)) \leq \gamma[d(x, f(y)) + d(y, f(x))]$, $0 \leq \gamma < \frac{1}{2+\delta}$

Then f has a unique fixed point.

Following example illustrates above corollary.

Example 4.10: Let $X = [0, 1]$, define $d : X \times X \to \mathbb{R}$ by $d(x, y) = |x|$. Then d is complete dislocated quasi metric. Let $f : X \to X$ be a mapping, defined by $fx = \frac{x}{2}$, then f satisfies (1) of corollary4.9 but f does not satisfy (2) of corollary4.9, since $d(f(1), f(0)) \not\leq \beta[d(1, \frac{1}{2}) + d(0, 0)]$.

Hence f satisfies all the conditions of the above corollary and '0' is the unique fixed point of f in X.

Conclusion: In this work, we discussed topological properties of b-dislocated quasi metric space and introduced s-z-quasi-contraction. Also, we derived the existence of fixed point theorems for b-dislocated quasi metric space. Moreover, some examples are provided wherever necessary. Our results may be the motivation to other authors for extending and improving these results to be suitable tools for their applications.

Acknowledgements: The author would like to express her sincere gratitude to M.Anand Prabhakar for his invaluable support and motivation.

References:

[1] S. Banach, Sur les operations dans les ensembles abstraits et leur applications aux equations integrales, Fundamenta Math., 3 (1922), pp. 133-181.

[2] I. A. Bakhtin, The contraction mapping principle in quasimetric spaces, Functional Analysis (Ulyanovsk), vol. 30, pp. 26-37, 1989.

[3] V. Berinde, Approximating fixed points of weak contractions using the Picard iteration, Nonlinear Anal. Forum 9 (2004), 43-53.

[4] V. Berinde, On the approximation of fixed points of weak contractive mappings, Carpathian J. Math. 19 (2003), 7-22.

[5] M Boriceanu , M Bota , A Petrusel , Mutivalued fractals in b-metric spaces, Cen. Eur. J. Math. 8(2)(2010),

367-377.
[6] M Boriceanu , Strict fixed point theorems for multivalued operators in b-metric spaces, International J. Modern Math., 4(3),285-301, 2009.
[7] M Bota , A Molnar and V Csaba, On Ekelands variational principle in b-metric spaces, Fixed Point Theory, 12(2011), 21-28.
[8] S.K. Chatterjea, Fixed-point theorems, C. R. Acad. Bulgare Sci. 25 (1972), 727-730.
[9] P. Hitzler. Generalized metrics and topology in logic programming semantics. Ph. D Thesis, School of Mathematics, Applied Mathematics and Statistics, National University Ireland, University college Cork, 2001.
[10] Hussain et al: Common fixed point results for weak contractive mappings in ordered b-dislocated metric spaces with applications. Journal of Inequalities and Applications 2013, 2013:486
[11] R. Kannan, Some results on fixed points, Bull. Calcutta Math. Soc. 60 (1968), 71-76.
[12] J. L. Kelley. General Topology. D. Van Nostrand Company, Inc., 1960.
[13] C Klin-eam and C Suanoom, Dislocated quasi-b-metric spaces and fixed point theorems for cyclic contractions, Fixed point Theory and Applications 2015, 2015:74.
[14] Kumari PS. On dislocated quasi metrics. Journal of Advanced Studies in Topology. 2012; 3(2):66-74.
[15] Kumari et al.: Common fixed point theorems on weakly compatible maps on dislocated metric spaces. Mathematical Sciences 2012 6:71.
[16] M.A Kutbi,Marshad,J.Ahmad and A.Azam, Generalized common fixed point results with applications,Abstract and Applied Analysis,Volume 2014,article ID 363925,7 pages.
[17] B. E. Rhoades, A comparison of various definitions of contractive mappings, Transactions of the American Mathematical Society, vol. 226, pp. 257-290, 1977.
[18] IR Sarma , PS Kumari . On dislocated metric spaces. International Journal of Mathematical Achieve. 2012; 3(1):727.
[19] T. Zamfirescu, Fixed point theorems in metric spaces, Arch. Math. 23 (1972), 292-298.
[20] C. Klin-eam, C. Suanoom, Dislocated quasi-b-metric spaces and fixed point theorems for cyclic contractions, Fixed Point Theory and Applications (2015) 2015:74.
[21]Kumari, P. Sumati, and Muhammad Sarwar. "Some fixed point theorems in generating space of b-quasi-metric family." SpringerPlus 5.1 (2016): 1-15.
[22] Kumari, Panda Sumati, and Dinesh Panthi. "Cyclic contractions and fixed point theorems on various generating spaces." Fixed Point Theory and Applications 1.2015 (2015): 1-17.