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# Convergence and almost sure (S,T)-stability for random iterative schemes

R. A. Rashwan[1]*, H. A. Hammad[2] and G. A. Okeke[3]

[1]Department of Mathematics, Faculty of Science, Assuit University, Assuit 71516, Egypt

E-mail: rr_rashwan54@yahoo.com

[2]Department of Mathematics, Faculty of Science, Sohag University, Sohag 82524, Egypt

E-mail:h_elmagd89@yahoo.com

[3]Department of Mathematics, Michael Okpara University of Agriculture, Umudike, 7267, Umuahia, Nigeria

E-mail: ga.okeke@mouau.edu.ng

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ABSTRACT. In this paper, we study the convergence and almost sure $(S,T)-$stability of Jungck-Noor type, Jungck-SP type, Jungck-Ishikawa type and Jungck-Mann type random iterative algorithms for some kind of a general contractive type random operators (2.14) in a separable Banach spaces. The Bochner integrability of random fixed point of this kind of random operators, the convergence and almost sure $(S,T)-$stability for these kind of random iterative algorithms under condition (18) are obtained. Our results are stochastic generalizations of Zhang et al. [1], Okeke and Eke [2] and many others in deterministic verse.

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## 1. Introduction

It is known that random fixed point theorems are stochastic generalization of classical fixed point theorems which are called deterministic results. These theorems was initiated in 1950s by Prague school of probabilistic. Many authors are impressed by random fixed point theory especially, when Bharucha-Reid [3, 4] presented his papers which lead to the development of these theorems. Interests in random fixed point theory stems in its vast applicability in stochastic functional analysis and various probabilistic models.

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* Corresponding Author.

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There are many new questions of measurability, probabilistic and statistical aspects of random solution were answered by the introduction of randomness. After papers of Bharucha-Reid, Špaček [5] and Hanš [6] established stochastic analogue of the Banach fixed point theorem in a separable metric space. Itoh [7] in 1979, generalized and extended Špaček and Hanš's theorem to a multivalued contraction random operators. Papageorgiou [8] proved several random fixed point theorems for measurable closed and nonclosed valued multifunctions satisfying general continuity conditions. His results improved the results of Engl [9], Itoh [7] and Reich [10]. In 1999, Shahzad and Latif [11] introduced a general random fixed point theorem for continuous random operators. As applications, they stated and proved a number of random fixed points theorems for various classes of 1-set and 1-ball contractive random operators. Chang et al. [12], Beg and Abbas [13] proved some convergence theorems of random Ishikawa scheme and random Mann iterative scheme for strongly pseudo-contractive operators and contraction operators, respectively, in separable reflexive Banach spaces.

Recently, Zhang et al. [1] studied the almost sure $T-$ stability and convergence of Ishikawa-type and Mann-type random algorithms for certain $\phi-$weakly contractive type random operators in a separable Banach space. They established the Bochner integrability of random fixed point for this kind of random operators and the almost sure $T-$stability and convergence for these two kinds of random iterative algorithms under suitable conditions. Okeke and Abbas [14], studied convergence and almost sure $T-$stability for a random iterative sequence generated by a generalized random operator. Okeke and Kim [15], introduced convergence and summable almost $T-$stability of the random Picard-Mann hybrid iterative process. Okeke and Eke [2], extended the results of Zhang by introducing a Noor-type random iterative scheme and studying the same results. These results are stochastic generalization of the deterministic fixed point theorems of Berinde [16, 17] and Rhoades [18, 19].

Meshra [30] studied some problems on approximations of functions in Banach spaces. Meshra et al. [28, 29, 31] proved fixed point theorems for generalized contractive and $S$-contractive mappings in partial metric spaces, in general they introduced Trigonometric approximation of signals (Functions) in $L_p(p \geq 1)$-norm. As application Deepmala [32] introduced study on fixed point theorems for nonlinear contractions while Rashwan and Hammad [33] studied random fixed point theorems with an application to a random nonlinear integral equation.

In 2005, Singh et al. [20] proved the stability of Jungck type iterative procedure as:

**Definition 1.1.** *(Jungck-Mann iteration process)* Let $(X, \|.\|)$ be a normed linear space and $Y$ be arbitrary set $S, T : Y \to X$ such that $T(Y) \subseteq S(Y)$, then for $x_\circ \in Y$, the sequence $\{Sx_n\}_{n=0}^{\infty}$ defined by

$$Sx_{n+1} = (1 - \alpha_n)Sx_n + \alpha_n Tx_n, \ n \geq 0, \tag{1.1}$$

where $\{\alpha_n\}_{n=0}^{\infty}$ is a sequence of real numbers in $[0, 1]$.

**Remark 1.1.** If we put $S = I$ (where $I$ is the identity mapping), $Y = X$, in (1.1), we obtain Mann iteration process [21].

In 2008, Great work published by Olatinwo and Imoru [22] which shows that convergence results of Jungck-Ishikawa iterations as:

**Definition 1.2.** *(Jungck-Ishikawa iteration process)* Let $(X, \|.\|)$ be a Banach space and $Y$ be arbitrary set. Let $S, T :$

$Y \to X$ be a nonself mappings such that $T(Y) \subseteq S(Y)$, $S(Y)$ is a complete subspace of $X$ and $S$ is injective, then for $x_\circ \in Y$, define the sequence $\{Sx_n\}_{n=0}^{\infty}$ iteratively by

$$\begin{cases} Sx_{n+1} = (1-\alpha_n)Sx_n + \alpha_n Ty_n \\ Sy_n = (1-\beta_n)Sx_n + \beta_n Tx_n, \end{cases} \tag{1.2}$$

where $\{\alpha_n\}_{n=0}^{\infty}$ and $\{\beta_n\}_{n=0}^{\infty}$ are real sequences in $[0,1]$.

**Remark 1.2.**

(i)If we take $S = I$ and $Y = X$ in (1.2), we have the Ishikawa iteration process [23].

(ii) Taking $\beta_n = 0$ in (1.2), we get Jungck-Mann iterative scheme (1.1).

The convergence results using Jungck-Noor three step iteration scheme were introduced by Olatinwo [24] as:

**Definition 1.3.** *(Jungck-Noor iteration process)* Let $(X, \|.\|)$ be a Banach space and $Y$ be arbitrary set. Let $S, T : Y \to X$ be a nonself mappings such that $T(Y) \subseteq S(Y)$, $S(Y)$ is a complete subspace of $X$ and $S$ is injective, then for $x_\circ \in Y$, define the sequence $\{Sx_n\}_{n=0}^{\infty}$ iteratively by

$$\begin{cases} Sx_{n+1} = (1-\alpha_n)Sx_n + \alpha_n Tz_n \\ Sz_n = (1-\beta_n)Sx_n + \beta_n Ty_n \\ Sy_n = (1-\gamma_n)Sx_n + \gamma_n Tx_n, \end{cases} \tag{1.3}$$

where $\{\alpha_n\}_{n=0}^{\infty}$, $\{\beta_n\}_{n=0}^{\infty}$ and $\{\gamma_n\}_{n=0}^{\infty}$ are sequences in $[0,1]$.

Chugh and Kumar [25] introduced strong convergence and stability results for Jungck-SP iterative scheme as:

**Definition 1.4.** *(Jungck-SP iteration process)* Let $(X, \|.\|)$ be a Banach space and $Y$ be arbitrary set. Let $S, T : Y \to X$ be a nonself mappings such that $T(Y) \subseteq S(Y)$, $S(Y)$ is a complete subspace of $X$ and $S$ is injective, then for $x_\circ \in Y$, define the sequence $\{Sx_n\}_{n=0}^{\infty}$ iteratively by

$$\begin{cases} Sx_{n+1} = (1-\alpha_n)Sy_n + \alpha_n Ty_n \\ Sy_n = (1-\beta_n)Sz_n + \beta_n Tz_n \\ Sz_n = (1-\gamma_n)Sx_n + \gamma_n Tx_n, \end{cases} \tag{1.4}$$

where $\{\alpha_n\}_{n=0}^{\infty}$, $\{\beta_n\}_{n=0}^{\infty}$ and $\{\gamma_n\}_{n=0}^{\infty}$ are real sequences in $[0,1]$.

**Remark 1.3.**

(i)If we take $S = I$ and $Y = X$ in (1.3), we obtain Noor-three iteration process [26].

(ii) The iterations process (1.1) and (1.2) are a special cases of the iteration (1.3).

(iii) Putting $\beta_n = \gamma_n = 0$ in (1.4), we get the iteration (1.1).

# 2. Preliminaries

In order to prove our main results, we need to recall the following concepts and results. Let $(X, \Sigma)$ be a separable Banach space where $\Sigma$ is $\sigma-$algebra of Borel subset of $X$ and let $(\Omega, \Sigma, \mu)$ denote a complete probability measure space with measure $\mu$ and $\Sigma$ be a $\sigma-$algebra subset of $\Omega$, $C$ is a nonempty subset of $X$.

**Definition 2.1. ([27])** A random variable $x(\omega)$ is Bochner integrable if $\|x(\omega)\| \in L^1(\Omega, \Sigma, \mu)$ meaning that

$$\int_{\Omega} \|x(\omega)\| \, d\mu(\omega) < \infty. \tag{2.1}$$

**Proposition 2.1. ([1])** A random variable $x(\omega)$ is Bochner integrable iff the sequence of random variables $\{x_n(\omega)\}_{n=1}^{\infty}$ converges strongly to $x(\omega)$ almost surely such that

$$\lim_{n\to\infty} \int_{\Omega} \|x_n(\omega) - x(\omega)\| \, d\mu(\omega) = 0. \tag{2.2}$$

**Definition 2.2. ([1])** Assume that $(\Omega, \Sigma, \mu)$ be a complete probability measure space and $C$ be a nonempty subset of a separable Banach space $X$. Let $T : \Omega \times C \to C$ be a random operator and $x^*(\omega) \in C$ is called random fixed point of $T$ (i.e. for all $\omega \in \Omega$, $T(\omega, x^*(\omega)) = x^*(\omega)$). For any given random variable $x_\circ(\omega) \in C$, define an iterative scheme $\{x_n(\omega)\}_{n=0}^{\infty} \subset C$ by

$$x_{n+1}(\omega) = f(T, x_n(\omega)), \quad n = 0, 1, 2, ... \tag{2.3}$$

where $f$ is some function measurable in the second variable. Let $T$ has a random fixed point (say $x^*(\omega)$) which is Bochner integrable with respect to $\{x_n(\omega)\}_{n=0}^{\infty}$. Let $\{y_n(\omega)\}_{n=0}^{\infty} \subset C$ be an arbitrary sequence of a random variable. Assume that

$$\varepsilon_n(\omega) = \|y_{n+1}(\omega) - f(T, y_n(\omega))\|, \tag{2.4}$$

and consider $\|\varepsilon_n(\omega)\| \in L^1(\Omega, \Sigma, \mu)$, $n = 0, 1, 2, ..$, then the iterative scheme (2.3) is stable with respect to $T$ almost surely ($T-$stable almost surely) iff

$$\lim_{n\to\infty} \int_{\Omega} \|\varepsilon_n(\omega)\| \, d\mu(\omega) = 0, \tag{2.5}$$

implies that $x^*(\omega)$ is Bochner integrable with respect to $\{y_n(\omega)\}_{n=0}^{\infty}$.

**Definition 2.3. ([1])** Let $(\Omega, \Sigma, \mu)$ be a complete probability measure space and $C$ is a nonempty subset of $X$. A random operator $T : \Omega \times C \to C$ is the $\phi$-weakly contractive type if there exists a continuous and nondecreasing function $\phi : R^+ \to R^+$ with $\phi(0) = 0$ and $\phi(t) > 0$ for every $t \in (0, \infty)$ such that $\forall x, y \in C, \omega \in \Omega$,

$$\int_{\Omega} \|T(\omega, x) - T(\omega, y)\| \, d\mu(\omega) \le \int_{\Omega} \|x - y\| \, d\mu(\omega) - \phi(\int_{\Omega} \|x - y\| \, d\mu(\omega)). \tag{2.6}$$

The main aim of this paper is to introduce the following four random iterations as Jungck-Noor type, Jungck-SP type, Jungck-Ishikawa type and Jungck-Mann type, also we prove that the random fixed point of this kind of random operators is Bochner integrable in addition, we prove the convergence and some stability results of these random iterative algorithms under contractive condition (2.14) in a separable Banach space $(X, \|.\|)$.

Let $(\Omega, \Sigma, \mu)$ be a complete probability measure space and $C$ is a nonempty subset of a separable Banach space $X$ and let $S, T : \Omega \times C \to C$ be two random operator defined on $C$, such that $S$ is injective. Let $x_\circ(\omega) \in C$ be arbitrary measurable mapping for $\omega \in \Omega$, $n = 0, 1, ..$ with $T(\omega, Y) \subseteq S(\omega, Y)$, $S$ is injective, then the sequence

$\{S(\omega, x_n(\omega))\}_{n=0}^{\infty}$ iteratively defined by

$$\begin{cases} S(\omega, x_{n+1}(\omega)) = (1-\alpha_n)S(\omega, x_n(\omega)) + \alpha_n T(\omega, z_n(\omega)) \\ S(\omega, z_n(\omega)) = (1-\beta_n)S(\omega, x_n(\omega)) + \beta_n T(\omega, y_n(\omega)) \\ S(\omega, y_n(\omega)) = (1-\gamma_n)S(\omega, x_n(\omega)) + \gamma_n T(\omega, x_n(\omega)) \end{cases}, \tag{2.7}$$

where $\{\alpha_n\}_{n=0}^{\infty}$, $\{\beta_n\}_{n=0}^{\infty}$ and $\{\gamma_n\}_{n=0}^{\infty}$ are real sequences in $(0,1)$, which it called Jungck-Noor type random iterative scheme.

Also the sequence $\{S(\omega, x_n(\omega))\}_{n=0}^{\infty}$ iteratively defined by

$$\begin{cases} S(\omega, x_{n+1}(\omega)) = (1-\alpha_n)S(\omega, y_n(\omega)) + \alpha_n T(\omega, y_n(\omega)) \\ S(\omega, y_n(\omega)) = (1-\beta_n)S(\omega, z_n(\omega)) + \beta_n T(\omega, z_n(\omega)) \\ S(\omega, z_n(\omega)) = (1-\gamma_n)S(\omega, x_n(\omega)) + \gamma_n T(\omega, x_n(\omega)) \end{cases}, \tag{2.8}$$

where $\{\alpha_n\}_{n=0}^{\infty}$, $\{\beta_n\}_{n=0}^{\infty}$ and $\{\gamma_n\}_{n=0}^{\infty}$ are real sequences in $(0,1)$, which it called Jungck-SP type random iterative scheme.

If we take $\gamma_n = 0$ for each $n \in \mathbb{N}$ in (2.7), then we have Jungck-Ishikawa type random iterative scheme

$$\begin{cases} S(\omega, x_{n+1}(\omega)) = (1-\alpha_n)S(\omega, x_n(\omega)) + \alpha_n T(\omega, z_n(\omega)) \\ S(\omega, z_n(\omega)) = (1-\beta_n)S(\omega, x_n(\omega)) + \beta_n T(\omega, x_n(\omega)) \end{cases}, \tag{2.9}$$

where $\{\alpha_n\}_{n=0}^{\infty}$ and $\{\beta_n\}_{n=0}^{\infty}$ are real sequences in $(0,1)$.

If we put $\beta_n = \gamma_n = 0$ for each $n \in \mathbb{N}$ in (2.7), then we have Jungck-Mann type random iterative scheme.

$$S(\omega, x_{n+1}(\omega)) = (1-\alpha_n)S(\omega, x_n(\omega)) + \alpha_n T(\omega, x_n(\omega)), \tag{2.10}$$

where $\{\alpha_n\}_{n=0}^{\infty}$ is real sequence in $(0,1)$.

**Remark 2.1.** If we take $\Omega$ is a singleton in (2.7), (2.8), (2.9) and (2.10) then we get the nonrandom cases defined in (1.3), (1.4), (1.2) and (1.1) respectively.

According to the Definition 2.2 and Definition 2.3, we investigate the following definitions which are used in the sequel.

**Definition 2.4.** Let $(\Omega, \Sigma, \mu)$ be a complete probability measure space and $C$ be a nonempty subset of a separable Banach space $X$. Let $S, T : \Omega \times C \to C$ such that $T(Y) \subseteq S(Y)$, for every $x_\circ(\omega) \in C$, let the sequence $\{S(\omega, x_n(\omega))\}_{n=0}^{\infty}$ generated by the iteration procedure

$$S(\omega, x_{n+1}(\omega)) = f(T, x_n(\omega)), \; n \geq 0, \tag{2.11}$$

Let $x^*(\omega)$ is a common random fixed point of $(S, T)$ (i.e. for all $\omega \in \Omega$, $T(\omega, x^*(\omega)) = S(\omega, x^*(\omega)) = x^*(\omega)$) and Bochner integrable with respect to $\{x_n(\omega)\}_{n=0}^{\infty}$. Let $\{y_n(\omega)\}_{n=0}^{\infty} \subset C$ be an arbitrary sequence of a random variable. Denote

$$\varepsilon_n(\omega) = \| S(\omega, y_{n+1}(\omega)) - f(T, y_n(\omega)) \|, \tag{2.12}$$

and consider $\|\varepsilon_n(\omega)\| \in L^1(\Omega,\Sigma,\mu)$, $n = 0,1,2,..$, then the iterative scheme (2.11) is $(S,T)-$stable almost surely iff

$$\lim_{n\to\infty} \int\limits_{\Omega} \|\varepsilon_n(\omega)\| \, d\mu(\omega) = 0, \tag{2.13}$$

implies that $x^*(\omega)$ is Bochner integrable with respect to $\{y_n(\omega)\}_{n=0}^{\infty}$.

**Definition 2.5.** Let $(\Omega,\Sigma,\mu)$ be a complete probability measure space and $C$ is a nonempty subset of a separable Banach space $X$. Let $S,T : \Omega \times C \to C$ are two random operators with $T(\omega,X) \subseteq S(\omega,X)$ satisfy the following contractive condition for all $x,y \in C$ and $\omega \in \Omega$

$$\int\limits_{\Omega} \|T(\omega,x) - T(\omega,y)\| \, d\mu(\omega) \le \varphi\left(\int\limits_{\Omega} \|S(\omega,x) - T(\omega,x)\| \, d\mu(\omega)\right) + a \int\limits_{\Omega} \|S(\omega,x) - S(\omega,y)\| \, d\mu(\omega), \tag{2.14}$$

where $a \in [0,1)$ and $\varphi : R^+ \to R^+$ is a monotone increasing function with $\varphi(0) = 0$.

**Example 2.1.** Consider the following nonlinear stochastic integral equation:

$$x(t;\omega) = \int\limits_0^\infty \frac{e^{-t-s}}{8(1+|x(s;\omega)|)} ds \le t^2 \left[\int\limits_0^\infty \frac{e^{-t-s}}{8(1+|x(s;\omega)|)} ds\right] + \frac{1}{2}\int\limits_0^\infty \frac{e^{-t-s}}{4(1+|x(s;\omega)|)} ds. \tag{2.15}$$

From (2.15), we have $\varphi = t^2$ for each $t \in R^+ = [0,\infty)$ and $a = \frac{1}{2} \in [0,1)$. Hence the conditions of relation (2.14) are satisfied.

The following lemma will be needed in this study.

**Lemma 2.1.** **([24])** If $\delta$ be a real number such that $0 \le \delta < 1$ and $\{\epsilon_n\}_{n=0}^{\infty}$ is a sequence of positive numbers such that $\lim_{n\to\infty} \epsilon_n = 0$, then for any sequence of positive numbers $\{\rho_n\}_{n=0}^{\infty}$ satisfying

$$\rho_{n+1} \le \delta\rho_n + \epsilon_n \;\; n = 0,1,2,..$$

we have $\lim_{n\to\infty} \rho_n = 0$.

# 3. Some Convergence Results

**Theorem 3.1.** *Let $(X,\|.\|)$ be a separable Banach space and $T,S : \Omega \times C \to C$ be two random operators satisfying (2.14) with $T(\omega,X) \subseteq S(\omega,X)$ and $F(T) \cap S(T) \neq \varnothing$. Assume that $x^*(\omega)$ be a common random fixed point of $(S,T)$ and $\{S(\omega,x_n(\omega))\}_{n=0}^{\infty}$ be a Jungck-Noor type random iterative sequence defined by (2.7) where $\{\alpha_n\}$, $\{\beta_n\}$ and $\{\gamma_n\}$ are real sequences in $(0,1)$ such that $\lim_{n\to\infty} \beta_n = 0$ and $\sum\limits_{n=1}^{\infty} \alpha_n\beta_n\gamma_n = \infty$. Then the common random fixed point $x^*(\omega)$ is Bochner integrable.*

*Proof.* For each $x_\circ(\omega) \in C$and $T(\omega,X) \subseteq S(\omega,X)$, we choose $x_1(\omega) \in C$ such that $x_1(\omega) = T(\omega,x_\circ(\omega)) = S(\omega,x_1(\omega))$ and $x_2(\omega) = T(\omega,x_1(\omega)) = S(\omega,x_2(\omega))$, by continuing this process we conclude a sequence $\{x_{n+1}(\omega)\} \in C$ such that

$$x_{n+1}(\omega) = T(\omega,x_n(\omega)) = S(\omega,x_{n+1}(\omega)). \tag{3.1}$$

To prove our theorem, it is sufficient to prove that from (3.1)

$$\lim_{n\to\infty}\int_{\Omega}\|x_n(\omega)-x^*(\omega)\|\,d\mu(\omega)=\lim_{n\to\infty}\int_{\Omega}\|S(\omega,x_n(\omega))-x^*(\omega)\|\,d\mu(\omega)=0.$$

Using (2.14) and (2.7), we get

$$\begin{aligned}
&\int_{\Omega}\|S(\omega,x_{n+1}(\omega))-x^*(\omega)\|\,d\mu(\omega)\\
\leq\;&(1-\alpha_n)\int_{\Omega}\|S(\omega,x_n(\omega))-x^*(\omega)\|\,d\mu(\omega)+\alpha_n\int_{\Omega}\|T(\omega,z_n(\omega))-x^*(\omega)\|\,d\mu(\omega)\\
=\;&(1-\alpha_n)\int_{\Omega}\|S(\omega,x_n(\omega))-x^*(\omega)\|\,d\mu(\omega)+\alpha_n\int_{\Omega}\|T(\omega,x^*(\omega))-T(\omega,z_n(\omega))\|\,d\mu(\omega)\\
\leq\;&(1-\alpha_n)\int_{\Omega}\|S(\omega,x_n(\omega))-x^*(\omega)\|\,d\mu(\omega)+\alpha_n\begin{pmatrix}\varphi\left(\int_{\Omega}\|S(\omega,x^*(\omega))-T(\omega,x^*(\omega))\|\,d\mu(\omega)\right)\\+a\int_{\Omega}\|S(\omega,x^*(\omega))-S(\omega,z_n(\omega))\|\,d\mu(\omega)\end{pmatrix},
\end{aligned}$$

since $\varphi(0)=0$, then we have

$$\int_{\Omega}\|S(\omega,x_{n+1}(\omega))-x^*(\omega)\|\,d\mu(\omega)\leq\begin{pmatrix}(1-\alpha_n)\int_{\Omega}\|S(\omega,x_n(\omega))-x^*(\omega)\|\,d\mu(\omega)\\+a\alpha_n\int_{\Omega}\|S(\omega,z_n(\omega))-x^*(\omega)\|\,d\mu(\omega)\end{pmatrix}. \tag{3.2}$$

Similarly,

$$\begin{aligned}
\int_{\Omega}\|S(\omega,z_n(\omega))-x^*(\omega)\|\,d\mu(\omega)\;\leq\;&(1-\beta_n)\int_{\Omega}\|S(\omega,x_n(\omega))-x^*(\omega)\|\,d\mu(\omega)\\
&+\beta_n\int_{\Omega}\|T(\omega,y_n(\omega))-x^*(\omega)\|\,d\mu(\omega)\\
\leq\;&(1-\beta_n)\int_{\Omega}\|S(\omega,x_n(\omega))-x^*(\omega)\|\,d\mu(\omega)\\
&+\beta_n\begin{bmatrix}\varphi\left(\int_{\Omega}\|S(\omega,x^*(\omega))-T(\omega,x^*(\omega))\|\,d\mu(\omega)\right)\\+a\int_{\Omega}\|S(\omega,x^*(\omega))-S(\omega,y_n(\omega))\|\,d\mu(\omega)\end{bmatrix}\\
\leq\;&(1-\beta_n)\int_{\Omega}\|S(\omega,x_n(\omega))-x^*(\omega)\|\,d\mu(\omega)\\
&+a\beta_n\int_{\Omega}\|S(\omega,y_n(\omega))-x^*(\omega)\|\,d\mu(\omega).
\end{aligned}\tag{3.3}$$

Applying (3.3) in (3.2) we have

$$\int_{\Omega}\|S(\omega,x_{n+1}(\omega))-x^*(\omega)\|\,d\mu(\omega)\leq\begin{pmatrix}(1-\alpha_n(1-a)-a\alpha_n\beta_n)\int_{\Omega}\|S(\omega,x_n(\omega))-x^*(\omega)\|\,d\mu(\omega)\\+a^2\alpha_n\beta_n\int_{\Omega}\|S(\omega,y_n(\omega))-x^*(\omega)\|\,d\mu(\omega)\end{pmatrix}. \tag{3.4}$$

Also,

$$\int_{\Omega}\|S(\omega,y_n(\omega))-x^*(\omega)\|\,d\mu(\omega)\leq\begin{pmatrix}(1-\gamma_n)\int_{\Omega}\|S(\omega,x_n(\omega))-x^*(\omega)\|\,d\mu(\omega)\\+\gamma_n\int_{\Omega}\|T(\omega,x_n(\omega))-x^*(\omega)\|\,d\mu(\omega)\end{pmatrix}. \tag{3.5}$$

Using (3.5) in (3.4), we obtain that

$$\begin{aligned}
&\int_\Omega \|S(\omega, x_{n+1}(\omega)) - x^*(\omega)\| \, d\mu(\omega) \\
\leq &\left( \begin{array}{c} (1-\alpha_n(1-a) - a\alpha_n\beta_n(1-a) - a^2\alpha_n\beta_n\gamma_n) \int_\Omega \|S(\omega, x_n(\omega)) - x^*(\omega)\| \, d\mu(\omega) \\ +a^2\alpha_n\beta_n\gamma_n \int_\Omega \|T(\omega, x_n(\omega)) - x^*(\omega)\| \, d\mu(\omega) \end{array} \right).
\end{aligned}$$

Since $a \in [0, 1)$, $\alpha_n \in (0, 1)$, $\sum_{n=1}^{\infty} \alpha_n\beta_n\gamma_n = \infty$ and $\lim_{n\to\infty} \beta_n = 0$ then $0 \leq 1 - \alpha_n(1-a) = \delta < 1$. If we take $\epsilon_n = a^2\alpha_n\beta_n\gamma_n \int_\Omega \|T(\omega, x_n(\omega)) - x^*(\omega)\| \, d\mu(\omega)$ and $\rho_n = \int_\Omega \|S(\omega, x_n(\omega)) - x^*(\omega)\| \, d\mu(\omega)$, therefore $\lim_{n\to\infty} \epsilon_n = 0$, we see that all conditions of Lemma 2.1 are satisfied, hence we get

$$\lim_{n\to\infty} \int_\Omega \|S(\omega, x_n(\omega)) - x^*(\omega)\| \, d\mu(\omega) = 0.$$

The proof is completed. □

**Theorem 3.2.** *Let $(X, \|.\|)$ be a separable Banach space and $T, S : \Omega \times C \to C$ be two random operators satisfying (2.14) with $T(\omega, X) \subseteq S(\omega, X)$ and $F(T) \cap S(T) \neq \varnothing$. Assume that $x^*(\omega)$ be a common random fixed point of $(S, T)$ and $\{S(\omega, x_n(\omega))\}_{n=0}^{\infty}$ be a Jungck-SP type random iterative sequence defined by (2.8) where $\{\alpha_n\}$, $\{\beta_n\}$ and $\{\gamma_n\}$ are real sequences in $(0, 1)$ such that $\sum_{n=1}^{\infty} \alpha_n = \infty$. Then the common random fixed point $x^*(\omega)$ is Bochner integrable.*

*Proof.* By a similar way of proof Theorem 3.1 and using (3.1), (2.14) and (2.8), we can write

$$\begin{aligned}
&\int_\Omega \|S(\omega, x_{n+1}(\omega)) - x^*(\omega)\| \, d\mu(\omega) \\
\leq\ & (1-\alpha_n) \int_\Omega \|S(\omega, y_n(\omega)) - x^*(\omega)\| \, d\mu(\omega) + \alpha_n \int_\Omega \|T(\omega, y_n(\omega)) - x^*(\omega)\| \, d\mu(\omega) \\
=\ & (1-\alpha_n) \int_\Omega \|S(\omega, y_n(\omega)) - x^*(\omega)\| \, d\mu(\omega) + \alpha_n \int_\Omega \|T(\omega, x^*(\omega)) - T(\omega, y_n(\omega))\| \, d\mu(\omega) \\
\leq\ & (1-\alpha_n) \int_\Omega \|S(\omega, y_n(\omega)) - x^*(\omega)\| \, d\mu(\omega) + \alpha_n \left( \begin{array}{c} \varphi\left( \int_\Omega \|S(\omega, x^*(\omega)) - T(\omega, x^*(\omega))\| \, d\mu(\omega) \right) \\ +a \int_\Omega \|S(\omega, x^*(\omega)) - S(\omega, y_n(\omega))\| \, d\mu(\omega) \end{array} \right),
\end{aligned}$$

since $\varphi(0) = 0$, then we have

$$\int_\Omega \|S(\omega, x_{n+1}(\omega)) - x^*(\omega)\| \, d\mu(\omega) \leq \left( [1-\alpha_n(1-a)] \int_\Omega \|S(\omega, y_n(\omega)) - x^*(\omega)\| \, d\mu(\omega) \right). \tag{3.6}$$

Similarly,

$$\begin{aligned}
\int_{\Omega} \|S(\omega, y_n(\omega)) - x^*(\omega)\| \, d\mu(\omega) &\leq (1-\beta_n) \int_{\Omega} \|S(\omega, z_n(\omega)) - x^*(\omega)\| \, d\mu(\omega) \\
&\quad + \beta_n \int_{\Omega} \|T(\omega, z_n(\omega)) - x^*(\omega)\| \, d\mu(\omega) \\
&\leq (1-\beta_n) \int_{\Omega} \|S(\omega, z_n(\omega)) - x^*(\omega)\| \, d\mu(\omega) \\
&\quad + \beta_n \left[ \begin{array}{c} \varphi\left( \int_{\Omega} \|S(\omega, x^*(\omega)) - T(\omega, x^*(\omega))\| \, d\mu(\omega) \right) \\ + a \int_{\Omega} \|S(\omega, x^*(\omega)) - S(\omega, z_n(\omega))\| \, d\mu(\omega) \end{array} \right] \\
&\leq [1 - \beta_n(1-a)] \int_{\Omega} \|S(\omega, z_n(\omega)) - x^*(\omega)\| \, d\mu(\omega). \qquad (3.7)
\end{aligned}$$

Applying (3.7) in (3.6) we have

$$\int_{\Omega} \|S(\omega, x_{n+1}(\omega)) - x^*(\omega)\| \, d\mu(\omega) \leq \left( [1 - \alpha_n(1-a)][1 - \beta_n(1-a)] \int_{\Omega} \|S(\omega, z_n(\omega)) - x^*(\omega)\| \, d\mu(\omega) \right). \qquad (3.8)$$

Also,

$$\begin{aligned}
\int_{\Omega} \|S(\omega, z_n(\omega)) - x^*(\omega)\| \, d\mu(\omega) &\leq \left( \begin{array}{c} (1-\gamma_n) \int_{\Omega} \|S(\omega, x_n(\omega)) - x^*(\omega)\| \, d\mu(\omega) \\ + \gamma_n \int_{\Omega} \|T(\omega, x_n(\omega)) - x^*(\omega)\| \, d\mu(\omega) \end{array} \right) \\
&\leq \left( [1 - \gamma_n(1-a)] \int_{\Omega} \|S(\omega, x_n(\omega)) - x^*(\omega)\| \, d\mu(\omega) \right). \qquad (3.9)
\end{aligned}$$

Using (3.9) in (3.8), we obtain that

$$\begin{aligned}
&\int_{\Omega} \|S(\omega, x_{n+1}(\omega)) - x^*(\omega)\| \, d\mu(\omega) \\
\leq\ & \left( [1 - \alpha_n(1-a)][1 - \beta_n(1-a)][1 - \gamma_n(1-a)] \int_{\Omega} \|S(\omega, x_n(\omega)) - x^*(\omega)\| \, d\mu(\omega) \right) \\
\leq\ & [1 - \alpha_n(1-a)] \int_{\Omega} \|S(\omega, x_n(\omega)) - x^*(\omega)\| \, d\mu(\omega) \\
\leq\ & \prod_{k=0}^{n} [1 - \alpha_k(1-a)] \int_{\Omega} \|S(\omega, x_\circ(\omega)) - x^*(\omega)\| \, d\mu(\omega) \\
\leq\ & e^{-(1-a) \sum_{k=0}^{\infty} \alpha_k} \int_{\Omega} \|S(\omega, x_\circ(\omega)) - x^*(\omega)\| \, d\mu(\omega). \qquad (3.10)
\end{aligned}$$

Since $a \in [0, 1)$, $\alpha_k \in (0, 1)$, $\sum_{n=0}^{\infty} \alpha_n = \infty$ and $\lim_{n\to\infty} e^{-(1-a)\sum_{k=0}^{\infty} \alpha_k} = 0$, hence it follows from (3.10) that $\lim_{n\to\infty} \int_{\Omega} \|S(\omega, x_{n+1}(\omega)) - x^*($
0, but every subsequence of convergence sequence being convergence, then

$$\lim_{n\to\infty} \int_{\Omega} \|S(\omega, x_n(\omega)) - x^*(\omega)\| \, d\mu(\omega) = 0.$$

The proof is completed. □

By the same method as in Theorem 3.1, we can prove the following theorems.

**Theorem 3.3.** *Let $(X, \|.\|)$ be a separable Banach space and $T, S : \Omega \times C \to C$ be two random operators satisfying (2.14) with $T(\omega, X) \subseteq S(\omega, X)$ and $F(T) \cap S(T) \neq \varnothing$. Assume that $x^*(\omega)$ be a common random fixed point of $(S, T)$ and $\{S(\omega, x_n(\omega))\}_{n=0}^{\infty}$ be a Jungck-Ishikawa type random iterative sequence defined by (2.9) where $\{\alpha_n\}$ and $\{\beta_n\}$ are real sequences in $(0,1)$ such that $\lim_{n\to\infty} \beta_n = 0$ and $\sum_{n=1}^{\infty} \alpha_n\beta_n = \infty$. Then the common random fixed point $x^*(\omega)$ is Bochner integrable.*

**Theorem 3.4.** *Let $(X, \|.\|)$ be a separable Banach space and $T, S : \Omega \times C \to C$ be two random operators satisfying (2.14) with $T(\omega, X) \subseteq S(\omega, X)$ and $F(T) \cap S(T) \neq \varnothing$. Assume that $x^*(\omega)$ be a common random fixed point of $(S, T)$ and $\{S(\omega, x_n(\omega))\}_{n=0}^{\infty}$ be a Jungck-Mann type random iterative sequence defined by (2.10) where $\{\alpha_n\}$ is real sequence in $(0,1)$ such that $\sum_{n=1}^{\infty} \alpha_n = \infty$. Then the common random fixed point $x^*(\omega)$ is Bochner integrable.*

# 4. Some Stability Results

**Theorem 4.1.** *Let $(X, \|.\|)$ be a separable Banach space and $T, S : \Omega \times C \to C$ be two random operators satisfying (2.14) with $T(\omega, X) \subseteq S(\omega, X)$ and $F(T) \cap S(T) \neq \varnothing$. Assume that $x^*(\omega)$ be a common random fixed point of $(S, T)$. Let $\{S(\omega, x_n(\omega))\}_{n=0}^{\infty}$ be a Jungck-Noor type random iterative scheme defined by (2.7) converging strongly to $x^*(\omega)$ almost surely, where $\{\alpha_n\}$, $\{\beta_n\}$ and $\{\gamma_n\}$ are real sequences in $(0,1)$ such that $\lim_{n\to\infty} \beta_n = 0$ and $\sum_{n=1}^{\infty} \alpha_n\beta_n\gamma_n = \infty$. Then $\{S(\omega, x_n(\omega))\}_{n=0}^{\infty}$ is $(S, T)-$stable almost surely.*

*Proof.* Consider that $\{S(\omega, y_n(\omega))\}_{n=0}^{\infty}$ be any sequence of random variable in $C$ and

$$\|\varepsilon_n\| = \|S(\omega, y_{n+1}(\omega)) - (1-\alpha_n)S(\omega, y_n(\omega)) - \alpha_n T(\omega, k_n(\omega))\|, \tag{4.1}$$

where $S(\omega, k_n(\omega)) = (1-\beta_n)S(\omega, y_n(\omega)) + \beta_n T(\omega, z_n(\omega))$ and $S(\omega, z_n(\omega)) = (1-\gamma_n)S(\omega, y_n(\omega)) - \gamma_n T(\omega, y_n(\omega))$ and $\lim_{n\to\infty} \int_\Omega \|\varepsilon_n\| \, d\mu(\omega) = 0$ for every $\omega \in \Omega$. Now we prove that $x^*(\omega)$ is Bochner integrable with respect to the sequence $\{S(\omega, y_n(\omega))\}_{n=0}^{\infty}$. It follows from (4.1) that

$$\begin{aligned}
&\int_\Omega \|S(\omega, y_{n+1}(\omega)) - x^*(\omega)\| \, d\mu(\omega) \\
\leq\ & \int_\Omega \|S(\omega, y_{n+1}(\omega)) - (1-\alpha_n)S(\omega, y_n(\omega)) - \alpha_n T(\omega, k_n(\omega))\| \, d\mu(\omega) \\
& +(1-\alpha_n)\int_\Omega \|S(\omega, y_n(\omega)) - x^*(\omega)\| \, d\mu(\omega) + \alpha_n \int_\Omega \|T(\omega, k_n(\omega)) - x^*(\omega)\| \, d\mu(\omega) \\
=\ & \int_\Omega \|\varepsilon_n\| \, d\mu(\omega) + (1-\alpha_n)\int_\Omega \|S(\omega, y_n(\omega)) - x^*(\omega)\| \, d\mu(\omega) \\
& +\alpha_n \int_\Omega \|T(\omega, x^*(\omega)) - T(\omega, k_n(\omega))\| \, d\mu(\omega).
\end{aligned} \tag{4.2}$$

By (2.14), we obtain

$$\begin{aligned}
& \int_{\Omega} \|T(\omega, x^*(\omega)) - T(\omega, k_n(\omega))\| \, d\mu(\omega) \\
\leq\ & \varphi\left( \int_{\Omega} \|S(\omega, x^*(\omega)) - T(\omega, x^*(\omega))\| \, d\mu(\omega) \right) + a \int_{\Omega} \|S(\omega, x^*(\omega)) - S(\omega, k_n(\omega))\| \, d\mu(\omega) \\
=\ & a \int_{\Omega} \|S(\omega, k_n(\omega)) - x^*(\omega)\| \, d\mu(\omega) \\
\leq\ & a \left( (1-\beta_n) \int_{\Omega} \|S(\omega, y_n(\omega)) - x^*(\omega)\| \, d\mu(\omega) + \beta_n \int_{\Omega} \|T(\omega, z_n(\omega)) - x^*(\omega)\| \, d\mu(\omega) \right) \\
\leq\ & a(1-\beta_n) \int_{\Omega} \|S(\omega, y_n(\omega)) - x^*(\omega)\| \, d\mu(\omega) + a^2\beta_n \int_{\Omega} \|S(\omega, z_n(\omega)) - x^*(\omega)\| \, d\mu(\omega) \\
=\ & a(1-\beta_n) \int_{\Omega} \|S(\omega, y_n(\omega)) - x^*(\omega)\| \, d\mu(\omega) + a^2\beta_n \left( \begin{array}{c} (1-\gamma_n) \int_{\Omega} \|S(\omega, y_n(\omega)) - x^*(\omega)\| \, d\mu(\omega) \\ +\gamma_n \int_{\Omega} \|T(\omega, y_n(\omega)) - x^*(\omega)\| \, d\mu(\omega) \end{array} \right) \\
\leq\ & (a(1-\beta_n) + a^2\beta_n) \int_{\Omega} \|S(\omega, y_n(\omega)) - x^*(\omega)\| \, d\mu(\omega). \qquad (4.3)
\end{aligned}$$

Applying (4.3) in (4.2), we have

$$\int_{\Omega} \|S(\omega, y_{n+1}(\omega)) - x^*(\omega)\| \, d\mu(\omega) \leq \left( \begin{array}{c} [1-\alpha_n(1-a) - a\alpha_n\beta_n(1-a)] \int_{\Omega} \|S(\omega, y_n(\omega)) - x^*(\omega)\| \, d\mu(\omega) \\ + \int_{\Omega} \|\varepsilon_n\| \, d\mu(\omega) \end{array} \right). \qquad (4.4)$$

Using the assumptions that $\lim_{n\to\infty} \int_{\Omega} \|\varepsilon_n\| \, d\mu(\omega) = 0$, $0 \leq 1-\alpha_n(1-a) = \delta < 1$, $\lim_{n\to\infty} \beta_n = 0$ and $\sum_{n=1}^{\infty} \alpha_n\beta_n\gamma_n = \infty$. Clearly, all conditions of Lemma 2.1 are satisfied. Hence, we have

$$\lim_{n\to\infty} \int_{\Omega} \|S(\omega, y_n(\omega)) - x^*(\omega)\| \, d\mu(\omega) = 0. \qquad (4.5)$$

*Conversely.* If $x^*(\omega)$ is Bochner integrable with respect to the sequence $\{S(\omega, y_n(\omega))\}_{n=0}^{\infty}$, we get

$$\begin{aligned}
\int_{\Omega} \|\varepsilon_n\| \, d\mu(\omega) &= \int_{\Omega} \|S(\omega, y_{n+1}(\omega)) - (1-\alpha_n)S(\omega, y_n(\omega)) - \alpha_n T(\omega, k_n(\omega))\| \, d\mu(\omega) \\
&\leq \int_{\Omega} \|S(\omega, y_{n+1}(\omega)) - x^*(\omega)\| \, d\mu(\omega) + (1-\alpha_n) \int_{\Omega} \|S(\omega, y_n(\omega)) - x^*(\omega)\| \, d\mu(\omega) \\
&\quad + \alpha_n \int_{\Omega} \|T(\omega, y_n(\omega)) - x^*(\omega)\| \, d\mu(\omega). \qquad (4.6)
\end{aligned}$$

Using (2.14) and (4.3) we have

$$\begin{aligned}
\int_\Omega \|T(\omega, y_n(\omega)) - x^*(\omega)\| \, d\mu(\omega) &= \int_\Omega \|T(\omega, x^*(\omega)) - T(\omega, y_n(\omega))\| \, d\mu(\omega) \\
&\leq \varphi\left(\int_\Omega \|S(\omega, x^*(\omega)) - T(\omega, x^*(\omega))\| \, d\mu(\omega)\right) \\
&\quad + a\int_\Omega \|S(\omega, x^*(\omega)) - S(\omega, k_n(\omega))\| \, d\mu(\omega) \\
&= a\int_\Omega \|S(\omega, k_n(\omega)) - x^*(\omega)\| \, d\mu(\omega) \\
&\leq (a(1-\beta_n) + a^2\beta_n)\int_\Omega \|S(\omega, y_n(\omega)) - x^*(\omega)\| \, d\mu(\omega). \qquad (4.7)
\end{aligned}$$

Using (4.7) in (4.6), we can write

$$\int_\Omega \|\varepsilon_n\| \, d\mu(\omega) \leq \left(\begin{array}{c} \int_\Omega \|S(\omega, y_{n+1}(\omega)) - x^*(\omega)\| \, d\mu(\omega) \\ +[1 - \alpha_n(1-a) - a\alpha_n\beta_n(1-a)] \int_\Omega \|S(\omega, y_n(\omega)) - x^*(\omega)\| \, d\mu(\omega). \end{array}\right) \qquad (4.8)$$

Hence, we get

$$\lim_{n\to\infty} \int_\Omega \|\varepsilon_n\| \, d\mu(\omega) = 0.$$

This leads to the Jungck-Noor type random iterative scheme $\{S(\omega, x_n(\omega))\}_{n=0}^{\infty}$ is $(S, T)-$stable almost surely.

The proof is completed. □

**Theorem 4.2.** *Let $(X, \|.\|)$ be a separable Banach space and $T, S : \Omega \times C \to C$ be two random operators satisfying (2.14) with $T(\omega, X) \subseteq S(\omega, X)$ and $F(T) \cap S(T) \neq \varnothing$. Assume that $x^*(\omega)$ be a common random fixed point of $(S, T)$. Let $\{S(\omega, x_n(\omega))\}_{n=0}^{\infty}$ be a Jungck-SP type random iterative scheme defined by (2.8) converging strongly to $x^*(\omega)$ almost surely, where $\{\alpha_n\}$, $\{\beta_n\}$ and $\{\gamma_n\}$ are real sequences in $(0,1)$ such that $\sum_{n=1}^{\infty} \alpha_n = \infty$ and $0 < \alpha \leq \alpha_n$. Then $\{S(\omega, x_n(\omega))\}_{n=0}^{\infty}$ is $(S, T)-$stable almost surely.*

*Proof.* Consider that $\{S(\omega, y_n(\omega))\}_{n=0}^{\infty}$ be any sequence of random variable in $C$ and

$$\|\varepsilon_n\| = \|S(\omega, y_{n+1}(\omega)) - (1-\alpha_n)S(\omega, k_n(\omega)) - \alpha_n T(\omega, k_n(\omega))\|, \qquad (4.9)$$

where $S(\omega, k_n(\omega)) = (1-\beta_n)S(\omega, z_n(\omega)) + \beta_n T(\omega, z_n(\omega))$ and $S(\omega, z_n(\omega)) = (1-\gamma_n)S(\omega, y_n(\omega)) - \gamma_n T(\omega, y_n(\omega))$ and $\lim_{n\to\infty} \int_\Omega \|\varepsilon_n\| \, d\mu(\omega) = 0$ for every $\omega \in \Omega$. Now we prove that $x^*(\omega)$ is Bochner integrable with respect to the sequence $\{S(\omega, y_n(\omega))\}_{n=0}^{\infty}$. It follows from (4.9) that

$$\begin{aligned}
& \int_{\Omega} \|S(\omega, y_{n+1}(\omega)) - x^*(\omega)\| \, d\mu(\omega) \\
\leq \; & \int_{\Omega} \|S(\omega, y_{n+1}(\omega)) - (1-\alpha_n)S(\omega, k_n(\omega)) - \alpha_n T(\omega, k_n(\omega))\| \, d\mu(\omega) \\
& +(1-\alpha_n)\int_{\Omega} \|S(\omega, k_n(\omega)) - x^*(\omega)\| \, d\mu(\omega) + \alpha_n \int_{\Omega} \|T(\omega, k_n(\omega)) - x^*(\omega)\| \, d\mu(\omega) \\
= \; & \int_{\Omega} \|\varepsilon_n\| \, d\mu(\omega) + (1-\alpha_n)\int_{\Omega} \|S(\omega, k_n(\omega)) - x^*(\omega)\| \, d\mu(\omega) \\
& +\alpha_n \int_{\Omega} \|T(\omega, x^*(\omega)) - T(\omega, k_n(\omega))\| \, d\mu(\omega). \qquad (4.10)
\end{aligned}$$

By (2.14), we obtain

$$\begin{aligned}
& \int_{\Omega} \|T(\omega, x^*(\omega)) - T(\omega, k_n(\omega))\| \, d\mu(\omega) \\
\leq \; & \varphi\left(\int_{\Omega} \|S(\omega, x^*(\omega)) - T(\omega, x^*(\omega))\| \, d\mu(\omega)\right) + a\int_{\Omega} \|S(\omega, x^*(\omega)) - S(\omega, k_n(\omega))\| \, d\mu(\omega) \\
= \; & a\int_{\Omega} \|S(\omega, k_n(\omega)) - x^*(\omega)\| \, d\mu(\omega). \qquad (4.11)
\end{aligned}$$

Applying (4.11) in (4.10), we have

$$\begin{aligned}
& \int_{\Omega} \|S(\omega, y_{n+1}(\omega)) - x^*(\omega)\| \, d\mu(\omega) \\
\leq \; & \int_{\Omega} \|\varepsilon_n\| \, d\mu(\omega) + [1-\alpha_n(1-a)]\int_{\Omega} \|S(\omega, k_n(\omega)) - x^*(\omega)\| \, d\mu(\omega) \\
= \; & \int_{\Omega} \|\varepsilon_n\| \, d\mu(\omega) + [1-\alpha_n(1-a)]\begin{pmatrix} (1-\beta_n)\int_{\Omega} \|S(\omega, z_n(\omega)) - x^*(\omega)\| \, d\mu(\omega) \\ \beta_n \int_{\Omega} \|T(\omega, z_n(\omega)) - x^*(\omega)\| \, d\mu(\omega) \end{pmatrix} \\
\leq \; & \int_{\Omega} \|\varepsilon_n\| \, d\mu(\omega) + [1-\alpha_n(1-a)][1-\beta_n(1-a)]\int_{\Omega} \|S(\omega, z_n(\omega)) - x^*(\omega)\| \, d\mu(\omega) \\
\leq \; & \int_{\Omega} \|\varepsilon_n\| \, d\mu(\omega) + [1-\alpha_n(1-a)][1-\beta_n(1-a)][1-\gamma_n(1-a)]\int_{\Omega} \|S(\omega, y_n(\omega)) - x^*(\omega)\| \, d\mu(\omega). \quad (4.12)
\end{aligned}$$

Using $0 < \alpha \leq \alpha_n$ and $a \in [0,1)$, we have $0 \leq [1-\alpha_n(1-a)][1-\beta_n(1-a)][1-\gamma_n(1-a)] = \delta < 1$, $\lim_{n\to\infty} \int_{\Omega} \|\varepsilon_n\| \, d\mu(\omega) = \lim_{n\to\infty} \epsilon_n = 0$. Clearly, all the conditions of Lemma 2.1 are satisfied. Hence, we have

$$\lim_{n\to\infty} \int_{\Omega} \|S(\omega, y_n(\omega)) - x^*(\omega)\| \, d\mu(\omega) = 0. \qquad (4.13)$$

*Conversely.* If $x^*(\omega)$ is Bochner integrable with respect to the sequence $\{S(\omega, y_n(\omega))\}_{n=0}^{\infty}$, we get

$$\begin{aligned}\int_{\Omega} \|\varepsilon_n\|\, d\mu(\omega) &= \int_{\Omega} \|S(\omega, y_{n+1}(\omega)) - (1-\alpha_n)S(\omega, k_n(\omega)) - \alpha_n T(\omega, k_n(\omega))\|\, d\mu(\omega) \\ &\leq \int_{\Omega} \|S(\omega, y_{n+1}(\omega)) - x^*(\omega)\|\, d\mu(\omega) + (1-\alpha_n)\int_{\Omega} \|S(\omega, k_n(\omega)) - x^*(\omega)\|\, d\mu(\omega) \\ &\quad + \alpha_{n\Omega} \|T(\omega, k_n(\omega)) - x^*(\omega)\|\, d\mu(\omega). \end{aligned} \tag{4.14}$$

Using the same calculations above, it follows from (4.11) and (4.12) in (4.14), one can write

$$\int_{\Omega} \|\varepsilon_n\|\, d\mu(\omega) \leq [1-\alpha_n(1-a)][1-\beta_n(1-a)][1-\gamma_n(1-a)] \int_{\Omega} \|S(\omega, y_n(\omega)) - x^*(\omega)\|\, d\mu(\omega).$$

Using (4.12), we have

$$\int_{\Omega} \|\varepsilon_n\|\, d\mu(\omega) \leq e^{-(1-a)\sum\limits_{k=0}^{\infty}\alpha_k} \int_{\Omega} \|S(\omega, y_\circ(\omega)) - x^*(\omega)\|\, d\mu(\omega) \to 0 \text{ as } n \to \infty. \tag{4.15}$$

Hence, we get

$$\lim_{n\to\infty} \int_{\Omega} \|\varepsilon_n\|\, d\mu(\omega) = 0.$$

This leads to the Jungck-SP type random iterative scheme $\{S(\omega, x_n(\omega))\}_{n=0}^{\infty}$ is $(S, T)-$stable almost surely.

The proof is completed. □

By the same manner as in Theorem 4.1, we can present the following theorems.

**Theorem 4.3.** *Let $(X, \|.\|)$ be a separable Banach space and $T, S : \Omega \times C \to C$ be two random operators satisfying (2.14) with $T(\omega, X) \subseteq S(\omega, X)$ and $F(T) \cap S(T) \neq \varnothing$. Assume that $x^*(\omega)$ be a common random fixed point of $(S, T)$. Let $\{S(\omega, x_n(\omega))\}_{n=0}^{\infty}$ be a Jungck-Ishikawa type random iterative scheme defined by (2.9) converging strongly to $x^*(\omega)$ almost surely, where $\{\alpha_n\}$ and $\{\beta_n\}$ are real sequences in $(0, 1)$ such that $\lim_{n\to\infty} \beta_n = 0$ and $\sum\limits_{n=1}^{\infty} \alpha_n\beta_n = \infty$. Then $\{S(\omega, x_n(\omega))\}_{n=0}^{\infty}$ is $(S, T)-$stable almost surely.*

**Theorem 4.4.** *Let $(X, \|.\|)$ be a separable Banach space and $T, S : \Omega \times C \to C$ be two random operators satisfying (2.14) with $T(\omega, X) \subseteq S(\omega, X)$ and $F(T) \cap S(T) \neq \varnothing$. Assume that $x^*(\omega)$ be a common random fixed point of $(S, T)$. Let $\{S(\omega, x_n(\omega))\}_{n=0}^{\infty}$ be a Jungck-Mann type random iterative scheme defined by (2.10) converging strongly to $x^*(\omega)$ almost surely, where $\{\alpha_n\}$ is real sequences in $(0, 1)$ such that $\sum\limits_{n=1}^{\infty} \alpha_n = \infty$. Then $\{S(\omega, x_n(\omega))\}_{n=0}^{\infty}$ is $(S, T)-$stable almost surely.*

**Remark 4.1.** If we put $S = I$ (where $I$ is the identity mapping) in (2.7) we get the results of Okeke and Eke [2] and in (2.9), (2.10) we obtain the results of Zhang et al. [1].

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# References

[1] S. S. Zhang, X. R. Wang and M. Liu, *Almost sure T-stability and convergence for random iterative algorithms*, Appl. Math. Mech. Engl. Ed., **32** (2011), 805-810.

[2] G. A. Okeke and K. S. Eke, *Convergence and almost sure T-stability for random Noor-type iterative scheme*, Int. J. Pure Appl. Math., **107** (2016), 1-16.

[3] A. T. Bharucha-Reid, *Random integral equations*, Academic Press, New York (1972).

[4] A. T. Bharucha-Reid, *Fixed point theorems in probabilistic analysis*, Bull. Amer. Math. Soc., **82** (1976) 641-657.

[5] A. Špaček, *Zufällige Gleichungen*, Czechoslovak Math. J., **5** (1955), 462-466.

[6] O. Hanš, *Reduzierende zufällige transformationen*, Czechoslov. Math. J., **7** (1957), 154-158.

[7] S. Itoh, *Random fixed-point theorems with an application to random differential equations in Banach spaces*, J. Math. Anal. Appl., **67** (1979), 261-273.

[8] N. S. Papageorgiou, *Random fixed point theorems for measurable multifunctions in Banach spaces*, Proc. Amer. Math. Soc., **97** (1986), 507-514.

[9] H. Engl, *Random fixed point theorems for multivalued mappings*, Pacific J. Math., **76** (1976), 351-360.

[10] S. Reich, *Approximate selections, best approximations, fixed points and invariant sets*, J. Math. Anal. Appl., **62** (1978), 104-112.

[11] N. Shahzad and S. Latif, *Random fixed points for several classes of 1-Ball-contractive and 1-set-contractive random maps*, J. Math. Anal. Appl., **237** (1999), 83-92.

[12] S. S. Chang, Y. J. Cho, J. K. Kim and H. Y. Zhou, *Random Ishikawa iterative sequence with applications*, Stoc. Anal. Appl., **23** (2005), 69-77.

[13] I. Beg and M. Abbas, *Equivalence and stability of random fixed point iterative procedures*, J. Appl. Math. Stoc. Anal., **2006** (2006), 1-19.

[14] G. A. Okeke and M. Abbas, *Convergence and almost sure T-stability for a random iterative sequence generated by a generalized random operator*, J. Inequalities appl., **2015:146**, (2015).

[15] G. A. Okeke and J. K. Kim, *Convergence and summable almost T-stability of the random Picard-Mann hybrid iterative process*, J. Inequalities appl., **2015:290**, (2015).

[16] V. Berinde, *On the stability of some fixed point procedures*, Buletinul Stiintifical Universitatii Baia Mare, Seria B, Fascicola Matematica-Informatica, **18** (2002), 7-14.

[17] V. Berinde, *On the convergence of the Ishikawa iteration in the class of quasi-contractive operators*, Acta Math. Univ. Comenianae, **73** (2004), 119-126.

[18] B. E. Rhoades, *Fixed point theorems and stability results for fixed point iteration procedures I*, Indian J. Pure Appl. Math., **21** (1990), 1-9.

[19] B. E. Rhoades, *Fixed point theorems and stability results for fixed point iteration procedures II*, Indian J. Pure Appl. Math., **24** (1993), 691-703.

[20] S. L. Singh, C. S. Bhatanagar and N. Mishra, *Stability of Jungck-type iterative procedures*, Int. J. Math. and Math. Sci., **19** (2005), 3035-3043.

[21] W. R. Mann, *Mean value methods in iteration*, Proc. Amer. Math. Soc., **4** (1953), 506-510.

[22] M. O. Olatinwo and C. O. Imoru, *Some convergence result for the Jungck-Mann and Jungck-Ishikawa iteration process in the class of generalized Zamfirescu operators*, Acta Math. Univ. Comenianae, **77** (2008), 299-304.

[23] S. Ishikawa, *Fixed points by new iteration method*, Proc. Amer. Math. Soc., **44** (1974), 147-150.

[24] M. O. Olatinwo, *A Generalization of some convergence results using the Jungck-Noor three step iteration process in arbitrary Banach space*, Fasciculi Math., **40** (2008) 37-43.

[25] R. Chugh and V. Kumar, *Strong convergence and stability results for Jungck-SP iterative scheme*, Int. J. Computer Appl., **36** (2011), 40-46.

[26] M. A. Noor, *Three-step iterative algorithms for multivalued quasi variational inclusion*, J. Math. Anal. Appl., **255** (2001), 589-604.

[27] M. C. Joshi and R. K. Bose, *Some topics in nonlinear functional analysis*, Wiley Eastern Limited, New Delhi, (1985).

[28] L. N. Mishra, S. K. Tiwari, V. N. Mishra and I. A. Khan, *Unique fixed point theorems for generalized contractive mappings in partial metric spaces*, J. Function Spaces, **2015** (2015), 1-8.

[29] L. N. Mishra, S. K. Tiwari and V. N. Mishra, *Fixed point theorems for generalized weakly S-contractive mappings in partial metric spaces*, J. Appl. Anal. Comput., **5 (4)** (2015), 600-612.

[30] V. N. Mishra, *Some problems on approximations of functions in banach sapaces*, Ph.D. Thesis, Indian Institute of Technology, Roorkee 247667, Uttarakhand, India, (2007).

[31] V. N. Mishra and L. N. Mishra, *Trigonometric approximation of signals (Functions) in* $L_p(p \geq 1)$*-norm*, Int. J. Contemporary Math. Sci., **7 (19)** (2012) 909-918.

[32] Deepmala, *A study on fixed point theorems for nonlinear contractions and its applications*, Ph.D. Thesis, Pt. Ravishankar Shukla University, Raipur 492 010, Chhatisgarh, India, (2014).

[33] R. A. Rashwan and H. A. Hammad, *Random fixed point theorems with an application to a random nonlinear integral equation*, Journal of Linear and Topological Algebra, **5 (2)**, 119-133, (2016).